DHANALAKSHMI COLLEGE OF ENGINEERING

(Dr.VPR Nagar, Manimangalam, Tambaram)

Chennai - 601 301

DEPARTMENT OF MECHANICAL ENGINEERING

III YEAR MECHANICAL - V SEMESTER

GE6503 – DESIGN OF MACHINE ELEMENTS

ACADEMIC YEAR (2017 - 2018) - ODD SEMESTER

UNIT – 4 (STUDY NOTES)

Prepared by:

R. SENDIL KUMAR, AP/MECH

P. V. ELUMALAI, AP/MECH

P. SIVA KUMAR, AP/MECH

Unit 4 - DESIGN OF ENERY STORING ELEMENTS

PART – A

1. What is a spring?

A spring is an elastic member, which deflects, or distorts under the action of load and regains its original shape after the load is removed.

2. State any two functions of springs.

i. To measure forces in spring balance, meters and engine indicators.
ii. To store energy.

3. What are the various types of springs? **(MAY/JUNE 2012)**

i. Helical springs
ii. Spiral springs
iii. Leaf springs
iv. Disc spring or Belleville springs

4. Classify the helical springs.

a. Close – coiled or tension helical spring.
b. Open –coiled or compression helical spring.

5. Define: Leaf springs

A leaf spring consists of flat bars of varying lengths clamped together and Supported at both ends, thus acting as a simply supported beam.

6. Define: Belleville Springs

They are made in the form of a cone disc to carry a high compressive force. In order to improve their load carrying capacity, they may be stacked up together. The major stresses are tensile and compressive.

7. What is spring index (C)?

The ratio of mean or pitch diameter to the diameter of wire for the spring is called the spring index.

8. What is pitch?

The axial distance between adjacent coils in uncompressed state.

9. What is solid length?

The length of a spring under the maximum compression is called its solid length. It is the product of total number of coils and the diameter of wire.

Ls = nt x d

Where, nt = total number of coils.

10. What are the requirements of spring while designing?

a. Spring must carry the service load without the stress exceeding the safe value.
b. The spring rate must be satisfactory for the given application.

11 What are the end conditions of spring?

(a) Plain end. (b) Plain and ground end (c). Squared end (d) Squared and ground end.

12. What is buckling of springs?

The helical compression spring behaves like a column and buckles at a comparative small load when the length of the spring is more than 4 times the mean coil diameter.

13. What is surge in springs?

The material is subjected to higher stresses, which may cause early fatigue failure. This effect is called as spring surge.

14. What is a laminated leaf spring?

In order to increase, the load carrying capacity, number of flat plates are placed and below the other.

15. What semi – elliptical leaf springs?

The spring consists of number of leaves, which are held together by U-clips. The long leaf fastened to the supported is called master leaf. Remaining leaves are called as graduated leaves.

16. What is nipping of laminated leaf spring?

Prestressing of leaf springs is obtained by a difference of radii of curvature known as nipping.

17. What are the various applications of springs?

The springs are used in various applications, they are

a. Used to absorb energy or shocks (e.g. shock absorbers, buffers, e.t.c.)
b. To apply forces as in brakes clutches, spring-loaded valves, e.t.c.
c. To measure forces as in spring balances and engine indicators
d. To store energy as in watches

18. Define free length.

Free length of the spring is the length of the spring when it is free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression plus clash allowance.

$$L_f = \text{solid length} + Y_{max} + 0.15\, Y_{max}$$

19. Define spring index. (NOV/DEC 2011)

Spring index (C) is defined as the ratio of the mean diameter of the coil to the diameter of the wire.

$$C = D/d$$

20. Define spring rate (stiffness). (NOV/DEC 2011)

The spring stiffness or spring constant is defined as the load required per unit deflection of the spring.

$$K = W/y$$

Where W-load
Y-deflection

21. Define pitch.

Pitch of the spring is defined as the axial distance between the adjacent coils in uncompressed state. Mathematically

$$\text{Pitch} = \frac{\text{free length}}{n-1}$$

22. What are the points to be taken into consideration while selecting the pitch of the spring?

The points taken into consideration of selecting the pitch of the springs are (a)The pitch of the coil should be such that if the spring is accidentally compressed the stress does not increase the yield point stress in torsion.(b)The spring should not be close up before the maximum service load is reached.

23. Define active turns.

Active turns of the spring are defined as the number of turns, which impart spring action while loaded. As load increases the no of active coils decreases.

24. Define inactive turns.

An inactive turn of the spring is defined as the number of turns which does not contribute to the spring action while loaded. As load increases number of inactive coils increases from 0.5 to 1 turn.

25. What are the different kinds of end connections for compression helical springs?

The different kinds of end connection for compression helical springs are

a. Plain ends
b. Ground ends
c. Squared ends

d. Ground & square ends

26. Write about the eccentric loading of springs?

If the load acting on the spring does not coincide with the axis of the spring, then spring is said to be have eccentric load. In eccentric loading the safe load of the spring decreases and the stiffness of the spring is also affected.

27. Explain about surge in springs? (MAY/JUNE 2013)

When one end of the spring is resting on a rigid support and the other end is loaded suddenly, all the coils of spring does not deflect equally, because some time is required for the propagation of stress along the wire. Thus a wave of compression propagates to the fixed end from where it is reflected back to the deflected end this wave passes through the spring indefinitely. If the time interval between the load application and that of the wave to propagate are equal, then resonance will occur. This will result in very high stresses and cause failure. This phenomenon is called surge.

28. What are the methods used for eliminating surge in springs?

The methods used for eliminating surge are

a. By using dampers on the center coil so that the wave propagation dies out

b. By using springs having high natural frequency.

29. What are the disadvantages of using helical spring of non-circular wires?

a. The quality of the spring is not good

b. The shape of the wire does not remain constant while forming the helix. It reduces the energy absorbing capacity of the spring.

c. The stress distribution is not favorable as in circular wires. But this effect is negligible where loading is of static nature.

30. Why concentric springs are used?

a. To get greater spring force with in a given space

b. To insure the operation of a mechanism in the event of failure of one of the spring

31. What is the advantage of leaf spring over helical spring?

The advantage of leaf spring over helical spring is that the end of the spring may be guided along a definite path as it deflects to act a structural member in addition to energy absorbing device.

32. Write notes on the master leaf & graduated leaf?

The longest leaf of the spring is known as main leaf or master leaf has its ends in the form of an eye through which bolts are passed to secure the spring. The leaf of the spring other than master leaf is called the graduated leaves.

33. What is meant by nip in leaf springs?

By giving greater radius of curvature to the full length leaves than the graduated leaves, before the leaves are assembled to form a spring thus a gap or clearance will be left between the leaves. This initial gap is called nip.

34. What is the application of leaf spring?

The leaf springs are used in automobiles as shock absorbers for giving suspension to the automobile and it gives support to the structure.

35. Define flat spiral spring.

A flat spiral spring is a long thin strip of elastic material wound like a spiral. These springs are frequently used in watch springs, gramophones, e.t.c

36.What are the differences between helical torsion spring and tension helical Springs?

Helical torsion springs are wound similar to that of tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion spring is bending stress whereas in tension springs the stresses are torsional shear stresses.

37. Define helical springs.

The helical springs are made up of a wire coiled in the form of a helix and are primarily intended for compressive or tensile load.

38. What are the different types of helical springs?

The different types of helical springs are

a. Open coil helical spring

b. Closed coil helical spring

39. What are the differences between closed coil & open coil helical springs?

Closed coil helical spring	Open coil helical spring
The spring wires are coiled very closely, each turn is nearly at right angles to the axis of helix	The wires are coiled such that there is a gap between the two consecutive turns.
Helix angle is less than 10^{o}	Helix angle is large ($>10^{o}$)

40. What is the use of flywheel? (MAY/JUNE 2012)

Flywheel is used in machine serves as a reservoir which store energy during the period when the supply of energy is more than the requirement and release it during the period when the requirement of the energy is more than the supply.

