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# DEPARTMENT OF MECHANICAL ENGINEERING III YEAR MECHANICAL - VI SEMESTER ME 6601 – DESIGN OF TRANSMISSION SYSTEMS

# **EVEN SEMESTER**

# <u>UNIT - IV</u> <u>STUDY NOTES</u>

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### UNIT IV

### **DESIGN OF GEAR BOXES**

Geometric progression - Standard step ratio - Ray diagram, kinematics layout -Design of sliding mesh gear box -Constant mesh gear box. – Design of multi speed gear box.

### 1. Requirements of a speed gear boxes

A speed gear box should have the following requirements:

- > It should provide the designed series of spindle speeds
- > It should transmit the required amount of power to the spindle
- > It should provide smooth silent operation of the transmission
- ➢ It should have simple construction
- Mechanism of speed gear boxes should be easily accessible so that it is easier to carry out preventive maintenance

### 2. The speeds in machine tool gear boxes are in geometric progression. Why?

The speeds in gear boxes can be arranged in arithmetic progression (A.P.), geometric progression (G.P.), harmonic progression (H.P), and logarithmic progression (L.P.). However, when the speeds are arranged in G.P., it has the following advantages over the other progressions.

- 1. The speed loss is minimum i.e., Speed loss = Desired optimum speed – Available speed
- 2. The number of gears to be employed is minimum
- 3. G.P. provides a more even range of spindle speeds at each step
- 4. The layout is comparatively very compact
- 5. Productivity of a machining operation, i.e., surface area of the metal removed in unit time, is constant in the whole speed range
- 6. G.P. machine tool spindle speeds can be selected easily from preferred numbers. Because preferred numbers are in geometric progression.

### 3. Methods for changing speed in gear boxes

The two important methods widely used are:

- 1. Sliding mesh gear box, and
- 2. Constant mesh gear box

### 4. Preferred Numbers

Preferred numbers are the conventionally rounded off values derived from geometric series. There are five basic series, denoted as R 5, R 10, R 20, R 40 and R 80 series. Each series has its own step ratio i.e., series factor. The series factor for various series are given in table.

Basic series	Step ratio $(\phi)$
R 5	$\sqrt[5]{10} = 1.58$
R 10	$\sqrt[10]{10} = 1.26$
R 20	$\sqrt[20]{10} = 1.12$
R40	$\sqrt[40]{10} = 1.06$
R 80	<sup>80</sup> √1.03

The series of preferred numbers is obtained by multiplying a step ratio with the first number to get the second number. The third number is obtained by multiplying a step ratio with the second number. Similarly the procedure is continued until the series is completed.

### 5. Step ratio (or series ratio or progression ratio) $(\phi)$

When the spindle speeds are arranged in geometric progression, then the ratio between the two adjacent speeds is known as step ratio or progression ratio. It is denoted by  $\phi$ .

If  $N_1$ ,  $N_2$ ,  $N_3$ , ...,  $N_n$  are the spindle speeds arranged in geometric progression, then

$$\frac{N_2}{N_1} = \frac{N_3}{N_2} = \frac{N_4}{N_3} = \dots \frac{N_n}{N_n - 1} = \text{constant} = \phi$$

If 'n' is the number of steps of speed, then

$$\frac{N_n}{N_1} = \phi^{n-1} or \left[ \frac{N_{\max}}{N_{\min}} = \phi^{n-1} \right] or \phi = \left[ \frac{N_{\max}}{N_{\min}} \right]^{\frac{1}{n-1}}$$

**Note:** Permissible deviation =  $\pm 10 (\phi - 1)\%$ 

No of speeds to skip =  $\frac{Natural \ Log \ of \ caluclated \ \phi}{Natural \ Log \ of \ s \tan dared \ \phi} = \frac{l_n(\phi)}{l_n(std(\phi))}$ 

Calculation of non standard speeds:

$$N_1 = N_{min}$$

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 $N_2 = N_{\min} x \phi^1$  $N_3 = N_{\min} x \phi^2$ ..

 $N_n = N_{\min} x \phi^{n-1}$ 

### <u>PART – A</u>

### 1. What is step ratio in a gear box?

When the spindle speeds are arranged in geometric progression, then the ratio between the two adjacent speeds is known as step ratio.

### 2. What does the ray-diagram indicate?

The ray diagram is a graphical representation of the drive arrangement in general form. It serves to determine the specific values of all the transmission ratios and speeds of all the shafts in the drive.

### **3.** Specify four types of gear boxes.

- i. Sliding mesh gear box
- ii. Constant mesh gear box
- iii. Synchromesh gear box
- iv. Planetary gear box

### 4. What are the possible arrangements to achieve 12 speeds from a gear box?

(A/M 2011)

The possible arrangements to achieve 12 speeds from a gear box are

- i.  $3 \times 2 \times 2$  scheme,
- ii.  $2 \times 3 \times 2$  scheme and
- iii.  $2 \times 3 \times 3$  scheme.

