DHANALAKSHMI COLLEGE OF ENGINEERING

(Dr.VPR Nagar, Manimangalam, Tambaram)

Chennai - 601 301

DEPARTMENT OF MECHANICAL ENGINEERING

III YEAR MECHANICAL - V SEMESTER

GE6503 – DESIGN OF MACHINE ELEMENTS

ACADEMIC YEAR (2017 - 2018) - ODD SEMESTER

UNIT – 2 (STUDY NOTES)

Prepared by:

R. SENDIL KUMAR, AP/MECH

P.V. ELUMALAI, AP/MECH

P. SIVA KUMAR, AP/MECH

UNIT 2 : SHAFTS AND COUPLING

PART - A

1. **List the various failures occurred in sunk keys? (A/M - 08)**

   Shear failure and Crushing failure.

2. **Why is maximum shear stress theory used for shaft? (A/M - 09) (N/D - 09)**

   Since the shaft is made of ductile materials, maximum shear stress theory is used.

3. **What is coupling? (A/M -09) (N/D - 09)**

   Couplings are used to connect sections of long transmission shafts and to connect the shaft of a driving machine to the shaft of a driven machine.

4. **Under what circumstances flexible couplings are used? (N/D - 07/08/09)**

   They are used to join the abutting ends of the shafts when they are not in exact alignment. They are used to permit an axial misalignment of the shafts without under absorption of power, which the shafts are transmitting.

5. Differentiate between keys and splines. (N/D - 11)

Keys – A shaft which is having single keyway
Keys are used in couplings.

Splines – A shaft which is having multiple keyways
Splines are used in automobiles and machine tools

6. At what angle of the crank the twisting moment is maximum in the crankshaft? (N/D - 11)

The angle of the crank the twisting moment is maximum in the crankshaft is between 25^o to 35^o from the TDC.

7. Why a hallow shaft has greater strength and stiffness than solid shaft of equal weight? (N/D - 12)

Stresses are maximum at the outer surface of a shaft. A hollow shaft has almost all the material concentrated at the outer circumferences and so has a better strength and stiffness for equal weight.

8. Under what circumstances flexible couplings are used. (N/D - 12)

(i) They are used to join the abrutting ends of shafts when they are not in exact alignment.
(ii) They are used to permit an axial misalignment of the shaft without under absorption of the power, which the shafts are transmitting.

9. What is meant by woodruff keys? (M/J - 13)

A woodruff key is used to transmit small value of Torque in automotive and machine tool industries. The keyway in the shaft is milled in a curves shape whereas the key way in the hub is usually straight.

10. Name any two of the rigid and flexible couplings. **(M/J - 13)**

Rigid coupling

Sleeve and Flange coupling

Flexible Coupling

Universal and Oldham’s coupling

11. Define the term critical speed.

The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite is known as critical or whirling speed.

12. What is key?

A key is a device, which is used for connecting two machine parts for preventing relative motion of rotation with respect to each other.

13. What are the purposes in machinery for which couplings are used?

1. To provide misalignment of the shafts (or) to introduce mechanical flexibility.
2. To reduce the transmission of shock from one shaft to another.
3. To introduce protection against over load.

14. What are the factors to be considered to design a shaft?

Strength and Stiffness

15. What is the main use of woodruff keys?

Woodruff key is used to transmit small value in automotive and machine tool industries. The key is the shaft is in a curved shape where as keyway in the hub is usually straight.

16. What is simple torsion?

When a shaft is subjected to pure torsional moment M, the shaft diameter can be found from torsional shear strength equation. Shear strength = 16*M/3.14.

17. What is simple bending?

When a shaft is subjected to a pure bending load, the principal stresses induced in the shaft are tension and compression. The maximum stress induced in the shaft can be determined by the theory of simple bending moment relation.

18. What are the types of Rigid Coupling?

(i)Sleeve, (ii) Flange, (iii) Clamp Coupling.

19. What material used for flange coupling?

Cast iron

20. What are the types of Flexible Coupling?

Universal, Oldham’s and Push pin type coupling

21. List the various failures occurred in sunk keys? **(A/M 2008)**

Shear failure and Crushing failure.

22. Write bending and torsion equation.

1. $\frac{M}{I} = \frac{E}{R} = \frac{\tau_s}{y}$ (Bending)
2. $\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau_s}{r}$ (Torsion)

23. Differentiate between keys and splines. **(N/D 2011)**

Keys – A shaft which is having single keyway
Keys are used in couplings.

Splines – A shaft which is having multiple keyways
Splines are used in automobiles and machine tools

24. At what angle of the crank the twisting moment is maximum in the crankshaft? **(N/D 2011)**

The angle of the crank the twisting moment is maximum in the crankshaft is between 25^o to 35^o from the TDC.

25. What is meant by equivalent bending moment? **(M/J 2012)**

26. Sketch the cross section of a splined shaft. **(M/J 2012)**

27. Why a hallow shaft has greater strength and stiffness than solid shaft of equal weight? **(N/D 2012)**

Stresses are maximum at the outer surface of a shaft. A hollow shaft has almost all the material concentrated at the outer circumferences and so has a better strength and stiffness for equal weight.

PART - B

1.

Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 100$ r.p.m. ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $n = 6$; $\sigma_t = 70$ MPa $= 70$ N/mm^2 ; $\mu = 0.3$

1. ***Design for shaft***

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3$ or $d = 71.4$ say 75 mm **Ans.**

2. ***Design for muff***

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm } \textbf{Ans.}$$

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm } \textbf{Ans.}$$

3. ***Design for key***

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, $w = 22$ mm **Ans.**

Thickness of key, $t = 14$ mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. ***Design for bolts***

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu\,(d_b)^2\,\sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\,(d_b)^2\;\; 70 \times 6 \times 75 = 5830 (d_b)^2$$

$\therefore$ $(d_b)^2 = 2865 \times 10^3 / 5830 = 492$ or $d_b = 22.2$ mm

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

2. A rigid type of coupling is used to connect two shafts transmitting 15 kW at 200 rpm. The shaft, keys and bolts are made of C45 steel and the coupling is of cast iron. Design the coupling. (MAY/JUNE 2013)

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 100$ r.p.m. ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $n = 6$; $\sigma_t = 70$ MPa $= 70$ N/mm$_2$; $\mu = 0.3$