41. Write the formula for natural frequency of spring. (NOV/DEC 2012)

$$f = \frac{d}{\pi n D^2}\sqrt{\frac{Gg}{8\rho}}$$ Where ρ- density of material.

42. How does the function of flywheel differ from that of governer? (NOV/DEC 2012) (NOV/DEC 2011)

A governor regulates the mean speed of an engine when there are variations in the mean loads. It automatically controls the supply of working fluid to engine with the varying load condition and keeps the mean speed within the limits. It does not control the speed variation caused by the varying load. A flywheel does not maintain constant speed.

43. Define Co-efficient of fluctuation of speed in flywheel. (MAY/JUNE 2013)

It is the ratio of the maximum change of speed to mean speed of the flywheel.

$$K_S = \frac{w_{max} - w_{min}}{w_{mean}}$$

PART – B

1.

A locomotive semi-elliptical laminated spring has an overall length of 1 m and sustains a load of 70 kN at its centre. The spring has 3 full length leaves and 15 graduated leaves with a central band of 100 mm width. All the leaves are to be stressed to 400 MPa, when fully loaded. The ratio of the total spring depth to that of width is 2. E = 210 kN/mm². Determine :

1. The thickness and width of the leaves.

2. The initial gap that should be provided between the full length and graduated leaves before the band load is applied.

3. The load exerted on the band after the spring is assembled. **(NOV/DEC 2011)**

Solution. Given : $2L_1$ = 1 m = 1000 mm ; $2W$ = 70 kN or W = 35 kN = 35 × 10^3 N ; n_F = 3 ; n_G = 15 ; l = 100 mm ; σ = 400 MPa = 400 N/mm² ; E = 210 kN/mm² = 210 × 10^3 N/mm²

1. *Thickness and width of leaves*

Let t = Thickness of leaves, and

b = Width of leaves.

We know that the total number of leaves,

$$n = n_F + n_G = 3 + 15 = 18$$

Since it is given that ratio of the total spring depth ($n \times t$) and width of leaves is 2, therefore

$$\frac{n \times t}{b} = 2 \quad \text{or} \quad b = n \times t / 2 = 18 \times t / 2 = 9\,t$$

We know that the effective length of the leaves,

$$2L = 2L_1 - l = 1000 - 100 = 900 \text{ mm} \quad \text{or} \quad L = 900 / 2 = 450 \text{ mm}$$

Since all the leaves are equally stressed, therefore final stress (σ),

$$400 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 35 \times 10^3 \times 450}{18 \times 9\,t \times t^2} = \frac{583 \times 10^3}{t^3}$$

$\therefore$ $t^3 = 583 \times 10^3 / 400 = 1458$ or $t = 11.34$ say 12 mm **Ans.**

and $b = 9\,t = 9 \times 12 = 108$ mm **Ans.**

2. *Initial gap*

We know that the initial gap (C) that should be provided between the full length and graduated leaves before the band load is applied, is given by

$$C = \frac{2\,W.L^3}{n.E.b.t^3} = \frac{2 \times 35 \times 10^3\,(450)^3}{18 \times 210 \times 10^3 \times 108\,(12)^3} = 9.04 \text{ mm} \quad \textbf{Ans.}$$

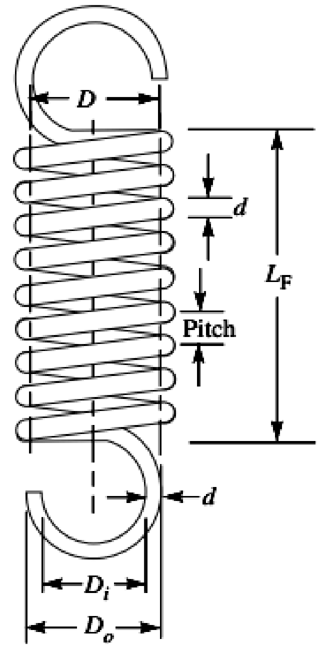
3. *Load exerted on the band after the spring is assembled*

We know that the load exerted on the band after the spring is assembled,

$$W_b = \frac{2\,n_F.n_G.W}{n(2n_G + 3n_F)} = \frac{2 \times 3 \times 15 \times 35 \times 10^3}{18\,(2 \times 15 + 3 \times 3)} = 4487 \text{ N} \quad \textbf{Ans.}$$

2. Design a helical spring for as spring loaded spring loaded safety value of the following conditions: Diameter of valve seat = 65 mm, Operating pressure = 0.7 N/mm², Maximum pressure when the valve blows freely = 0.75 N/mm², Maximum lift of the valve when the pressure = 3.5 mm rises from 0.7 to 0.75 N/mm², Maximum allowable stress = 550 MPa, Modulus of rigidity = 84 kN/mm², Spring index = 6, Draw a neat sketch of the free spring showing the main dimensions.

(MAY/JUNE 2012) (NOV/DEC 2012)



Solution. Given : D_1 = 65 mm ; p_1 = 0.7 N/mm² ; p_2 = 0.75 N/mm² ; δ = 3.5 mm ; τ = 550 MPa = 550 N/mm² ; G = 84 kN/mm² = 84 × 10³ N/mm² ; C = 6

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

Since the safety valve is a Ramsbottom safety valve, therefore the spring will be under tension. We know that initial tensile force acting on the spring (*i.e.* before the valve lifts),

$$W_1 = \frac{\pi}{4}(D_1)^2\, p_1 = \frac{\pi}{4}(65)^2\, 0.7 = 2323 \text{ N}$$

and maximum tensile force acting on the spring (*i.e.* when the valve blows off freely),

$$W_2 = \frac{\pi}{4}(D_1)^2\, p_2 = \frac{\pi}{4}(65)^2\, 0.75 = 2489 \text{ N}$$

∴ Force which produces the deflection of 3.5 mm,

$$W = W_2 - W_1 = 2489 - 2323 = 166 \text{ N}$$

Since the diameter of the spring wire is obtained for the maximum spring load (W_2), therefore maximum twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2489 \times \frac{6\,d}{2} = 7467\,d \qquad \text{... } (\because C = D/d = 6)$$

We know that maximum twisting moment (T),

$$7467\,d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 550 \times d^3 = 108\,d^3$$

$$\therefore \quad d^2 = 7467 / 108 = 69.14 \quad \text{or} \quad d = 8.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 2/0 having diameter (d) = 8.839 mm **Ans.**

∴ Mean diameter of the coil,

$$D = 6\,d = 6 \times 8.839 = 53.034 \text{ mm } \textbf{Ans.}$$

Outside diameter of the coil,

$$D_o = D + d = 53.034 + 8.839 = 61.873 \text{ mm } \textbf{Ans.}$$

and inside diameter of the coil,

$$D_i = D - d = 53.034 - 8.839 = 44.195 \text{ mm } \textbf{Ans.}$$

2. ***Number of turns of the coil***

Let n = Number of active turns of the coil.

We know that the deflection of the spring (δ),

$$3.5 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 166 \times 6^3 \times n}{84 \times 10^3 \times 8.839} = 0.386\,n$$

∴ $n = 3.5 / 0.386 = 9.06$ say 10 **Ans.**

For a spring having loop on both ends, the total number of turns,

$$n' = n + 1 = 10 + 1 = 11 \textbf{ Ans.}$$

3. ***Free length of the spring***

Taking the least gap between the adjacent coils as 1 mm when the spring is in free state, the free length of the tension spring,

$$L_F = n.d + (n - 1)\,1 = 10 \times 8.839 + (10 - 1)\,1 = 97.39 \text{ mm } \textbf{Ans.}$$

4. ***Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n - 1} = \frac{97.39}{10 - 1} = 10.82 \text{ mm } \textbf{Ans.}$$

3.

