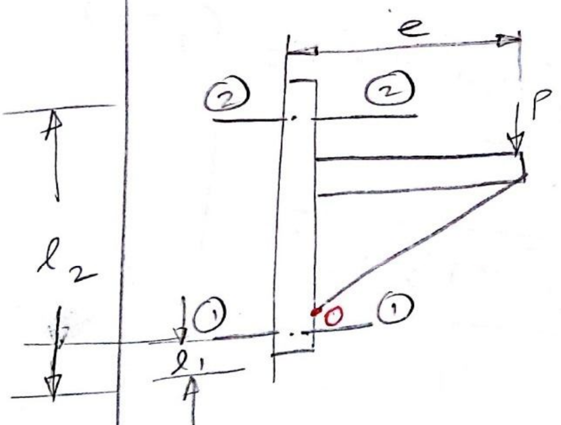
Eccentrically Loaded bolted joints.

(i) Loading in a plane different from the plane of bolts:



P - External force

O - Fulcrum about which the bracket may tilt

1-1, 2-2 - Rows of bolts

e - Eccentricity

l_1 - distance between first row of bolts and the fulcrum.

l_2 - distance between

n_1 - Number of bolts in first row.

n_2 - number of bolts in second row.

q - stiffness of bolts.

F_1 - Force induced in first row bolts $\} = n_1 \times q \times l_1$

F_2 - Force induced in second row bolts $\} = n_2 \times q \times l_2$

Taking moments about o,

$$P \times e = F_1 l_1 + F_2 l_2$$
$$= n_1 q l_1 \times l_1 + n_2 q l_2 \times l_2$$
$$= n_1 q l_1^2 + n_2 q l_2^2$$