1. *Design for shaft*

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3 \text{ or } d = 71.4 \text{ say } 75 \text{ mm } \textbf{Ans.}$$

2. *Design for muff*

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm } \textbf{Ans.}$$

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm } \textbf{Ans.}$$

3. *Design for key*

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, $w = 22$ mm **Ans.**

Thickness of key, $t = 14$ mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. *Design for bolts*

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2\, \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\, (d_b)^2\ 70 \times 6 \times 75 = 5830 (d_b)^2$$

$$\therefore \quad (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

3. Design a cast iron protective type flange coupling to transmit 15 kW at 900 rpm from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stress may be used: Shear stress for the shaft, bolt and key material = 40 MPa, Crushing stress for bolt and key = 80 MPa, Shear stress for cast iron = 8 MPa. **(Nov/Dec 2012)**

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 8 MPa = 8 N/mm^2

The protective type flange coupling is designed as discussed below :

1. ***Design for hub***

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

$$T_{max} = 1.35 \times 159.13 = 215 \text{ N-m} = 215 \times 10^3 \text{ N-mm}$$

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm}$$ **Ans.**

We know that outer diameter of the hub,

$$D = 2d = 2 \times 35 = 70 \text{ mm}$$ **Ans.**

and length of hub,

$$L = 1.5\, d = 1.5 \times 35 = 52.5 \text{ mm}$$ **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\ 147\ \tau_c$$

$$\therefore \quad \tau_c = 215 \times 10^3 / 63\ 147 = 3.4 \text{ N/mm}^2 = 3.4 \text{ MPa}$$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 35 mm diameter,

Width of key, w = 12 mm **Ans.**

and thickness of key, $t = w$ = 12 mm **Ans.**

The length of key (l) is taken equal to the length of hub.

$\therefore$ $l = L$ = 52.5 mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\ \tau_k$$

$$\therefore \quad \tau_k = 215 \times 10^3/11\,025 = 19.5 \text{ N/mm}^2 = 19.5 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 215 \times 10^3/\ 5512.5 = 39 \text{ N/mm}^2 = 39 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of flange (t_f) is taken as 0.5 d.

$\therefore$ $t_f = 0.5\,d = 0.5 \times 35 = 17.5$ mm **Ans.**

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\,713\ \tau_c$$

$$\therefore \quad \tau_c = 215 \times 10^3/134\,713 = 1.6 \text{ N/mm}^2 = 1.6 \text{ MPa}$$

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

$$n = 3$$

and pitch circle diameter of bolts,

$$D_1 = 3d = 3 \times 35 = 105 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4}(d_1)^2 \ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2 \ 40 \times 3 \times \frac{105}{2} = 4950\,(d_1)^2$$

$\therefore \quad (d_1)^2 = 215 \times 10^3/4950 = 43.43$ or $d_1 = 6.6$ mm

Assuming coarse threads, the nearest standard size of bolt is M 8. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 35 = 140 \text{ mm} \textbf{ Ans.}$$

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 35 = 8.75 \text{ say } 10 \text{ mm} \textbf{ Ans.}$$

4.

The shaft and the flange of a marine engine are to be designed for flange coupling, in which the flange is forged on the end of the shaft. The following particulars are to be considered in the design :

Power of the engine	*= 3 MW*
Speed of the engine	*= 100 r.p.m.*
Permissible shear stress in bolts and shaft	*= 60 MPa*
Number of bolts used	*= 8*
Pitch circle diameter of bolts	*= 1.6 × Diameter of shaft*

Find : 1. diameter of shaft ; 2. diameter of bolts ; 3. thickness of flange ; and 4. diameter of flange.

Solution. Given : $P = 3$ MW $= 3 \times 10^6$ W ; $N = 100$ r.p.m. ; $\tau_b = \tau_s = 60$ MPa $= 60$ N/mm^2 ; $n = 8$; $D_1 = 1.6\,d$

1. ***Diameter of shaft***

Let $\quad d$ = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{3 \times 10^6 \times 60}{2\pi \times 100} = 286 \times 10^3 \text{ N-m} = 286 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$286 \times 10^6 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\,d^3$$

$\therefore \quad d^3 = 286 \times 10^6 / 11.78 = 24.3 \times 10^6$

or $\quad d = 2.89 \times 10^2 = 289$ say 300 mm **Ans.**

2. ***Diameter of bolts***

Let $\quad d_1$ = Nominal diameter of bolts.

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$286 \times 10^6 = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} \times (d_1)^2\, 60 \times 8 \times \frac{1.6 \times 300}{2}$$

$$= 90\,490\,(d_1)^2 \qquad \ldots (\because D_1 = 1.6\, d)$$

$\therefore \qquad (d_1)^2 = 286 \times 10^6 / 90\,490 = 3160 \qquad$ or $\qquad d_1 = 56.2$ mm

Assuming coarse threads, the standard diameter of the bolt is 60 mm (M 60). The taper on the bolt may be taken from 1 in 20 to 1 in 40. **Ans.**

3. ***Thickness of flange***

The thickness of flange (t_f) is taken as d / 3.

$\therefore \qquad t_f = d / 3 = 300/3 = 100$ mm **Ans.**

Let us now check the induced shear stress in the flange by considering the flange at the junction of the shaft in shear. We know that the torque transmitted (T),

$$286 \times 10^6 = \frac{\pi d^2}{2} \times \tau_s \times t_f = \frac{\pi (300)^2}{2} \times \tau_s \times 100 = 14.14 \times 10^6\, \tau_s$$

$\therefore \qquad \tau_s = 286 \times 10^6 / 14.14 \times 10^6 = 20.2\ \text{N/mm}^2 = 20.2$ MPa

Since the induced shear stress in the *flange is less than the permissible shear stress of 60 MPa, therefore the thickness of flange (t_f = 100 mm) is safe.

4. ***Diameter of flange***

The diameter of flange (D_2) is taken as 2.2 d.