A safety valve of 60 mm diameter is to blow off at a pressure of 1.2 N/mm². It is held on its seat by a close coiled helical spring. The maximum lift of the valve is 10 mm. Design a suitable compression spring of spring index 5 and providing an initial compression of 35 mm. The maximum shear stress in the material of the wire is limited to 500 MPa. The modulus of rigidity for the spring material is 80 kN/mm². Calculate : 1. Diameter of the spring wire, 2. Mean coil diameter, 3. Number of active turns, and 4. Pitch of the coil. **(MAY/JUNE 2013)**

Solution. Given : Valve dia. = 60 mm ; Max. pressure = 1.2 N/mm² ; δ_2 = 10 mm ; C = 5 ; δ_1 = 35 mm ; τ = 500 MPa = 500 N/mm² ; G = 80 kN/mm² = 80 × 10³ N/mm²

1. ***Diameter of the spring wire***

Let d = Diameter of the spring wire.

We know that the maximum load acting on the valve when it just begins to blow off,

$$W_1 = \text{Area of the valve} \times \text{Max. pressure}$$

$$= \frac{\pi}{4}(60)^2\,1.2 = 3394 \text{ N}$$

and maximum compression of the spring,

$$\delta_{max} = \delta_1 + \delta_2 = 35 + 10 = 45 \text{ mm}$$

Since a load of 3394 N keeps the valve on its seat by providing initial compression of 35 mm, therefore the maximum load on the spring when the valve is oepn (*i.e.* for maximum compression of 45 mm),

2. *Mean coil diameter*

Let D = Mean coil diameter.

We know that the spring index,

$$C = D/d \quad \text{or} \quad D = C.d = 5 \times 12.7 = 63.5 \text{ mm } \textbf{Ans.}$$

3. *Number of active turns*

Let n = Number of active turns.

We know that the maximum compression of the spring (δ),

$$45 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 4364 \times 5^3 \times n}{80 \times 10^3 \times 12.7} = 4.3\,n$$

$$\therefore \quad n = 45 / 4.3 = 10.5 \text{ say } 11 \textbf{ Ans.}$$

Taking the ends of the coil as squared and ground, the total number of turns,

$$n' = n + 2 = 11 + 2 = 13 \textbf{ Ans.}$$

Note : The valve of n may also be calculated by using

$$\delta_1 = \frac{8\,W_1.C^3.n}{G.d}$$

$$35 = \frac{8 \times 3394 \times 5^3 \times n}{80 \times 10^3 \times 12.7} = 3.34\,n \quad \text{or} \quad n = 35 / 3.34 = 10.5 \text{ say } 11$$

$$W = \frac{3394}{35} \times 45 = 4364 \text{ N}$$

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

We also know that the maximum shear stress (τ),

$$500 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.31 \times \frac{8 \times 4364 \times 5}{\pi\,d^2} = \frac{72\ 780}{d^2}$$

$$\therefore \quad d^2 = 72\ 780 / 500 = 145.6 \quad \text{or} \quad d = 12.06 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 7/0 having diameter (d) = 12.7 mm. **Ans.**

4. *Pitch of the coil*

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max} = 13 \times 12.7 + 45 + 0.15 \times 45$$

$$= 216.85 \text{ mm } \textbf{Ans}$$

$$\therefore \quad \text{Pitch of the coil} = \frac{\text{Free length}}{n' - 1} = \frac{216.85}{13 - 1} = 18.1 \text{ mm } \textbf{Ans.}$$

4.

A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.

Solution. Given : d = 6 mm ; D_o = 75 mm ; τ = 350 MPa = 350 N/mm² ; G = 84 kN/mm² = 84 × 10^3 N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

∴ Spring index,

$$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. ***Neglecting the effect of curvature***

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\, d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

∴ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. ***Considering the effect of curvature***

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\, d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

∴ $W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$

5.

Example 23.6. *Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, $G = 84\ kN/mm^2$.*

Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Solution. Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420\ N/mm^2$; $G = 84\ kN/mm^2 = 84 \times 10^3\ N/mm^2$

1. ***Mean diameter of the spring coil***

Let D = Mean diameter of the spring coil for a maximum load of $W_2 = 2750$ N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\ d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment (T),

$$6875\ d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\ d^3$$

$\therefore$ $d^2 = 6875 / 82.48 = 83.35$ or $d = 9.13$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm } \textbf{Ans.}$$

2. ***Number of turns of the spring coil***

Let n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for $W = 500$ N) is 6 mm.

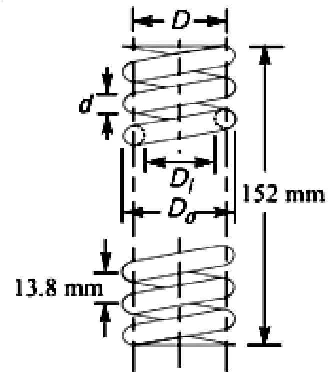
We know that the deflection of the spring (δ),

$$6 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 500\,(5)^3\,n}{84 \times 10^3 \times 9.49} = 0.63\,n$$

$\therefore \quad n = 6 / 0.63 = 9.5$ say 10 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \textbf{ Ans.}$$



3. *Free length of the spring*

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm} \textbf{ Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm} \textbf{ Ans.}$$

6.

Example 23.17. *A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 400 N to 1000 N. The spring index is 6 and the design factor of safety is 1.25. If the yield stress in shear is 770 MPa and endurance stress in shear is 350 MPa, find : 1. Size of the spring wire, 2. Diameters of the spring, 3. Number of turns of the spring, and 4. Free length of the spring.*

The compression of the spring at the maximum load is 30 mm. The modulus of rigidity for the spring material may be taken as 80 kN/mm².

Solution. Given : W_{min} = 400 N ; W_{max} = 1000 N ; C = 6 ; $F.S.$ = 1.25 ; τ_y = 770 MPa = 770 N/mm² ; τ_e = 350 MPa = 350 N/mm² ; δ = 30 mm ; G = 80 kN/mm² = 80×10^3 N/mm²

1. ***Size of the spring wire***

Let d = Diameter of the spring wire, and

D = Mean diameter of the spring = $C.d = 6\,d$ ... ($\because D/d = C = 6$)

We know that the mean load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{1000 + 400}{2} = 700 \text{ N}$$

and variable load,

$$W_v = \frac{W_{max} - W_{min}}{2} = \frac{1000 - 400}{2} = 300 \text{ N}$$

Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 6} = 1.083$$

Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

We know that mean shear stress,

$$\tau_m = K_S \times \frac{8\,W_m \times D}{\pi\,d^3} = 1.083 \times \frac{8 \times 700 \times 6\,d}{\pi\,d^3} = \frac{11\,582}{d^2} \text{ N/mm}^2$$

and variable shear stress,

$$\tau_v = K \times \frac{8\,W_v \times D}{\pi\,d^3} = 1.2525 \times \frac{8 \times 300 \times 6\,d}{\pi\,d^3} = \frac{5740}{d^2} \text{ N/mm}^2$$

We know that

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\,\tau_v}{\tau_e}$$

$$\frac{1}{1.25} = \frac{\frac{11\,582}{d^2} - \frac{5740}{d^2}}{770} + \frac{2 \times \frac{5740}{d^2}}{350} = \frac{7.6}{d^2} + \frac{32.8}{d^2} = \frac{40.4}{d^2}$$

$\therefore$ $d^2 = 1.25 \times 40.4 = 50.5$ or $d = 7.1$ mm **Ans.**

2. *Diameters of the spring*

We know that mean diameter of the spring,

$$D = C.d = 6 \times 7.1 = 42.6 \text{ mm } \textbf{Ans.}$$

Outer diameter of the spring,

$$D_o = D + d = 42.6 + 7.1 = 49.7 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring,

$$D_i = D - d = 42.6 - 7.1 = 35.5 \text{ mm } \textbf{Ans.}$$

3. *Number of turns of the spring*

Let n = Number of active turns of the spring.

We know that deflection of the spring (δ),

$$30 = \frac{8\, W.D^3.n}{G.d^4} = \frac{8 \times 1000\ (42.6)^3\ n}{80 \times 10^3\ (7.1)^4} = 3.04\ n$$

$\therefore$ $n = 30 / 3.04 = 9.87$ say 10 **Ans.**

Assuming the ends of the spring to be squared and ground, the total number of turns of the spring,

$$n' = n + 2 = 10 + 2 = 12 \textbf{ Ans.}$$

4. *Free length of the spring*

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\ \delta = 12 \times 7.1 + 30 + 0.15 \times 30 \text{ mm}$$

$$= 119.7 \text{ say } 120 \text{ mm } \textbf{Ans.}$$

7.