$$P \times e = q (n_1 l_1^2 + n_2 l_2^2)$$

$$\boxed{q = \frac{P \times e}{n_1 l_1^2 + n_2 l_2^2}}$$

$$q = \frac{F_1}{l_1}$$

$$F_1 = q \times l_1 = \frac{P \times e \times l_1}{n_1 l_1^2 + n_2 l_2^2}$$

$$F_2 = \frac{P \times e \times l_2}{n_1 l_1^2 + n_2 l_2^2}$$

Note:-

Bolts in the second row are put into higher stress, because $l_2 > l_1$

$$\sigma_1 = \frac{F_1}{A_s} \qquad \sigma_2 = \frac{F_2}{A_s}$$

shear stress induced in the bolt,

$$\tau = \frac{P}{(n_1+n_2) A_s}$$

$$\tau_{max} = \frac{1}{2} \sqrt{\sigma^2 + 4\tau^2}$$

For designing $\tau_{max} = \frac{\sigma_y}{\cancel{A}(or)\ FOS}$

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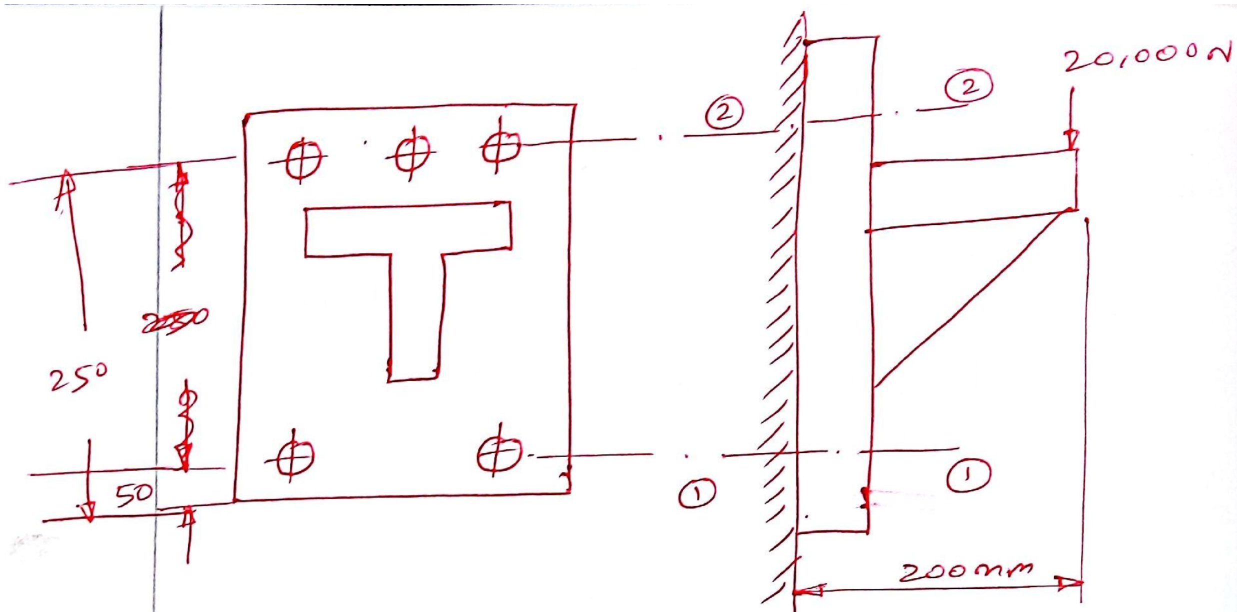
1. A bracket as shown in figure. It's fitted to a wall with 5 bolts, three at the top and two at the bottom, with all the bolts equally spaced. A load of 20,000 N is acting at an eccentricity of 200mm vertical distances of first and second rows from the hinge point are 50mm and 250mm respectively. select a suitable bolt size for this application.



$n_1 = 2$ $\quad l_1 = 50mm$ $\quad e = 200mm$

$n_2 = 3$ $\quad l_2 = 250mm$ $\quad P = 20,000N$

To find:-

$d = ?$

Solution:-

Force on first row of bolts

$$F_1 = \frac{P \times e \times l_1}{n_1 \times l_1^2 + n_2 l_2^2}$$

$$F_1 = \frac{20000 \times 200 \times 50}{2 \times (50)^2 + (3 \times 250^2)}$$

$$\boxed{F_1 = 1038.96\ N}$$

Force on second row of bolts

$$F_2 = \frac{P \times e \times l_2}{n_1 \times l_1^2 + n_2 \times l_2^2}$$

$$F_2 = \frac{20000 \times 200 \times 250}{(2 \times 50^2) + (3 \times 250^2)}$$

$$\boxed{F_2 = 5194.8 N}$$

$$F_2 > F_1$$

$$\sigma = \frac{F_2}{A_s} = \frac{5194.8}{A_s}$$

$$\tau = \frac{P}{(n_1 + n_2) A_s} = \frac{20,000}{5 \times A_s} = \frac{4000}{A_s}$$

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{5194.8}{A_s}\right)^2 + 4\left(\frac{4000}{A_s}\right)^2}$$

$$= \frac{1}{2}\sqrt{\frac{26.98 \times 10^6}{A_s^2} + \frac{64 \times 10^6}{A_s^2}}$$

$$= \frac{1}{2} \times \frac{9538.34}{A_s}$$

$$\frac{9538.65}{A_s}$$

4769.33.

$$\tau_{max} = \frac{4769.17}{A_s} \; N/mm^2$$

Assume $\sigma_y = 300 \, N/mm^2$, $FOS = 3$

$$\tau_{max} = \frac{\sigma_y}{FOS} = \frac{300}{3} = 100 \, N/mm^2$$

$$\boxed{\tau_{max} = 100 \, N/mm^2}$$

$$A_s = \frac{4769.17}{100} = 47.69 \; mm^2$$

$$A_s = \pi/4 \times d_c^2$$

$$d_c = \sqrt{\frac{A_s \times 4}{\pi}}$$

$$= \sqrt{\frac{47.69 \times 4}{\pi}}$$

$d_c = 7.79$ mm.