$\therefore \qquad D_2 = 2.2\, d = 2.2 \times 300 = 660$ mm **Ans.**

28. Design a bushed –pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 KW at 960 rpm the overall torque is 20 percent more than mean torque. The material properties are as follows:

(i)The allowable shear and crushing stress for shaft and key material is 40 Mpa and 80 Mpa respectively.
(ii) The allowable shear stress for cast iron is 15 Mpa.
(iii) The allowable bearing pressure for rubber bush is 0.8 N/mm2.
(iv) The material of the pin is same as that of shaft and key. Draw neat sketch of the coupling. (NOV/DEC 2012)

Solution. Given : $P = 32$ kW $= 32 \times 10^3$ W ; $N = 960$ r.p.m. ; $T_{max} = 1.2\, T_{mean}$; $\tau_s = \tau_k = 40$ MPa $= 40\ \text{N/mm}^2$; $\sigma_{cs} = \sigma_{ck} = 80$ MPa $= 80\ \text{N/mm}^2$; $\tau_c = 15$ MPa $= 15\ \text{N/mm}^2$; $p_b = 0.8\ \text{N/mm}^2$

The bushed-pin flexible coupling is designed as discussed below :

1. ***Design for pins and rubber bush***

First of all, let us find the diameter of the shaft (d). We know that the mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2 \pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \text{ N-m}$$

and the maximum or overall torque transmitted,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

We also know that the maximum torque transmitted by the shaft (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $$d^3 = 382 \times 10^3 / 7.86 = 48.6 \times 10^3 \text{ or } d = 36.5 \text{ say } 40 \text{ mm}$$

We have discussed in rigid type of flange coupling that the number of bolts for 40 mm diameter shaft are 3. In the flexible coupling, we shall use the number of pins (n) as 6.

$\therefore$ Diameter of pins, $$d_1 = \frac{0.5\,d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm}$$

In order to allow for the bending stress induced due to the compressibility of the rubber bush, the diameter of the pin (d_1) may be taken as 20 mm. **Ans.**

The length of the pin of least diameter *i.e.* $d_1 = 20$ mm is threaded and secured in the right hand coupling half by a standard nut and washer. The enlarged portion of the pin which is in the left hand coupling half is made of 24 mm diameter. On the enlarged portion, a brass bush of thickness 2 mm is pressed. A brass bush carries a rubber bush. Assume the thickness of rubber bush as 6 mm.

$\therefore$ Overall diameter of rubber bush,

$$d_2 = 24 + 2 \times 2 + 2 \times 6 = 40 \text{ mm } \textbf{Ans.}$$

and diameter of the pitch circle of the pins,

$$D_1 = 2\,d + d_2 + 2 \times 6 = 2 \times 40 + 40 + 12 = 132 \text{ mm } \textbf{Ans.}$$

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$$W = p_b \times d_2 \times l = 0.8 \times 40 \times l = 32\, l \text{ N}$$

and the maximum torque transmitted by the coupling (T_{max}),

$$382 \times 10^3 = W \times n \times \frac{D_1}{2} = 32\, l \times 6 \times \frac{132}{2} = 12\,672\, l$$

$\therefore$ $$l = 382 \times 10^3 / 12\,672 = 30.1 \text{ say } 32 \text{ mm}$$

and $$W = 32\, l = 32 \times 32 = 1024 \text{ N}$$

$\therefore$ Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2} = \frac{1024}{\frac{\pi}{4}(20)^2} = 3.26 \text{ N/mm}^2$$

Since the pin and the rubber bush are not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load (W) along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right) = 1024\left(\frac{32}{2} + 5\right) = 21\,504 \text{ N-mm}$$

and section modulus, $$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3 = 785.5 \text{ mm}^3$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{21\,504}{785.5} = 27.4 \text{ N/mm}^2$$

∴ Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[27.4 + \sqrt{(27.4)^2 + 4\,(3.26)^2}\right]$$

$$= 13.7 + 14.1 = 27.8 \text{ N/mm}^2$$

and maximum shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(27.4)^2 + 4\,(3.26)^2}\right] = 14.1 \text{ N/mm}^2$$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. *Design for hub*

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 40 = 80 \text{ mm}$$

and length of hub, $L = 1.5\,d = 1.5 \times 40 = 60$ mm

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(80)^4 - (40)^4}{80}\right] = 94.26 \times 10^3\, \tau_c$$

∴ $\tau_c = 382 \times 10^3 \,/\, 94.26 \times 10^3 = 4.05 \text{ N/mm}^2 = 4.05$ MPa

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\,\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 40 mm diameter,

Width of key, $w = 14$ mm **Ans.**

and thickness of key, $t = w = 14$ mm **Ans.**

The length of key (L) is taken equal to the length of hub, *i.e.*

$$L = 1.5\,d = 1.5 \times 40 = 60 \text{ mm}$$

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 60 \times 14 \times \tau_k \times \frac{40}{2} = 16\,800\, \tau_k$$

∴ $\tau_k = 382 \times 10^3 / 16\,800 = 22.74 \text{ N/mm}^2 = 22.74$ MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2} = 8400\, \sigma_{ck}$$

∴ $\sigma_{ck} = 382 \times 10^3 / 8400 = 45.48 \text{ N/mm}^2 = 45.48$ MPa

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. *Design for flange*

The thickness of flange (t_f) is taken as $0.5\,d$.

∴ $t_f = 0.5\,d = 0.5 \times 40 = 20$ mm

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (80)^2}{2} \times \tau_c \times 20 = 201 \times 10^3 \tau_c$$

$\therefore \quad \tau_c = 382 \times 10^3 / 201 \times 10^3 = 1.9 \text{ N/mm}^2 = 1.9 \text{ MPa}$

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.

6. Two 35 mm shafts are connected by a flanged coupling. The flanges are fitted with 6 bolts on 25 mm bolt circle. The shafts transmit a torque of 800 N-m at 350 rpm. For the safe stresses mentions below, calculate (i) diameter of bolts, (ii) thickness of flanges, (iii) key dimensions, (iv) hub length and (v) power transmitted. Safe stress for shaft material 63 MPa. Safe stress for bolt material 56 MPa. Safe stress for cast iron coupling 10 MPa and Safe stress for key material 46 MPa. (NOV/DEC 2011)

Solution. Given : $d = 35$ mm ; $n = 6$; $D_1 = 125$ mm ; $T = 800$ N-m $= 800 \times 10^3$ N-mm ; $N = 350$ r.p.m.; $\tau_s = 63$ MPa $= 63$ N/mm^2 ; $\tau_b = 56$ MPa $= 56$ N/mm^2 ; $\tau_c = 10$ MPa $= 10$ N/mm^2 ; $\tau_k = 46$ MPa $= 46$ N/mm^2

1. *Diameter of bolts*

Let $\quad d_1$ = Nominal or outside diameter of bolt.