A concentric spring for an aircraft engine valve is to exert a maximum force of 5000 N under an axial deflection of 40 mm. Both the springs have same free length, same solid length and are subjected to equal maximum shear stress of 850 MPa. If the spring index for both the springs is 6, find (a) the load shared by each spring, (b) the main dimensions of both the springs, and (c) the number of active coils in each spring.

Assume G = 80 kN/mm² and diametral clearance to be equal to the difference between the wire diameters.

Solution. Given : $W = 5000$ N ; $\delta = 40$ mm ; $\tau_1 = \tau_2 = 850$ MPa = 850 N/mm² ; $C = 6$; $G = 80$ kN/mm² = 80×10^3 N/mm²

The concentric spring is shown in Fig. 23.22 (*a*).

(*a*) *Load shared by each spring*

Let W_1 and W_2 = Load shared by outer and inner spring respectively,

d_1 and d_2 = Diameter of spring wires for outer and inner springs respectively, and

D_1 and D_2 = Mean diameter of the outer and inner springs respectively.

* The net clearance between the two springs is given by

$$2\,c = (D_1 - D_2) - (d_1 + d_2)$$

Since the diametral clearance is equal to the difference between the wire diameters, therefore

$$(D_1 - D_2) - (d_1 + d_2) = d_1 - d_2$$

or $$D_1 - D_2 = 2\,d_1$$

We know that $D_1 = C.d_1$, and $D_2 = C.d_2$

$\therefore$ $$C.d_1 - C.d_2 = 2\,d_1$$

or $$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5 \qquad ...(i)$$

We also know that $$\frac{W_1}{W_2} = \left(\frac{d_1}{d_2}\right)^2 = (1.5)^2 = 2.25 \qquad ...(ii)$$

and $$W_1 + W_2 = W = 5000 \text{ N} \qquad ...(iii)$$

From equations (*ii*) and (*iii*), we find that

$$W_1 = 3462 \text{ N, and } W_2 = 1538 \text{ N } \textbf{Ans.}$$

(*b*) *Main dimensions of both the springs*

We know that Wahl's stress factor for both the springs,

$$K_1 = K_2 = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stress induced in the outer spring (τ_1),

$$850 = K_1 \times \frac{8\,W_1.C}{\pi(d_1)^2} = 1.2525 \times \frac{8\times 3462\times 6}{\pi(d_1)^2} = \frac{66\,243}{(d_1)^2}$$

$\therefore$ $$(d_1)^2 = 66\,243 / 850 = 78 \text{ or } d_1 = 8.83 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

and $$D_1 = C.d_1 = 6\,d_1 = 6\times 10 = 60 \text{ mm } \textbf{Ans.}$$

Similarly, maximum shear stress induced in the inner spring (τ_2),

$$850 = K_2 \times \frac{8W_2.C}{\pi(d_2)^2} = 1.2525 \times \frac{8\times 1538\times 6}{\pi(d_2)^2} = \frac{29\,428}{(d_2)^2}$$

$\therefore$ $$(d_2)^2 = 29\,428 / 850 = 34.6 \text{ or } {}^*d_2 = 5.88 \text{ say } 6 \text{ mm } \textbf{Ans.}$$

and $$D_2 = C.d_2 = 6\times 6 = 36 \text{ mm } \textbf{Ans.}$$

(*c*) *Number of active coils in each spring*

Let n_1 and n_2 = Number of active coils of the outer and inner spring respectively.

We know that the axial deflection for the outer spring (δ),

$$40 = \frac{8\,W_1.C^3.n_1}{G.d_1} = \frac{8\times 3462\times 6^3\times n_1}{80\times 10^3\times 10} = 7.48\,n_1$$

$\therefore$ $$n_1 = 40 / 7.48 = 5.35 \text{ say } 6 \textbf{ Ans.}$$

Assuming square and ground ends for the spring, the total number of turns of the outer spring,

$$n_1' = 6 + 2 = 8$$

$\therefore$ Solid length of the outer spring,

$$L_{S1} = n_1'.\,d_1 = 8\times 10 = 80 \text{ mm}$$

Let n_2' be the total number of turns of the inner spring. Since both the springs have the same solid length, therefore,

$$n_2'.d_2 = n_1'.d_1$$

or $n_2' = \frac{n_1'.d_1}{d_2} = \frac{8 \times 10}{6} = 13.3$ say 14

and $n_2 = 14 - 2 = 12$ **Ans.** ... ($\because n_2' = n_2 + 2$)

Since both the springs have the same free length, therefore

Free length of outer spring

= Free length of inner spring

$= L_{S1} + \delta + 0.15\,\delta = 80 + 40 + 0.15 \times 40 = 126$ mm **Ans.**

Other dimensions of the springs are as follows:

Outer diameter of the outer spring

$= D_1 + d_1 = 60 + 10 = 70$ mm **Ans.**

Inner diameter of the outer spring

$= D_1 - d_1 = 60 - 10 = 50$ mm **Ans.**

Outer diameter of the inner spring

$= D_2 + d_2 = 36 + 6 = 42$ mm **Ans.**

Inner diameter of the inner spring

$= D_2 - d_2 = 36 - 6 = 30$ mm **Ans.**

8.

Design a leaf spring for the following specifications :

Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves = 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm.

Take Young's modulus, E = 200 kN/mm² and allowable stress in spring material as 600 MPa.

Solution. Given : Total load = 140 kN ; No. of springs = 4; $n = 10$; $2L = 1000$ mm or $L = 500$ mm ; $\delta = 80$ mm ; $E = 200$ kN/mm² $= 200 \times 10^3$ N/mm² ; $\sigma = 600$ MPa $= 600$ N/mm²

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$$\therefore \quad W = 35 / 2 = 17.5 \text{ kN} = 17\,500 \text{ N}$$

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$$\therefore \quad n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3 \qquad ...(i)$$

and deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$$\therefore \quad n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6 \qquad ...(ii)$$

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$$b = 87.5 \text{ say } 90 \text{ mm } \textbf{Ans.}$$

9.

A semi-elliptical laminated vehicle spring to carry a load of 6000 N is to consist of seven leaves 65 mm wide, two of the leaves extending the full length of the spring. The spring is to be 1.1 m in length and attached to the axle by two U-bolts 80 mm apart. The bolts hold the central portion of the spring so rigidly that they may be considered equivalent to a band having a width equal to the distance between the bolts. Assume a design stress for spring material as 350 MPa. Determine :

1. Thickness of leaves, 2. Deflection of spring, 3. Diameter of eye, 4. Length of leaves, and 5. Radius to which leaves should be initially bent.

Sketch the semi-elliptical leaf-spring arrangement.

The standard thickness of leaves are : 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11 etc. in mm.

Solution. Given : $2W = 6000$ N or $W = 3000$ N ; $n = 7$; $b = 65$ mm ; $n_F = 2$; $2L_1 = 1.1$ m $= 1100$ mm or $L_1 = 550$ mm ; $l = 80$ mm ; $\sigma = 350$ MPa $= 350$ N/mm^2

1. *Thickness of leaves*

Let t = Thickness of leaves.

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1100 - 80 = 1020 \text{ mm}$$

$$\therefore \quad L = 1020 / 2 = 510 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 7 - 2 = 5$$

Assuming that the leaves are not initially stressed, the maximum stress (σ_F),

$$350 = \frac{18\,W.L}{b.t^2\,(2n_G + 3n_F)} = \frac{18 \times 3000 \times 510}{65 \times t^2\,(2 \times 5 + 3 \times 2)} = \frac{26\,480}{t^2} \quad \text{... } (\sigma_F = \sigma)$$

$$\therefore \quad t^2 = 26\,480 / 350 = 75.66 \quad \text{or} \quad t = 8.7 \text{ say } 9 \text{ mm } \textbf{Ans.}$$

2. *Deflection of spring*

We know that deflection of spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\,(2n_G + 3n_F)} = \frac{12 \times 3000\,(510)^3}{210 \times 10^3 \times 65 \times 9^3\,(2 \times 5 + 3 \times 2)}$$

$$= 30 \text{ mm } \textbf{Ans.} \quad \text{... (Taking } E = 210 \times 10^3 \text{ N/mm}^2\text{)}$$

3. *Diameter of eye*

The inner diameter of eye is obtained by considering the pin in the eye in bearing, because the inner diameter of the eye is equal to the diameter of the pin.