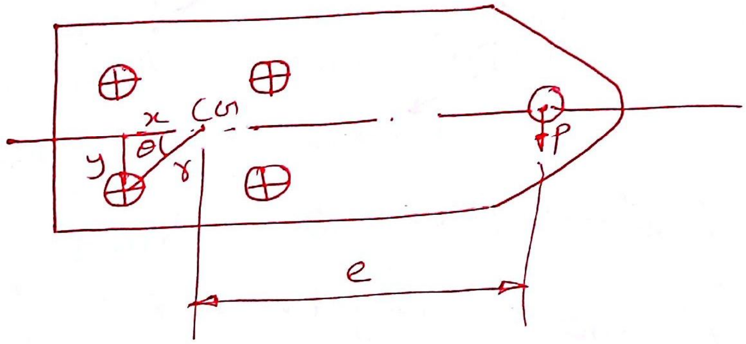
From PSGDB : 5.42

$$\boxed{d_c = 8.160\,mm}$$

From PSGDB: 5.42, $d_c = 8.160\,mm$

M10 bolt can be used.

(ii) Loading on the same Plane:-



r - radius of bolt centre from C.G

There are two forces acting on the bolts

(i) Direct shear force (F_1)

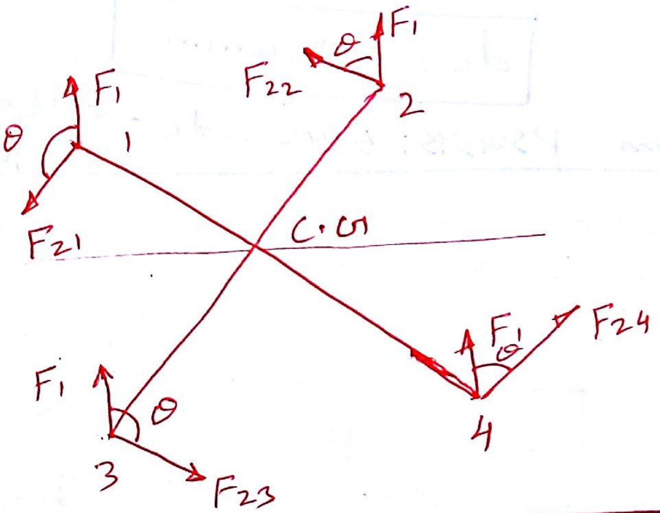
(ii) secondary shear force (F_2). It's due to the location of the with respect to the C.G.

$$F_1 = P/n$$

$$F_2 = \frac{P \times e}{n \times r} \quad (or) \quad F_{21} = \frac{P \times e \times r_1}{r_1^2 + r_2^2 + r_3^2}$$

F_{21} - Secondary shear force at bolt 1.

Forces on bolts



Resultant force $(F_R) = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

If $\cos\theta$ is negative, F_R will be minimum

$\therefore$ θ should be an acute angle.

bolt 2 & 4 have acute angle.

$\therefore$ there will ~~have~~ be maximum shear force.

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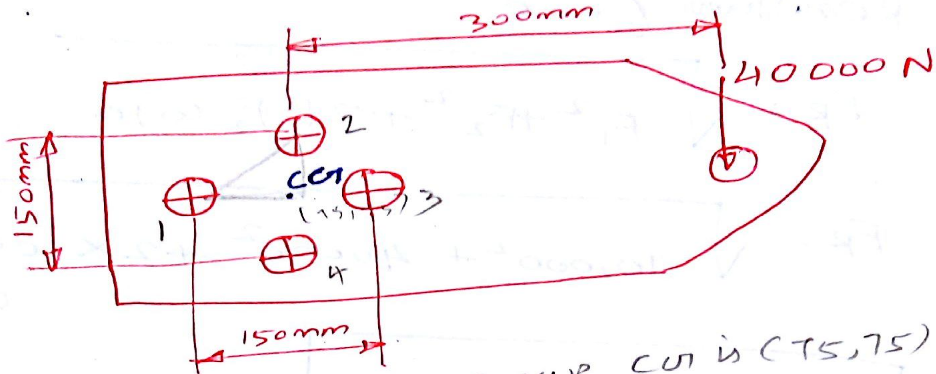
1. A bracket is fitted to a channel with 4 bolts as shown in figure. Distance between bolts 1 and 3 is equal to the distance between bolts 2 & 4 which is 150 mm. bolts 2 & 4 which is 150 mm. Find a suitable eccentricity is 300mm. Find a suitable bolt.

given:-

$e = 300mm$



bolt group CG is (75,75)

CG of bolt group is at (75,75) with bolt 1 as reference.