We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi}{4} (d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2\, 56 \times 6 \times \frac{125}{2} = 16\,495\, (d_1)^2$$

$\therefore \quad (d_1)^2 = 800 \times 10^3 / 16\,495 = 48.5$ or $d_1 = 6.96$ say 8 mm **Ans.**

2. *Thickness of flanges*

Let $\quad t_f$ = Thickness of flanges.

We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (2 \times 35)^2}{2} \times 10 \times t_f = 76\,980\, t_f \quad \ldots (\because D = 2d)$$

$\therefore \quad t_f = 800 \times 10^3 / 76\,980 = 10.4$ say 12 mm **Ans.**

3. *Key dimensions*

From Table 13.1, we find that the proportions of key for a 35 mm diameter shaft are :

Width of key, $\quad w = 12$ mm **Ans.**

and thickness of key, $\quad t = 8$ mm **Ans.**

The length of key (l) is taken equal to the length of hub (L).

$\therefore \quad l = L = 1.5\, d = 1.5 \times 35 = 52.5$ mm

Let us now check the induced shear stress in the key. We know that the torque transmitted (T),

$$800 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} \times = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\, \tau_k$$

$\therefore \quad \tau_k = 800 \times 10^3 / 11\,025 = 72.5$ N/mm^2

Since the induced shear stress in the key is more than the given safe stress (46 MPa), therefore let us find the length of key by substituting the value of $\tau_k = 46$ MPa in the above equation, *i.e.*

$$800 \times 10^3 = l \times 12 \times 46 \times \frac{35}{2} = 9660\, l$$

$\therefore$ $$l = 800 \times 10^3 / 9660 = 82.8 \text{ say } 85 \text{ mm } \textbf{Ans.}$$

4. *Hub length*

Since the length of key is taken equal to the length of hub, therefore we shall take hub length,

$$L = l = 85 \text{ mm } \textbf{Ans.}$$

5. *Power transmitted*

We know that the power transmitted,

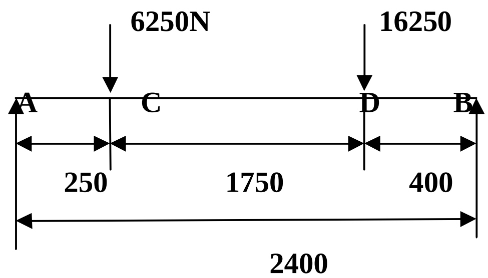
$$P = \frac{T \times 2\pi N}{60} = \frac{800 \times 2\pi \times 350}{60} = 29\,325 \text{ W} = 29.325 \text{ kW } \textbf{Ans.}$$

7. A Horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250 mm and 400 mm respectively from the centre line of the left and right bearings. The pitch diameter of the gear C is 600 mm and that of gear D is 200 mm. The pitch diameter of the gear C is 600 mm and that of gear D is 200 mm. The distance between the centre line of the bearings is 2400mm. The shaft transmits 20 KW at 120 rpm. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure FtC of the gear C and FtD of the gear D act vertically downwards.

Find the diameter of the shaft, if the working stress is 100 Mpa in tension and 56 Mpa in shear. The gear C and D weighs 950 N and 350 N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively.

(NOV/DEC 2012)

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 120} = 1590\ N-m$$



$$F_{tC} = \frac{T}{R_C} = \frac{1590 \times 10^3}{300} = 5300\ N$$

$$= F_{tC} + W_C = 5300 + 950 = 6250\ N$$

$$F_{tD} = \frac{T}{R_D} = \frac{1590 \times 10^3}{100} = 15900\ N$$

$$= F_{tD} + W_D = 15900 + 350 = 16250\ N$$

$$R_B = 34062.5 \times 10^3 / 2400 = 14190\ N$$

$$R_A = 22500 - 14190 = 8310\ N$$

$$M_C = R_A \times 250 = 8310 \times 250 = 2077.5 \times 10^3 N-mm$$

$$M_D = R_B \times 400 = 14190 \times 400 = 5676 \times 10^3 N-mm$$

$$T_e = \sqrt{(K_m \times M) + (K_t \times T)}^2$$

$=8725 \times 10^3 \, N-mm$

$8725 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 56 \times d^3 = 11d^3$

d = 92.5 mm

$M_e = \frac{1}{2}[K_m \times M] + \sqrt{(K_m \times M) + (K_t \times T)^2}]$

$$= \frac{1}{2}(K_m \times M + T_e)$$

d = 95.7mm

Taking the larger of the two values, we have d=95.7 mm Say 100 mm.

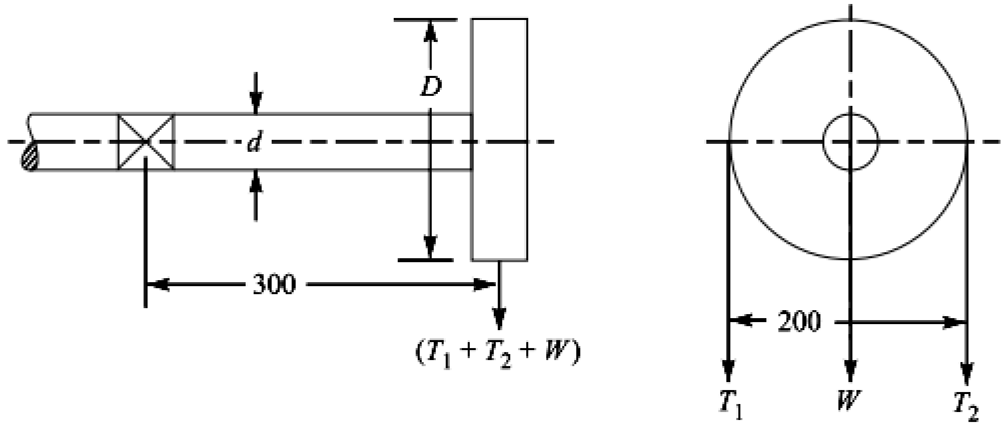
8. Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weights 200 N and is located at 300 mm from the center of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 rpm. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. the shock arid fatigue factors for bending and twisting are 1.5 and 2 respectively. The allowable shear stress in the shaft may be taken as 35 MPa. (NOV/DEC 2011)

Solution. Given : W = 200 N ; L = 300 mm ; D = 200 mm or R = 100 mm ; P = 1 kW = 1000 W ; N = 120 r.p.m. ; θ = 180° = π rad ; μ = 0.3 ; K_m = 1.5 ; K_t = 2 ; τ = 35 MPa = 35 N/mm^2

The shaft with pulley is shown in Fig.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{1000 \times 60}{2\pi \times 120} = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons.