Let d = Inner diameter of the eye or diameter of the pin,

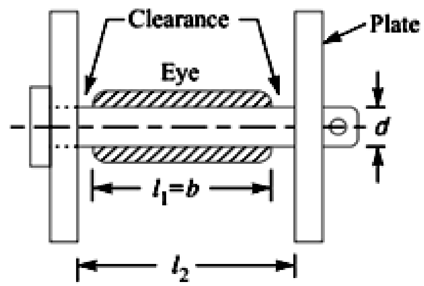
l_1 = Length of the pin which is equal to the width of the eye or leaf (*i.e.* b) = 65 mm ...(Given)

p_b = Bearing pressure on the pin which may be taken as 8 N/mm^2.

We know that the load on pin (W),

$$3000 = d \times l_1 \times p_b = d \times 65 \times 8 = 520\, d$$

$$\therefore \quad d = 3000 / 520 = 5.77 \text{ say } 6 \text{ mm}$$



Let us now consider the bending of the pin. Since there is a clearance of about 2 mm between the shackle (or plate) and eye as shown in Fig. 23.33, therefore length of the pin under bending,

$$l_2 = l_1 + 2 \times 2 = 65 + 4 = 69 \text{ mm}$$

Maximum bending moment on the pin,

$$M = \frac{W \times l_2}{4} = \frac{3000 \times 69}{4} = 51\,750 \text{ N-mm}$$

and section modulus, $$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{51\,750}{0.0982\, d^3} = \frac{527 \times 10^3}{d^3} \quad \text{... (Taking } \sigma_b = 80 \text{ N/mm}^2)$$

$$\therefore \quad d^3 = 527 \times 10^3 / 80 = 6587 \quad \text{or} \quad d = 18.7 \text{ say 20 mm } \textbf{Ans.}$$

We shall take the inner diameter of eye or diameter of pin (d) as 20 mm **Ans.**

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin (W),

$$3000 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2\ \tau = 628.4\ \tau$$

$$\therefore \quad \tau = 3000 / 628.4 = 4.77 \text{ N/mm}^2, \text{ which is safe.}$$

4. *Length of leaves*

We know that ineffective length of the spring

$= l = 80$ mm ... ($\because$ *U*-bolts are considered equivalent to a band)

$\therefore$ Length of the smallest leaf $= \dfrac{\text{Effective length}}{n-1} + \text{Ineffective length}$

$$= \frac{1020}{7-1} + 80 = 250 \text{ mm} \quad \textbf{Ans.}$$

Length of the 2nd leaf $= \dfrac{1020}{7-1} \times 2 + 80 = 420$ mm **Ans.**

Length of the 3rd leaf $= \dfrac{1020}{7-1} \times 3 + 80 = 590$ mm **Ans.**

Length of the 4th leaf $= \dfrac{1020}{7-1} \times 4 + 80 = 760$ mm **Ans.**

Length of the 5th leaf $= \dfrac{1020}{7-1} \times 5 + 80 = 930$ mm **Ans.**

Length of the 6th leaf $= \dfrac{1020}{7-1} \times 6 + 80 = 1100$ mm **Ans.**

The 6th and 7th leaves are full length leaves and the 7th leaf (*i.e.* the top leaf) will act as a master leaf.

We know that length of the master leaf

$$= 2L_1 + \pi (d + t) 2 = 1100 + \pi (20 + 9)2 = 1282.2 \text{ mm} \quad \textbf{Ans.}$$

5. *Radius to which the leaves should be initially bent*

Let R = Radius to which the leaves should be initially bent, and

y = Camber of the spring.

We know that

$$y(2R - y) = (L_1)^2$$

$$30(2R - 30) = (550)^2 \quad \text{or} \quad 2R - 30 = (550)^2/30 = 10\ 083 \qquad \ldots (\because y = \delta)$$

$$\therefore \quad R = \frac{10\ 083 + 30}{2} = 5056.5 \text{ mm} \quad \textbf{Ans.}$$

10.

A multi-cylinder engine is to run at a constant load at a speed of 600 r.p.m. On drawing the crank effort diagram to a scale of 1 m = 250 N-m and 1 mm = 3°, the areas in sq mm above and below the mean torque line are as follows:

+ 160, – 172, + 168, – 191, + 197, – 162 sq mm

The speed is to be kept within ± 1% of the mean speed of the engine. Calculate the necessary moment of inertia of the flywheel.

Determine suitable dimensions for cast iron flywheel with a rim whose breadth is twice its radial thickness. The density of cast iron is 7250 kg / m³, and its working stress in tension is 6 MPa. Assume that the rim contributes 92% of the flywheel effect.

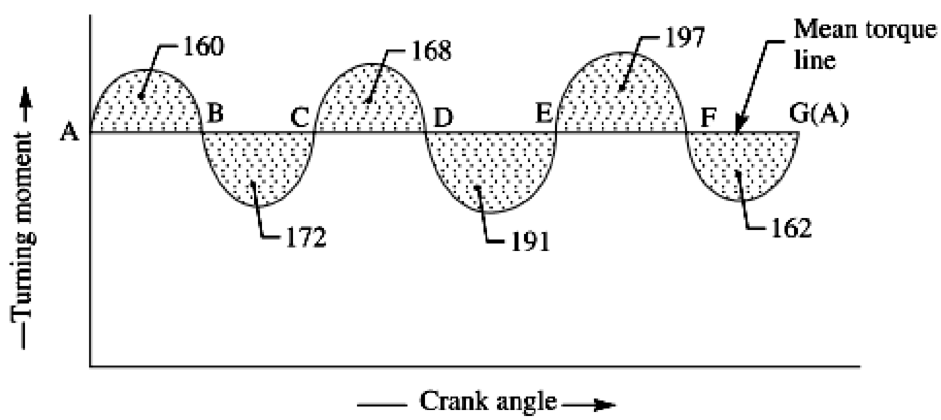
(NOV/DEC 2011)

Solution. Given : = N = 600 r.p.m. or
$\omega = 2\pi \times 600 / 60 = 62.84$ rad / s ; $\rho = 7250$ kg / m^3 ; $\sigma_t = 6$ MPa $= 6 \times 10^6$ N/m^2

Moment of inertia of the flywheel

Let I = Moment of inertia of the flywheel.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig.



Since the scale for the turning moment is 1 mm = 250 N-m and the scale for the crank angle is

1 mm = 3° = $\frac{\pi}{60}$ rad, therefore

1 mm^2 on the turning moment diagram

$$= 250 \times \frac{\pi}{60} = 13.1 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 22.12, we find that

$$\text{Energy at } B = E + 160$$

$$\text{Energy at } C = E + 160 - 172 = E - 12$$

$$\text{Energy at } D = E - 12 + 168 = E + 156$$

$$\text{Energy at } E = E + 156 - 191 = E - 35$$

$$\text{Energy at } F = E - 35 + 197 = E + 162$$

$$\text{Energy at } G = E + 162 - 162 = E = \text{Energy at } A$$

From above, we find that the energy is maximum at F and minimum at E.

$\therefore$ Maximum energy $= E + 162$

and minimum energy $= E - 35$

We know that the maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 162) - (E - 35) = 197 \text{ mm}^2 = 197 \times 13.1 = 2581 \text{ N-m}$$

Since the fluctuation of speed is ± 1% of the mean speed (ω), therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 2\% \, \omega = 0.02 \, \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.02$$

We know that the maximum fluctuation of energy (ΔE),

$$2581 = I.\omega^2.C_S = I \, (62.84)^2 \, 0.02 = 79 \, I$$

$\therefore$ $I = 2581 / 79 = 32.7 \text{ kg-m}^2$ **Ans.**

Dimensions of a flywheel rim

Let t = Thickness of the flywheel rim in metres, and

b = Breadth of the flywheel rim in metres = $2\,t$ ...(Given)

First of all let us find the peripheral velocity (v) and mean diameter (D) of the flywheel.