For bolt 1, $x_1 = 75$, $y = 0$ $\therefore r = \sqrt{x^2+y^2}$

$= \sqrt{75^2+0^2}$

$\boxed{r = 75}$

For bolt 2, $x = 0$, $y = 75$ $\therefore r = \sqrt{0+75^2}$

$r = 75$

For bolt 3, $x = 75$, $y = 0$ $\therefore r = 75$

For bolt 4, $x = 0$, $y = 75$ $\therefore r = 75$

$F_1 = \frac{P}{n} = \frac{40000}{4} = 10\,000\,N \Rightarrow \boxed{F_1 = 10000\,N}$

$F_2 = \frac{P \times e}{n \times r} = \frac{40000 \times 300}{4 \times 75} = 40{,}000\,N$

$\boxed{F_2 = 40{,}000\,N}$

Resultant force

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$F_R = \sqrt{10{,}000^2 + 40000^2 + 2 \times 10000 \times 40000 \cos 90}$$

$$\boxed{F_R = 41231.05\,N}$$

$$\frac{F_R}{A_s} = \frac{\sigma_y}{n \text{ (or) } FOS}$$

Assume n (or) $FOS = 3$

$\sigma_y = 300\ N/mm^2$

$$A_s = \frac{F_R \times n}{\sigma_y} = \frac{41231.05 \times 3}{300}$$

$$A_c \text{ (or) } A_s = 412.31\ mm^2$$

$$d_c = \sqrt{\frac{A_c \times 4}{\pi}} = \sqrt{\frac{412.31 \times 4}{\pi}}$$

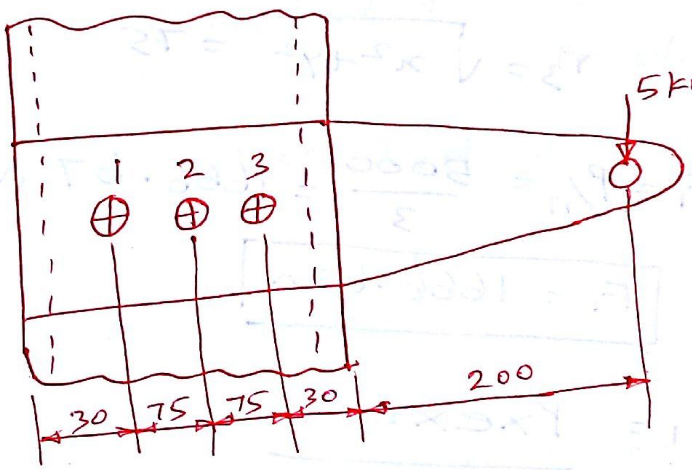
$$\boxed{d_c = 22.91\ mm}$$

From PSGDB P.No: 5.42

$d_c = 27.721\ mm$

M30 Bolt is used.

② A steel plate subjected to a force of 5 kN and fixed to a channel by means of three identical bolts is shown in Figure. The bolts are made from plain carbon steel 45C8 and the factor of safety is 3. Specify the size of bolts. [N/D 2010]



Given:-

$P = 5\,kN = 5000\,N$

$n = 3$

To find:-

Bolt size

Solution:-

CG of the bolt group is at (105, 0) with the left end as reference. i.e the CG is same as the location of bolt 2.

So for bolt 1, distance from c.g.