∴ Torque transmitted (T),

$$79.6 \times 10^3 = (T_1 - T_2) R = (T_1 - T_2)\, 100$$

$$\therefore \quad T_1 - T_2 = 79.6 \times 10^3 / 100 = 796 \text{ N} \quad ...(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\,\pi = 0.9426$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.9426}{2.3} = 0.4098 \text{ or } \frac{T_1}{T_2} = 2.57 \quad ...(ii)$$

...(Taking antilog of 0.4098)

From equations (*i*) and (*ii*), we get,

$$T_1 = 1303 \text{ N, and } T_2 = 507 \text{ N}$$

We know that the total vertical load acting on the pulley,

$$W_T = T_1 + T_2 + W = 1303 + 507 + 200 = 2010 \text{ N}$$

∴ Bending moment acting on the shaft,

$$M = W_T \times L = 2010 \times 300 = 603 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

$$= \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$918 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 35 \times d^3 = 6.87\, d^3$$

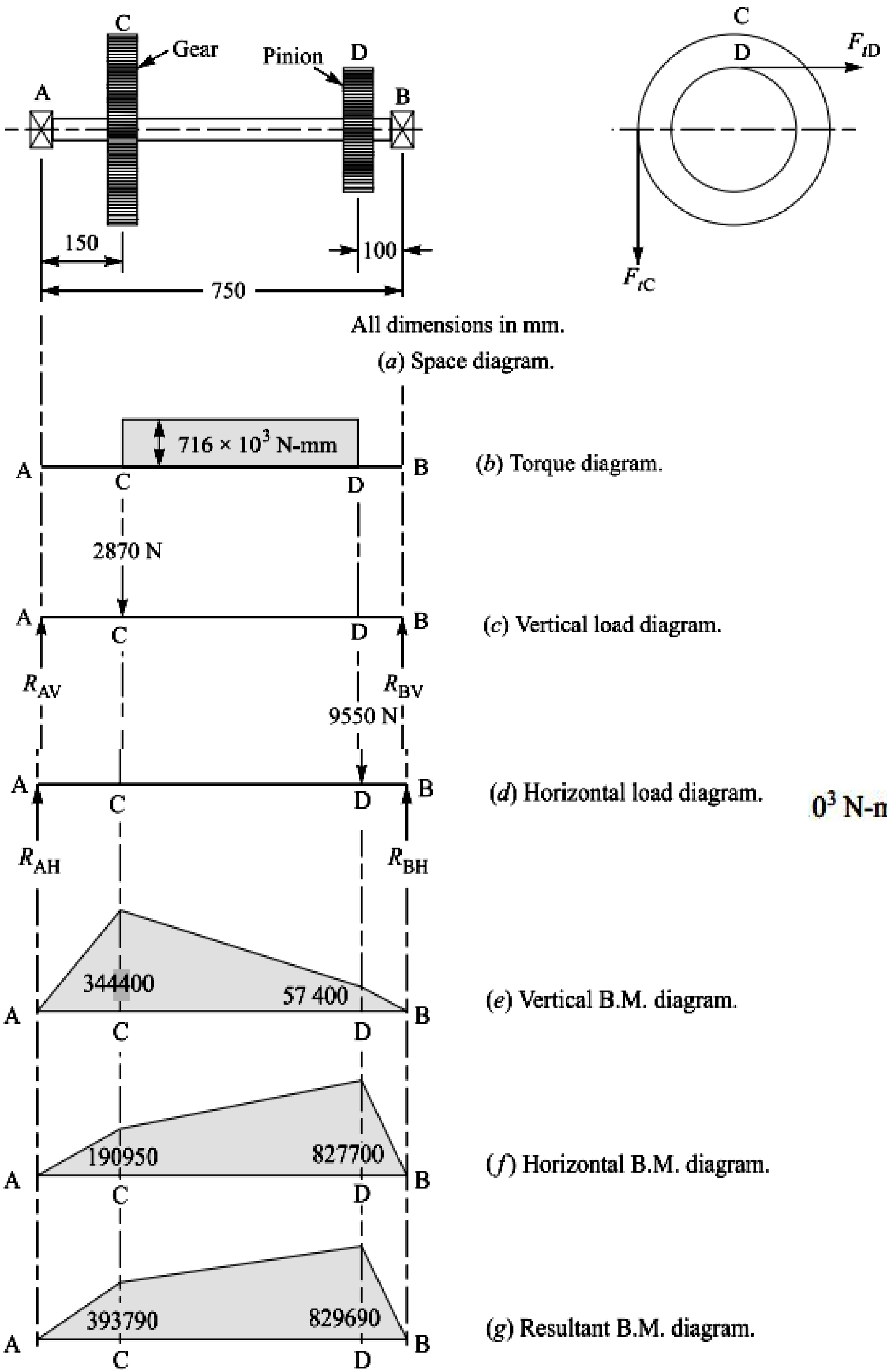
$$\therefore \quad d^3 = 918 \times 10^3 / 6.87 = 133.6 \times 10^3 \text{ or } d = 51.1 \text{ say } 55 \text{ mm } \textbf{Ans.}$$

9. A steel solid shaft transmitting 15 kW at 200 rpm is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 30 teeth of 5 mm module is located 100 min to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 5 mm module is located 150 mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear. Determine the diameter of the shaft. (MAY/JUNE 2013)

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 200$ r.p.m. ; $AB = 750$ mm ; $T_D = 30$; $m_D = 5$ mm ; $BD = 100$ mm ; $T_C = 100$; $m_C = 5$ mm ; $AC = 150$ mm ; $\tau = 54$ MPa $= 54$ N/mm^2

The space diagram of the shaft is shown in Fig. 14.8 (*a*).