We know that tensile stress (σ_t),

$$6 \times 10^6 = \rho.v^2 = 7250 \times v^2$$

$\therefore$ $v^2 = 6 \times 10^6 / 7250 = 827.6$ or $v = 28.76$ m/s

We also know that peripheral velocity (v),

$$28.76 = \frac{\pi D . N}{60} = \frac{\pi D \times 600}{60} = 31.42 D$$

$\therefore \quad D = 28.76 / 31.42 = 0.915 \text{ m} = 915 \text{ mm}$ **Ans.**

Now let us find the mass of the flywheel rim. Since the rim contributes 92% of the flywheel effect, therefore the energy of the flywheel rim (E_{rim}) will be 0.92 times the total energy of the flywheel (E). We know that maximum fluctuation of energy (ΔE),

$$2581 = E \times 2 C_S = E \times 2 \times 0.02 = 0.04 E$$

$\therefore \quad E = 2581 / 0.04 = 64\ 525 \text{ N-m}$

and energy of the flywheel rim,

$$E_{rim} = 0.92 E = 0.92 \times 64\ 525 = 59\ 363 \text{ N-m}$$

Let m = Mass of the flywheel rim.

We know that energy of the flywheel rim (E_{rim}),

$$59\ 363 = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times m\ (28.76)^2 = 413.6\ m$$

$\therefore \quad m = 59\ 363 / 413.6 = 143.5 \text{ kg}$

We also know that mass of the flywheel rim (m),

$$143.5 = b \times t \times \pi D \times \rho = 2\ t \times t \times \pi \times 0.915 \times 7250 = 41\ 686\ t^2$$

$\therefore \quad t^2 = 143.5 / 41\ 686 = 0.003\ 44$

or $\quad t = 0.0587$ say $0.06 \text{ m} = 60 \text{ mm}$ **Ans.**

and $\quad b = 2\ t = 2 \times 60 = 120 \text{ mm}$ **Ans.**

11. An engine runs at a constant load at a speed of 480 rpm. The crank effort diagram is drawn to a scale 1 mm = 200 N-m torque and 1 mm = 3.6^o crank angle. The areas of the diagram above and below the mean torque line in sq. mm are in the following order: +110, -132, +153, _166, +197,-162. Design the flywheel if the total fluctuation of speed is not to exceed 10 rpm and the centrifugal stress in the rim is not to exceed 5 MPa. Assume that the rim breadth is approximately 2.5 times the rim thickness and 90% of the moment of inertial is due to rim. The density of the material of the flywheel is 7250 kg/m^3. Make a sketch of the flywheel giving the dimensions of the rim, the mean diameter of the rim and other estimated dimensions of spoke, hub etc., (May/June 2012)

Solution. Given : $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s ; $\sigma_t = 5.6$ MPa $= 5.6 \times 10^6 \text{ N/m}^2$; $\rho = 7200 \text{ kg/m}^3$

Diameter of the flywheel

Let D = Diameter of the flywheel in metres.

We know that peripheral velocity of the flywheel,

$$v = \frac{\pi D . N}{60} = \frac{\pi D \times 300}{60} = 15.71 D \text{ m/s}$$

We also know that hoop stress (σ_t),

$$5.6 \times 10^6 = \rho \times v^2 = 7200\ (15.71\ D)^2 = 1.8 \times 10^6\ D^2$$

$\therefore \quad D^2 = 5.6 \times 10^6 / 1.8 \times 10^6 = 3.11$ or $D = 1.764$ m **Ans.**

Cross-section of the flywheel

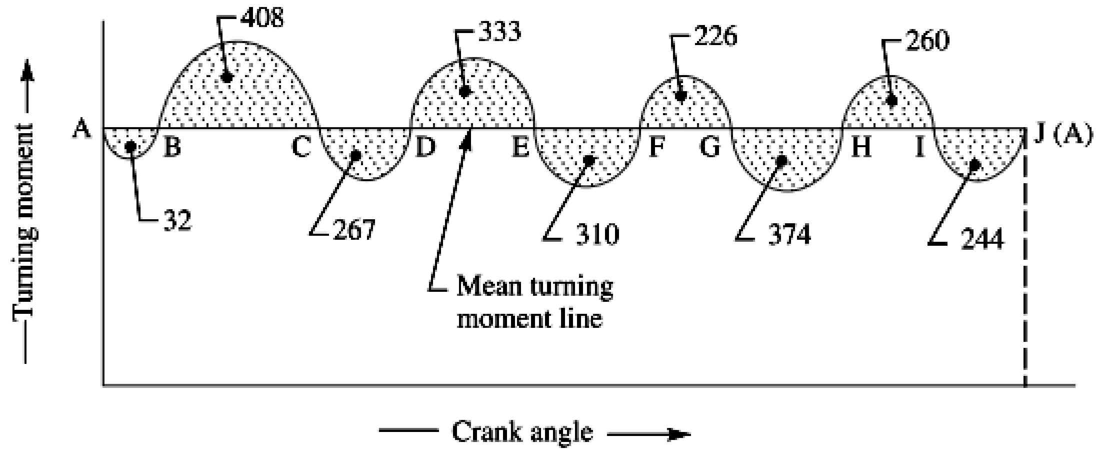
Let t = Thickness of the flywheel rim in metres, and

b = Width of the flywheel rim in metres = $4\,t$ ...(Given)

∴ Cross-sectional area of the rim,

$$A = b \times t = 4\,t \times t = 4\,t^2 \text{ m}^2$$

Now let us find the maximum fluctuation of energy. The turning moment diagram for one revolution of a multi-cylinder engine is shown in Fig



Since the scale of crank angle is 1 mm = 2.4° = $2.4 \times \frac{\pi}{180}$ = 0.042 rad, and the scale of the turning moment is 1 mm = 650 N-m, therefore

1 mm² on the turning moment diagram

$$= 650 \times 0.042 = 27.3 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 22.13, we find that

Energy at $B = E - 32$

Energy at $C = E - 32 + 408 = E + 376$

Energy at $D = E + 376 - 267 = E + 109$

Energy at $E = E + 109 + 333 = E + 442$

Energy at $F = E + 442 - 310 = E + 132$

Energy at $G = E + 132 + 226 = E + 358$

Energy at $H = E + 358 - 374 = E - 16$

Energy at $I = E - 16 + 260 = E + 244$

Energy at $J = E + 244 - 244 = E$ = Energy at A

From above, we see that the energy is maximum at E and minimum at B.

$\therefore$ Maximum energy $= E + 442$

and minimum energy $= E - 32$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 442) - (E - 32) = 474 \text{ mm}^2$$

$$= 474 \times 27.3 = 12\,940 \text{ N-m}$$

Since the fluctuation of speed is ± 1.5% of the mean speed, therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 3\% \text{ of mean speed} = 0.03\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.03$$

Let m = Mass of the flywheel rim.