$x = 75$, $y = 0$

$\therefore r_1 = \sqrt{x^2 + y^2} = 75$

For bolt 2, $x = y = 0$, $r_2 = 0$

For bolt 3, $x = 75$, $y = 0$

$r_3 = \sqrt{x^2 + y^2} = 75$

$F_1 = P/n = \frac{5000}{3} = 1666.67\ N$

$\boxed{F_1 = 1666.67 N}$

$$\boxed{F_2 = F_{21} = \frac{P \times e \times r_1}{r_1^2 + r_2^2 + r_3}}$$

$$= \frac{5000 \times (75 + 30 + 200) \times 75}{\cancel{r_1^2 + r_2^2} \; (75^2 + 0^2 + 75^2}$$

$\boxed{F_{21} = 10166.67 N}$

$F_{22} = 0$

$$F_{23} = \frac{P \times e \times r_3}{r_1^2 + r_2^2 + r_3^2} = F_{21} = 10166.67 N$$

$\boxed{F_{23} = 10166.67 N}$

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{1667.67^2 + 10166.67^2 + 2 \times 1667.67 \times 10166.67 \times \cos\theta}$$

$$\boxed{F_R = 11833.34\,N}$$

$$\frac{F_R}{A_s} = \frac{\sigma_y}{n}$$

assuming $\sigma_y = 300\,N/mm^2$ and $n = 3$

$$\frac{11833.34}{A_s} = \frac{300}{3}$$

$$\boxed{A_s = 118.33\,mm^2}$$

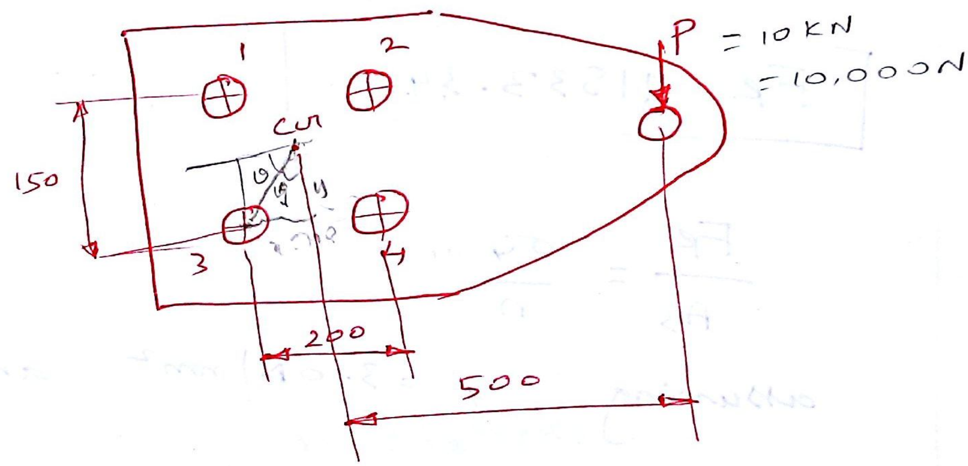
From PSGDB = 5.42

we choose M16 bolt

Result:-

Bolt chosen is M16

(3) The structural connections in fig. is subjected to an eccentric force P of 10 KN with an e = 50mm. The centre distance between bolts 1 and 2 is 200mm and 1 and 3 is 150mm. All the bolts are identical. Assume $\tau_y = 80\ N/mm^2$, n = 1



For bolt 1

$x = 100$, $y = 75$ $\quad r = \sqrt{100^2 + 75^2}$

$r = 125$

For bolt 2

$x = 100$ $\quad y = 75$ $\quad r = 125$

Similar 3 and 4

$$F_1 = P/n = \frac{10,000}{4} = 2500\ N$$

$$\boxed{F_1 = 2500N}$$

$$F_2 = \frac{P \times e}{n \times r} = \frac{10000 \times 500}{4 \times 125} = 10,000 N$$

$$\boxed{F_2 = 10,000 N}$$

$$\cos\theta = x/r = \frac{100}{125}$$

$$\boxed{\theta = 36.86^\circ}$$

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{2500^2 + 10,000^2 + 2 \times 2500 \times 10000 \times \cos 36.86}$$