We know that the torque transmitted by the shaft,


C
Gear
Pinion
D
A
B
C
D
F_{tD}
F_{tC}
150
100
750
All dimensions in mm.
(a) Space diagram.
716 × 10³ N-mm
A C D B
(b) Torque diagram.
2870 N
A C D B
R_{AV}
R_{BV}
(c) Vertical load diagram.
9550 N
A C D B
R_{AH}
R_{BH}
(d) Horizontal load diagram.
344400
57 400
A C D B
(e) Vertical B.M. diagram.
190950
827700
A C D B
(f) Horizontal B.M. diagram.
393790
829690
A C D B
(g) Resultant B.M. diagram.

0³ N-mm

The

We

∴ Radius of gear C,

$$R_C = \frac{T_C \times m_C}{2} = \frac{100 \times 5}{2} = 250 \text{ mm}$$

and radius of pinion D,

$$R_D = \frac{T_D \times m_D}{2} = \frac{30 \times 5}{2} = 75 \text{ mm}$$

Assuming that the torque at C and D is same (*i.e.* 716×10^3 N-mm), therefore tangential force on the gear C, acting downward,

$$F_{tC} = \frac{T}{R_C} = \frac{716 \times 10^3}{250} = 2870 \text{ N}$$

and tangential force on the pinion D, acting horizontally,

$$F_{tD} = \frac{T}{R_D} = \frac{716 \times 10^3}{75} = 9550 \text{ N}$$

The vertical and horizontal load diagram is shown in Fig. 14.8 (*c*) and (*d*) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at C. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2870 \text{ N}$$

Taking moments about A, we get

$$R_{BV} \times 750 = 2870 \times 150$$

$$\therefore \quad R_{BV} = 2870 \times 150 / 750 = 574 \text{ N}$$

and $$R_{AV} = 2870 - 574 = 2296 \text{ N}$$

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $$M_{CV} = R_{AV} \times 150 = 2296 \times 150 = 344\ 400 \text{ N-mm}$$

B.M. at D, $$M_{DV} = R_{BV} \times 100 = 574 \times 100 = 57\ 400 \text{ N-mm}$$

The B.M. diagram for vertical loading is shown in Fig. 14.8 (*e*).

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 9550 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 750 = 9550\ (750 - 100) = 9550 \times 650$$

$$\therefore \quad R_{BH} = 9550 \times 650 / 750 = 8277 \text{ N}$$

and $$R_{AH} = 9550 - 8277 = 1273 \text{ N}$$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $M_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190\ 950$ N-mm

B.M. at *D*, $M_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827\ 700$ N-mm

The B.M. diagram for horizontal loading is shown in Fig. 14.8 (*f*).

We know that resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(344\ 400)^2 + (190\ 950)^2}$$
$$= 393\ 790 \text{ N-mm}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(57\ 400)^2 + (827\ 700)^2}$$
$$= 829\ 690 \text{ N-mm}$$

The resultant B.M. diagram is shown in Fig. 14.8 (*g*). We see that the bending moment is maximum at *D*.

∴ Maximum bending moment,

$$M = M_D = 829\ 690 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(829\ 690)^2 + (716 \times 10^3)^2} = 1096 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1096 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 54 \times d^3 = 10.6\ d^3$$

∴ $d^3 = 1096 \times 10^3 / 10.6 = 103.4 \times 10^3$

or $d = 47$ say 50 mm **Ans.**

10.

A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution. Given : $M = 3000$ N-m $= 3 \times 10^6$ N-mm ; $T = 10\ 000$ N-m $= 10 \times 10^6$ N-mm ; $\sigma_{tu} = 700$ MPa $= 700$ N/mm^2 ; $\tau_u = 500$ MPa $= 500$ N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\ d^3$$

$\therefore$ $d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6$ or $d = 86$ mm

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\ d^3$$

$\therefore$ $d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6$ or $d = 83.7$ mm

Taking the larger of the two values, we have

$d = 86$ say 90 mm **Ans.**

11.

A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Solution. Given : $AB = 1$ m ; $D_C = 600$ mm or $R_C = 300$ mm $= 0.3$ m ; $AC = 300$ mm $= 0.3$ m ; $T_1 = 2.25$ kN $= 2250$ N ; $D_D = 400$ mm or $R_D = 200$ mm $= 0.2$ m ; $BD = 200$ mm $= 0.2$ m ; $\theta = 180° = \pi$ rad ; $\mu = 0.24$; $\sigma_b = 63$ MPa $= 63$ N/mm^2 ; $\tau = 42$ MPa $= 42$ N/mm^2

The space diagram of the shaft is shown in Fig. 14.5 (*a*).

Let T_1 = Tension in the tight side of the belt on pulley $C = 2250$ N ...(Given)

T_2 = Tension in the slack side of the belt on pulley C.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127 \qquad \text{...(Taking antilog of 0.3278)}$$

and

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

∴ Vertical load acting on the shaft at C,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

and vertical load on the shaft at D

$$= 0$$

The vertical load diagram is shown in Fig. 14.5 (*c*).

We know that torque acting on the pulley C,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig. 14.5 (*b*).

Let T_3 = Tension in the tight side of the belt on pulley D, and

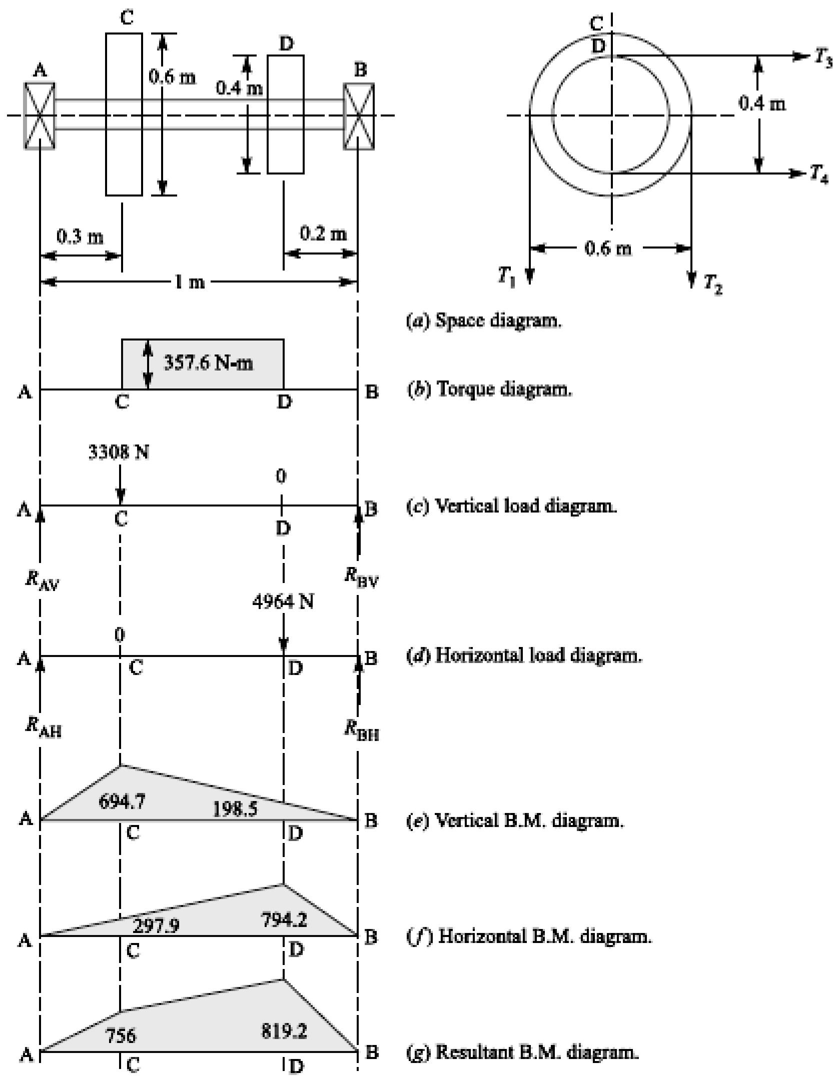
T_4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (*i.e.* C and D) is same, therefore