We know that maximum fluctuation of energy (ΔE),

$$12\,940 = m.R^2.\omega^2.C_S = m\left(\frac{1.764}{2}\right)^2 (31.42)^2\ 0.03 = 23\ m$$

$\therefore$ $m = 12\,940 / 23 = 563$ kg **Ans.**

We also know that mass of the flywheel rim (m),

$$563 = A \times \pi D \times \rho = 4\,t^2 \times \pi \times 1.764 \times 7200 = 159\,624\ t^2$$

$\therefore$ $t^2 = 563 / 159\,624 = 0.00353$

or $t = 0.0594$ m $= 59.4$ say 60 mm **Ans.**

and $b = 4\,t = 4 \times 60 = 240$ mm **Ans.**

12. A machine punching 35 mm holes in 32 mm thick plate requires a7 N m of energy per sq. mm of sheared area and punches one hole in every 10 seconds. Calculate the power on the motor required. The mean speed of the flywheel is 25 m/sec. the punch has a stroke of 100 mm. find the mass of the flywheel required if the total fluctuation of speed is not to exceed 3% of the mean speed. Assume that the motor supplies the energy to the machine at uniform rate. (MAY/JUNE 2013)

Solution. Given : No. of holes = 35 per min ; Energy per hole = 10 kN-m = 10 000 N-m ; d = 800 mm = 0.8 m ; N = 210 r.p.m. ; h = 80% = 0.8 ; $1/C_S = 5$ or $C_S = 1/5 = 0.2$; D_{max} = 1.3 m ; τ = 50 MPa = 50 N/mm² ; σ_t = 4 MPa = 4 N/mm² ; ρ = 7200 kg / m³

Power of the electric motor

We know that energy used for piercing holes per minute

= No. of holes pierced × Energy used per hole

= 35 × 10 000 = 350 000 N-m / min

∴ Power needed for the electric motor,

$$P = \frac{\text{Energy used per minute}}{60 \times \eta} = \frac{350\ 000}{60 \times 0.8} = 7292 \text{ W} = 7.292 \text{ kW}$$ **Ans.**

Design of cast iron flywheel

First of all, let us find the maximum fluctuation of energy.

Since the overall efficiency of the transmission unit is 80%, therefore total energy to be supplied during each revolution,

$$E_T = \frac{10\ 000}{0.8} = 12\ 500 \text{ N-m}$$

We know that velocity of the belt,

$$v = \pi\, d.N = \pi \times 0.8 \times 210 = 528 \text{ m/min}$$

∴ Net tension or pull acting on the belt

$$= \frac{P \times 60}{v} = \frac{7292 \times 60}{528} = 828.6 \text{ N}$$

Since each piercing takes 40 per cent of the time needed to make one revolution, therefore time required to punch a hole

= 0.4 / 35 = 0.0114 min

and the distance moved by the belt during punching a hole

= Velocity of the belt × Time required to punch a hole

= 528 × 0.0114 = 6.03 m

∴ Energy supplied by the belt during punching a hole,

E_B = Net tension × Distance travelled by belt

= 828.6 × 6.03 = 4996 N-m

Thus energy to be supplied by the flywheel for punching during each revolution or maximum fluctuation of energy,

$$\Delta E = E_T - E_B = 12\ 500 - 4996 = 7504 \text{ N-m}$$

1. *Mass of the flywheel*

Let m = Mass of the flywheel rim.

Since space considerations limit the maximum diameter of the flywheel as 1.3 m ; therefore let us take the mean diameter of the flywheel,

D = 1.2 m or R = 0.6 m

We know that angular velocity

$$\omega = \frac{2\,\pi \times N}{60} = \frac{2\,\pi \times 210}{60} = 22 \text{ rad / s}$$

We also know that the maximum fluctuation of energy (ΔE),

$$7504 = m.R^2.\omega^2.C_S = m\,(0.6)^2\,(22)^2\,0.2 = 34.85\,m$$

∴ m = 7504 / 34.85 = 215.3 kg **Ans.**

13.

Design and draw a cast iron flywheel used for a four stroke I.C engine developing 180 kW at 240 r.p.m. The hoop or centrifugal stress developed in the flywheel is 5.2 MPa, the total fluctuation of speed is to be limited to 3% of the mean speed. The work done during the power stroke is 1/3 more than the average work done during the whole cycle. The maximum torque on the shaft is twice the mean torque. The density of cast iron is 7220 kg/m³.

(NOV/DEC 2012)

Solution. Given: $P = 180$ kW $= 180 \times 10^3$ W;
$N = 240$ r.p.m. ; $\sigma_t = 5.2$ MPa $= 5.2 \times 10^6$ N/m^2 ;
$N_1 - N_2 = 3\%\, N$; $\rho = 7220$ kg/m^3

First of all, let us find the maximum fluctuation of energy (ΔE). The turning moment diagram of a four stroke engine is shown in Fig. 22.18.

We know that mean torque transmitted by the flywheel,

$$T_{mean} = \frac{P \times 60}{2\,\pi\,N} = \frac{180 \times 10^3 \times 60}{2\,\pi \times 240} = 7161 \text{ N-m}$$

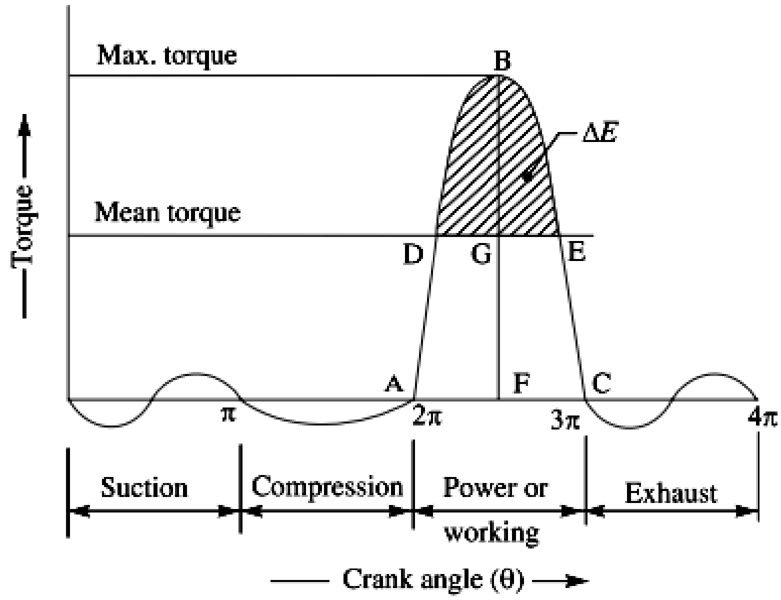
and *workdone per cycle $= T_{mean} \times \theta = 7161 \times 4\,\pi = 90\,000$ N-m

Since the workdone during the power stroke is 1/3 more than the average workdone during the whole cycle, therefore,

Workdone during the power (or working) stroke

$$= 90\,000 + \frac{1}{3} \times 90\,000 = 120\,000 \text{ N-m} \qquad ...(i)$$

The workdone during the power stroke is shown by a triangle *ABC* in Fig. 22.18 in which the base $AC = \pi$ radians and height $BF = T_{max.}$



∴ Workdone during power stroke

$$= \frac{1}{2} \times \pi \times T_{max} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{1}{2} \times \pi \times T_{max} = 120\,000$$

$$\therefore \quad T_{max} = \frac{120\,000 \times 2}{\pi} = 76\,384 \text{ N-m}$$

Height above the mean torque line,

$$BG = BF - FG = T_{max} - T_{mean} = 76\,384 - 7161 = 69\,223 \text{ N-m}$$

Since the area *BDE* shown shaded in Fig. 22.18 above the mean torque line represents the maximum fluctuation of energy (Δ *E*), therefore from geometrical relation,

$$= \frac{\text{Area of } \Delta\ BDE}{\text{Area of } \Delta\ ABC} = \frac{(BG)^2}{(BF)^2}, \text{ we have}$$

*Maximum fluctuation of energy (*i.e.* area of Δ *BDE*),

$$\Delta E = \text{Area of } \Delta\, ABC \times \left(\frac{BG}{BF}\right)^2 = 120\,000 \left(\frac{69\,223}{76\,384}\right)^2 = 98\,555 \text{ N-m}$$

1. *Diameter of the flywheel rim*

Let D = Diameter of the flywheel rim in metres, and

v = Peripheral velocity of the flywheel rim in m/s.

We know that the hoop stress developed in the flywheel rim (σ_t),

$$5.2 \times 10^6 = \rho.v^2 = 7220 \times v^2$$

$$\therefore \quad v^2 = 5.2 \times 10^6 / 7220 = 720 \quad \text{or} \quad v = 26.8 \text{ m/s}$$

We also know that peripheral velocity (v),

$$26.8 = \frac{\pi D.N}{60} = \frac{2\pi \times 250}{60} = 13.1\, D$$

$$\therefore \quad D = 26.8 / 13.1 = 2.04 \text{ m } \textbf{Ans.}$$

2. *Mass of the flywheel rim*

Let m = Mass of the flywheel rim in kg.