$$\boxed{F_R = 12093 N}$$

$$\frac{F_R}{A_S} = \sigma_y/n.$$

$$\frac{12093}{A_S} = 80 \Rightarrow A_S = \frac{12093}{80}$$

$$\boxed{A_S = 151.16 mm^2}$$

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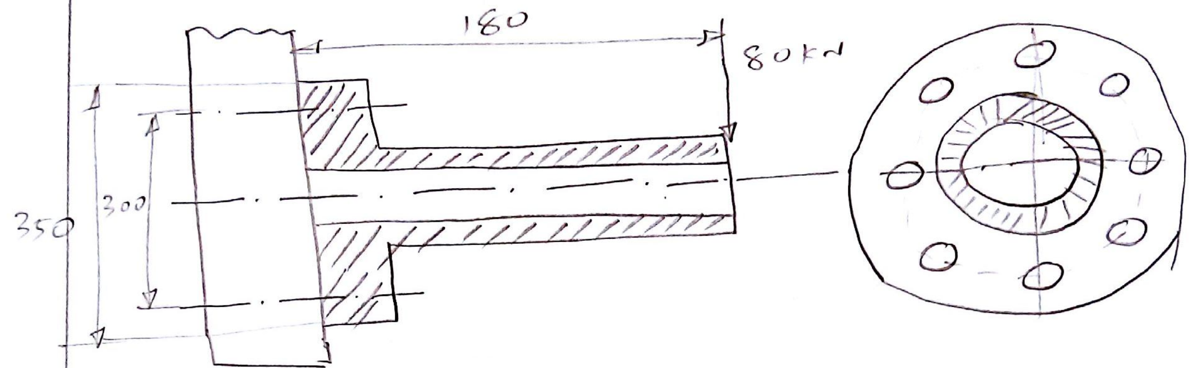
(4) A machine member has circular flange, which is connected to the frame by means of 8 bolts and loaded as shown in fig. Assuming that the bolt material has a allowable shear stress of 30 MPa, find the bolt size

required.



given:-

No. of bolts $n=8$

Load, $P = 80\,kN = 80{,}000\,N$

$\tau_{max} = 30\,MPa$

$e = 180\,mm$

$r_1 = 175\,mm$ & $r_2 = 150\,mm$.

To find:-

Bolt size

solution:-

(i) To find maximum load on each bolt due to tilting about '0' (F_1)

$$F_1 = F_{max} = \frac{2 \times P \times e \left[r_1 + r_2 \cos \frac{180}{n} \right]}{n \left[2r_1^2 + r_2^2 \right]}$$

$$= \frac{2 \times 80 \times 10^3 \times 180 \left(175 + 150 \cos \frac{180}{8}\right)}{8\,(2 \times 175^2 + 150^2)}$$

$$\boxed{F_1 = 13479 N}$$

(ii) Direct shear load on each bolt F_2

$$F_2 = \frac{\text{Total load}}{\text{No. of bolts}} = \frac{80 \times 10^3}{8} = 10,000 N$$

$$\boxed{F_2 = 10,000 N}$$

The maximum shear stress

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{13479}{A_s}\right)^2 + 4\left(\frac{10000}{A_s}\right)^2}$$

$$30 = \frac{12059}{A_s} N/mm^2$$

$$\sigma = \frac{F_1}{A_s} = \frac{13479}{A_s}$$

$$\tau = \frac{F_2}{A_s} = \frac{10000}{A_s}$$

$$\boxed{A_s = 401.97 mm^2}$$

$$d_c = \sqrt{\frac{A_s \times 4}{\pi}} = \sqrt{\frac{401.97 \times 4}{\pi}}$$

$$d_c = 22.62 mm$$

From PSGDB : 5.42 , $d_c = 27.727 mm$

$\therefore$ M30 Bolt can be used.