$$(T_3 - T_4)\, R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \qquad \text{...}(i)$$

We know that

$$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4 \qquad \text{...}(ii)$$



From equations (*i*) and (*ii*), we find that

$$T_3 = 3376 \text{ N, and } T_4 = 1588 \text{ N}$$

∴ Horizontal load acting on the shaft at *D*,

$$W_D = T_3 + T_4 = 3376 + 1588 = 4964 \text{ N}$$

and horizontal load on the shaft at $C = 0$

The horizontal load diagram is shown in Fig. 14.5 (*d*).

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 3308 \text{ N}$$

Taking moments about *A*,

$$R_{BV} \times 1 = 3308 \times 0.3 \text{ or } R_{BV} = 992.4 \text{ N}$$

and $$R_{AV} = 3308 - 992.4 = 2315.6 \text{ N}$$

Taking moments about A,

$$R_{BV} \times 1 = 3308 \times 0.3 \quad \text{or} \quad R_{BV} = 992.4 \text{ N}$$

and $$R_{AV} = 3308 - 992.4 = 2315.6 \text{ N}$$

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $$M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7 \text{ N-m}$$

B.M. at D, $$M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5 \text{ N-m}$$

The bending moment diagram for vertical loading in shown in Fig. 14.5 (*e*).

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 4964 \text{ N}$$

Taking moments about A,

$$R_{BH} \times 1 = 4964 \times 0.8 \quad \text{or} \quad R_{BH} = 3971 \text{ N}$$

and $$R_{AH} = 4964 - 3971 = 993 \text{ N}$$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $$M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9 \text{ N-m}$$

B.M. at D, $$M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2 \text{ N-m}$$

The bending moment diagram for horizontal loading is shown in Fig. 14.5 (*f*).

Resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 14.5 (*g*).

We see that bending moment is maximum at D.

$\therefore$ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let $$d = \text{Diameter of the shaft.}$$

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$
$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $$d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3 \text{ or } d = 47.6 \text{ mm}$$

Again we know that equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\, d^3$$

$\therefore$ $$d^3 = 856.6 \times 10^3 / 6.2 = 138.2 \times 10^3 \text{ or } d = 51.7 \text{ mm}$$

Taking larger of the two values, we have

$$d = 51.7 \text{ say } 55 \text{ mm } \textbf{Ans.}$$

12.

A shaft is supported on bearings A and B, 800 mm between centres. A 20° straight tooth spur gear having 600 mm pitch diameter, is located 200 mm to the right of the left hand bearing A, and a 700 mm diameter pulley is mounted 250 mm towards the left of bearing B. The gear is driven by a pinion with a downward tangential force while the pulley drives a horizontal belt having 180° angle of wrap. The pulley also serves as a flywheel and weighs 2000 N. The maximum belt tension is 3000 N and the tension ratio is 3 : 1. Determine the maximum bending moment and the necessary shaft diameter if the allowable shear stress of the material is 40 MPa.

Solution. Given : $AB = 800$ mm ; $\alpha_C = 20°$; $D_C = 600$ mm or $R_C = 300$ mm ; $AC = 200$ mm ; $D_D = 700$ mm or $R_D = 350$ mm ; $DB = 250$ mm ; $\theta = 180° = \pi$ rad ; $W = 2000$ N ; $T_1 = 3000$ N ; $T_1/T_2 = 3$; $\tau = 40$ MPa $= 40$ N/mm^2

The space diagram of the shaft is shown in Fig. 14.6 (*a*).

We know that the torque acting on the shaft at D,

$$T = (T_1 - T_2)\,R_D = T_1\left(1 - \frac{T_2}{T_1}\right) R_D$$

$$= 3000\left(1 - \frac{1}{3}\right) 350 = 700 \times 10^3 \text{ N-mm} \qquad \ldots(\because T_1/T_2 = 3)$$

The torque diagram is shown in Fig. 14.6 (*b*).

Assuming that the torque at D is equal to the torque at C, therefore the tangential force acting on the gear C,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear C,

$$W_C = \frac{F_{tc}}{\cos \alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. 14.7. Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at C,

$$W_{CV} = W_C \cos 20°$$
$$= 2483 \times 0.9397 = 2333 \text{ N}$$

and horizontal component of W_C *i.e.* the horizontal load acting on the shaft at C,

$$W_{CH} = W_C \sin 20°$$
$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and $T_1 = 3000$ N, therefore

$$T_2 = T_1 / 3 = 3000 / 3 = 1000 \text{ N}$$

∴ Horizontal load acting on the shaft at D,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at D,

$$W_{DV} = W = 2000 \text{ N}$$

The vertical and horizontal load diagram at C and D is shown in Fig. 14.6 (*c*) and (*d*) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all considering the vertical loading at C and D. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