We know that angular speed of the flywheel rim,

$$\omega = \frac{2\,\pi\,N}{60} = \frac{\pi\,D \times 250}{60} = 25.14 \text{ rad / s}$$

and coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = 0.03$$

We know that maximum fluctuation of energy (ΔE),

$$98\,555 = m.R^2.\omega^2.C_S = m\left(\frac{2.04}{2}\right)^2 (25.14)^2\, 0.03 = 19.73\, m$$

$$\therefore \quad m = 98\,555 / 19.73 = 4995 \text{ kg } \textbf{Ans.}$$

3. *Cross-sectional dimensions of the rim*

Let t = Depth or thickness of the rim in metres, and

b = Width of the rim in metres = $2\,t$ ...(Assume)

$\therefore$ Cross-sectional area of the rim,

$$A = b.t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$4995 = A \times \pi D \times \rho = 2\,t^2 \times \pi \times 2.04 \times 7220 = 92\,556\,t^2$$

$$\therefore \quad t^2 = 4995 / 92\,556 = 0.054 \quad \text{or} \quad t = 0.232 \text{ say } 0.235 \text{ m} = 235 \text{ mm } \textbf{Ans.}$$

and $$b = 2\,t = 2 \times 235 = 470 \text{ mm } \textbf{Ans.}$$

4. *Diameter and length of hub*

Let d = Diameter of the hub,

d_1 = Diameter of the shaft, and

l = Length of the hub.

Since the maximum torque on the shaft is twice the mean torque, therefore maximum torque acting on the shaft,

$$T_{max} = 2 \times T_{mean} = 2 \times 7161 = 14\ 322 \text{ N-m} = 14\ 322 \times 10^3 \text{ N-mm}$$

We know that the maximum torque acting on the shaft (T_{max}),

$$14\ 322 \times 10^3 = \frac{\pi}{16} \times \tau (d_1)^3 = \frac{\pi}{16} \times 40 (d_1)^3 = 7.855 (d_1)^3$$

...(Taking $\tau = 40$ MPa $= 40$ N/mm²)

$\therefore \quad (d_1)^3 = 14\ 322 \times 10^3 / 7.855 = 1823 \times 10^3$

or $\quad d_1 = 122$ say 125 mm **Ans.**

The diameter of the hub is made equal to twice the diameter of shaft and length of hub is equal to width of the rim.

$\therefore \quad d = 2\, d_1 = 2 \times 125 = 250 \text{ mm} = 0.25 \text{ m}$

and $\quad l = b = 470 \text{ mm} = 0.47$ mm **Ans.**

5. *Cross-sectional dimensions of the elliptical arms*

Let $\quad a_1$ = Major axis,

b_1 = Minor axis $= 0.5\, a_1$ ...(Assume)

n = Number of arms = 6 ...(Assume)

σ_b = Bending stress for the material of arms = 15 MPa = 15 N/mm² ...(Assume)

We know that the maximum bending moment in the arm at the hub end, which is assumed as cantilever is given by

$$M = \frac{T}{R.n} (R - r) = \frac{T}{D\,.\,n} (D - d) = \frac{14\ 322}{2.04 \times 6} (2.04 - 0.25) \text{ N-m}$$

$$= 2094.5 \text{ N-m} = 2094.5 \times 10^3 \text{ N-mm}$$

and section modulus for the cross-section of the arm,

$$Z = \frac{\pi}{32} \times b_1 (a_1)^2 = \frac{\pi}{32} \times 0.5\, a_1 (a_1)^2 = 0.05 (a_1)^3$$

We know that the bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{2094.5 \times 10^3}{0.05 (a_1)^3} = \frac{41\ 890 \times 10^3}{(a_1)^3}$$

$\therefore \quad (a_1)^3 = 41\ 890 \times 10^3 / 15 = 2793 \times 10^3$ or $a_1 = 140$ mm **Ans.**

and $\quad b_1 = 0.5\, a_1 = 0.5 \times 140 = 70$ mm **Ans.**

6. *Dimensions of key*

The standard dimensions of rectangular sunk key for a shaft of diameter 125 mm are as follows:

Width of key, $\quad w = 36$ mm **Ans.**

and thickness of key $\quad = 20$ mm **Ans.**

The length of key (L) is obtained by considering the failure of key in shearing.

We know that the maximum torque transmitted by the shaft (T_{max}),

$$14\,322 \times 10^3 = L \times w \times \tau \times \frac{a_1}{2} = L \times 36 \times 40 \times \frac{125}{2} = 90 \times 10^3\, L$$

$\therefore$ $$L = 14\,322 \times 10^3 / 90 \times 10^3 = 159 \text{ say } 160 \text{ mm } \textbf{Ans.}$$

Let us now check the total stress in the rim which should not be greater than 15 MPa. We know that total stress in the rim,

$$\sigma = \rho.v^2 \left(0.75 + \frac{4.935\ R}{n^2\ .\ t}\right)$$

$$= 7220\ (26.8)^2 \left[0.75 + \frac{4.935\ (2.04\ /\ 2)}{6^2 \times 0.235}\right] \text{ N/m}^2$$

$$= 5.18 \times 10^6\ (0.75 + 0.595) = 6.97 \times 10^6 \text{ N/m}^2 = 6.97 \text{ MPa}$$

Since it is less than 15 MPa, therefore the design is safe.

14.

A punching press pierces 35 holes per minute in a plate using 10 kN-m of energy per hole during each revolution. Each piercing takes 40 per cent of the time needed to make one revolution. The punch receives power through a gear reduction unit which in turn is fed by a motor driven belt pulley 800 mm diameter and turning at 210 r.p.m. Find the power of the electric motor if overall efficiency of the transmission unit is 80 per cent. Design a cast iron flywheel to be used with the punching machine for a coefficient of steadiness of 5, if the space considerations limit the maximum diameter to 1.3 m.

Allowable shear stress in the shaft material = 50 MPa

Allowable tensile stress for cast iron = 4 MPa

Density of cast iron = 7200 kg / m^3

Solution. Given : No. of holes = 35 per min ; Energy per hole = 10 kN-m = 10 000 N-m ; d = 800 mm = 0.8 m ; N = 210 r.p.m. ; η = 80% = 0.8 ; $1/C_S$ = 5 or C_S = 1/5 = 0.2 ; D_{max} = 1.3 m ; τ = 50 MPa = 50 N/mm^2 ; σ_t = 4 MPa = 4 N/mm^2 ; ρ = 7200 kg / m^3

Power of the electric motor

We know that energy used for piercing holes per minute

= No. of holes pierced × Energy used per hole

= 35 × 10 000 = 350 000 N-m / min

$\therefore$ Power needed for the electric motor,

$$P = \frac{\text{Energy used per minute}}{60 \times \eta} = \frac{350\,000}{60 \times 0.8} = 7292 \text{ W} = 7.292 \text{ kW } \textbf{Ans.}$$

Design of cast iron flywheel

First of all, let us find the maximum fluctuation of energy.

Since the overall efficiency of the transmission unit is 80%, therefore total energy to be supplied during each revolution,

$$E_T = \frac{10\,000}{0.8} = 12\,500 \text{ N-m}$$

We know that velocity of the belt,

$$v = \pi\, d.N = \pi \times 0.8 \times 210 = 528 \text{ m/min}$$

∴ Net tension or pull acting on the belt

$$= \frac{P \times 60}{v} = \frac{7292 \times 60}{528} = 828.6 \text{ N}$$

Since each piercing takes 40 per cent of the time needed to make one revolution, therefore time required to punch a hole

$$= 0.4 / 35 = 0.0114 \text{ min}$$

and the distance moved by the belt during punching a hole

= Velocity of the belt × Time required to punch a hole

$$= 528 \times 0.0114 = 6.03 \text{ m}$$