Assumptions:

1. There is no stress concentration.
2. The load is uniformly distributed over the joint.

Step 1:

Failure of solid rod in tension:

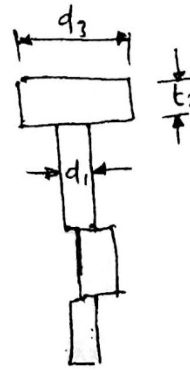
The rods are subjected to direct tensile load.

$$\therefore \; P = \frac{\pi}{4} d^2 \sigma_t$$

$$d = \sqrt{\frac{P \times 4}{\pi \times (\sigma_t)}}$$

Step 2:

Failure of knuckle pin by double shear:



The pin is in double shear.

$$P = 2 \times \frac{\pi}{4} d_1^2 \tau$$

$$P = \frac{\pi}{2} d_1^2 \tau$$

$$d_1 = \sqrt{\frac{P \times 2}{\pi \times (\tau)}}$$

but $\boxed{d_1 = d}$ ... a margin of strength is provided to allow for the bending of the pin.

Step 3:

Empirical Relations for knuckle Joint

diameter of the pin $(d_1) = d$

outer diameter of eye $(d_2) = 2d$

diameter of knuckle pin head and collar $\} (d_3) = 1.5d$

Thickness of single eye (or) rod end $(t) = 1.25d$

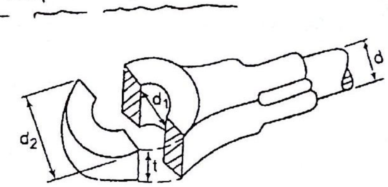
Thickness of fork $(t_1) = 0.75d$

Thickness of Pin head $(t_2) = 0.5d$

Step 4:

Failure of the single eye or rod end in tension:

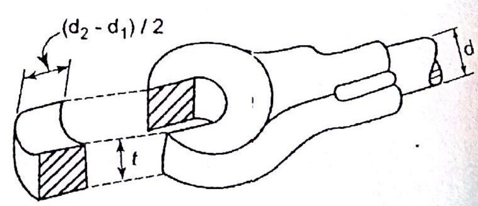
$$P = (d_2 - d_1)t \cdot \sigma_t$$



Step 5:

Failure of single eye (or) rod end in double shear:

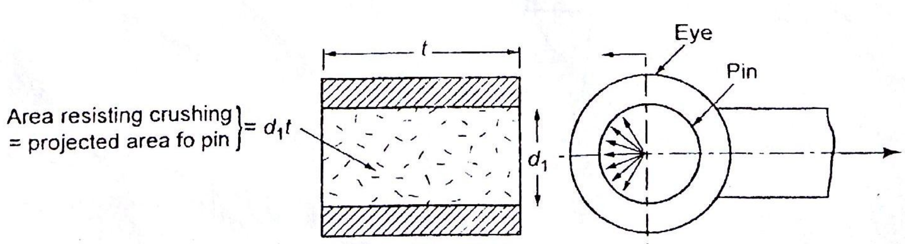
$$P = (d_2 - d_1) \cdot t \cdot \tau$$



step 6:

Failure of single eye (or) rod end in crushing:

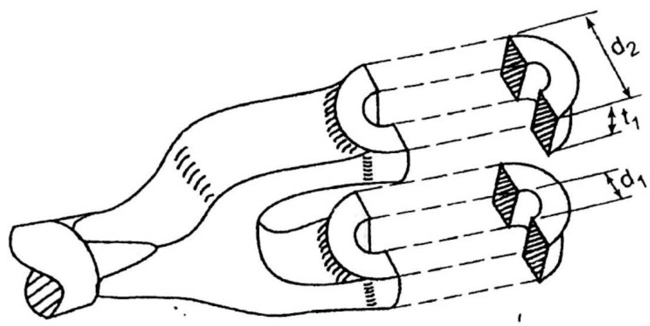
$$P = d_1 \times t \times \sigma_c$$



Step 7 :

Failure of the forked end in tension

$$P = (d_2 - d_1)\, t_1 \times 2 \times \sigma_t$$



Step 8 :

Failure of the forked end in shear :

$$P = (d_2 - d_1)\, t_1 \times 2 \times \tau$$

Step 9 :

Failure of forked end in crushing.

$$P = 2\, d_1 \times t_1 \times \sigma_c$$