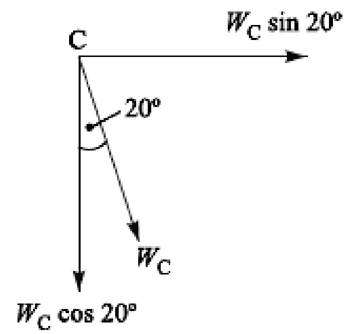
$$R_{AV} + R_{BV} = 2333 + 2000 = 4333 \text{ N}$$

Taking moments about A, we get

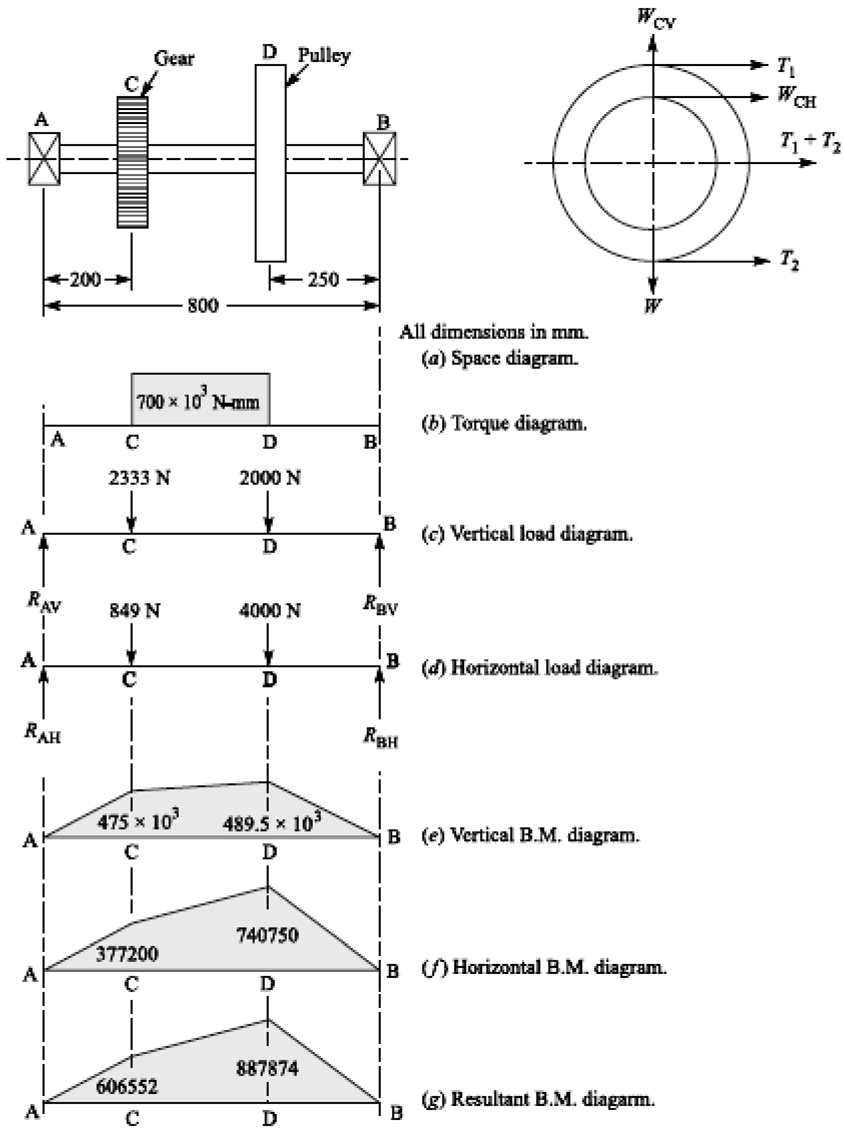
$$R_{BV} \times 800 = 2000\,(800 - 250) + 2333 \times 200$$
$$= 1\ 566\ 600$$

$$\therefore \quad R_{BV} = 1\ 566\ 600 / 800 = 1958 \text{ N}$$

and $$R_{AV} = 4333 - 1958 = 2375 \text{ N}$$



We know that B.M. at A and B,



All dimensions in mm.

(*a*) Space diagram.

(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.

(*g*) Resultant B.M. diagarm.

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $M_{CV} = R_{AV} \times 200 = 2375 \times 200$

$= 475 \times 10^3$ N-mm

B.M. at D, $M_{DV} = R_{BV} \times 250 = 1958 \times 250 = 489.5 \times 10^3$ N-mm

The bending moment diagram for vertical loading is shown in Fig. 14.6 (*e*).

Now consider the horizontal loading at C and D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 4849 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 800 = 4000\,(800 - 250) + 849 \times 200 = 2\,369\,800$$

$\therefore$ $R_{BH} = 2\,369\,800 / 800 = 2963$ N

and $R_{AH} = 4849 - 2963 = 1886$ N

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $M_{CH} = R_{AH} \times 200 = 1886 \times 200 = 377\,200$ N-mm

B.M. at D, $M_{DH} = R_{BH} \times 250 = 2963 \times 250 = 740\,750$ N-mm

The bending moment diagram for horizontal loading is shown in Fig. 14.6 (*f*).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(475 \times 10^3)^2 + (377\,200)^2}$$

$= 606\,552$ N-mm

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(489.5 \times 10^3)^2 + (740\,750)^2}$$

$= 887\,874$ N-mm

Maximum bending moment

The resultant B.M. diagram is shown in Fig. 14.6 (*g*). We see that the bending moment is maximum at D, therefore

Maximum B.M., $M = M_D = 887\,874$ N-mm **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\,874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3$ or $d = 52.4$ say 55 mm **Ans.**

13. Design a plain carbon steel center crankshaft for a single acting four stroke, single cylinder engine for the following data: Piston diameter = 200 mm; stroke = 400mm, Maximum combustion pressure = 2.0 N/mm^2, Weight of the flywheel = 15 kN, Total belt pull = 3 N, Length of connecting rod = 900 mm, when the crank has turned through 30^o from top dead center, the pressure on the piston is 1 N/mm^2 and the torque on the crank is maximum. Any other data required for the design may be assumed. **(May/June 2012)**

14. Design a muff coupling to connect two shafts transmitting 40 kW at 120 rpm. The permissible shear and crushing stress for the shaft and key material are 30 MPa and 80MPa respectively. The material of muff is cast iron with permissible shear stress of 15 MPa. Assume that the maximum torque transmitted is 25% greater than the mean torque.

(May/June 2012)