# 5. What are the points to be considered while designing a sliding-mesh type of multispeed gear box? (A/M 2010)

The basic rules to be followed while designing the gear boxes are as follows:

i. The transmission ratio (*i*) in a gear box is limited by

### (N/D 2014)

# (M/J 2012)

(M/J 2011)

$$\frac{1}{4} \leq i \leq 2$$

ii. For stable operation, the speed ratio at any stage not be greater than 8.

In other words,  $\frac{N_{max}}{N_{min}} \leq 8$ 

iii. In all stages except in the first stage,  $N_{max} \ge N_{input} > N_{min}$ .

iv. The sum of teeth of mating gears in a given stage must be the same for same module in a sliding gear set.

v. The minimum number of teeth on smallest gear in drives should be greater than or equal to 17.

6. List the ways by which the number of intermediate steps may be arranged in a gear box. (A/M 2010)

S.No.	Number of speeds		Preferred structural formula
1.	6 speeds	i.	3 (1) 2 (3)
		ii.	2 (1) 3 (2)
2.	8 speeds	i.	2 (1) 2 (2) 2 (4)
		ii.	4 (1) 2 (4)
3.	9 speeds	i.	3 (1) 3 (3)
4.	12 speeds	i.	3 (1) 2 (3) 2 (6)
		ii.	2 (1) 3 (2) 2 (6)
		iii.	2 (1) 2 (2) 3 (4)

### 7. Which type of gear is used in constant mesh gear box? Justify.

Helical gears are used in constant mesh type gear boxes tp provide quieter and smooth operation.

8. What is speed reducer?

Speed reducer is a gear mechanism with a constant speed ratio, to reduce the angular speed of output shaft as compared with that of input shaft.

9. What are preferred numbers?

Preferred number are the conventionally rounded off values derived from geometric series. There are five basis series, denoted as R 5, R 10, R 20, R 40 and R 80 series.

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(N/D 2009)

(N/D 2008)

(N/D 2014)

### 10. Name any two methods used for changing speeds in gear boxes. (M/J 2013)

The two methods used for changing speeds in gear boxes are

- i. Silding mesh gear box and
- ii. Constant mesh gear box.

# 11. What is step ratio? Name the series in which speeds of multi-speed gear box are arranged. (M/J 2014)

When the spindle speeds are arranged in geometric progression, then the ratio between the two adjacent speeds is known as step ratio or progression ratio.

R20 and R40 series are used in the design of multi-speed gear boxes.

### 12. Distinguish between structural diagram and speed diagram. (N/D 2011)

The structural diagram is a kinematic layout that shows the arrangement of gears in a gear box.

The speed diagram, also known as ray diagram, is a graphical representation of the structural formula.

### 13. What are the methods of lubrication in speed reducers?

- i. Splash or spray lubrication method
- ii. Pressure lubrication method

### 14. What is the function of spacers in a gear box?

The function of spacers is to provide the necessary distance between the gears and the bearings.

### 15. What are the possible arrangements to achieve 12 speeds from a gear box? (M/J 2013)

The possible arrangement are:

- i.  $3 \times 2 \times 2$  scheme
- ii.  $2 \times 3 \times 2$  scheme
- iii.  $2 \times 2 \times 3$  scheme

# **Problem.1:** Find the progression ratio for a 12 speed gear box having speeds between 100 and 3.55 r.p.m. Also find the spindle speeds.

Given data: n = 12;  $N_{min} = 100$  r.p.m.;  $N_{max} = 355$  r.p.m.

To find;

(**M/J 2011**)

(N/D 2011)

- 1. Progression ratio  $(\phi) = \left(\frac{N_{\text{max}}}{N_{\text{min}}}\right)^{\frac{1}{n-1}} = \left(\frac{355}{100}\right)^{\frac{1}{12-1}} = 1.122$
- 2. Spindle speeds: Since the calculated  $\phi$  (=1.12) is a standard step ratio for R 20 series. Therefore the spindle speed from R 20 series are 1 x 100 = 100, 1.12 x 100 = 112, 1.25 x 100 = 125, 1.40 x 100 = 140, 1.60 x 100 = 160, 1.80 x 100 = 180, 2.00 x 100 = 200, 2.24 x 100 = 224, 2.50 x 100 = 250, 2.80 x 100 = 280, 3.15 x 100 = 315 and 3.55 x 100 = 355 r.p.m.

Problem.2: Select spindle speeds, for 9 speed gear box, between 80 and 1285 r.p.m.

Given data: n = 9;  $N_{min} = 80$  r.p.m.;  $N_{max} = 1285$  r.p.m.

To find: Spindle speeds.

© Solution: We know that Progression ratio  $(\phi) = \left(\frac{N_{\text{max}}}{N_{\text{min}}}\right)^{\frac{1}{n-1}} = \left(\frac{1285}{80}\right)^{\frac{1}{9-1}} = 1.415$ 

No of speeds to skip = 
$$\frac{Natural \ Log \ of \ caluclated \ \phi}{Natural \ Log \ of \ s \tan dared \ \phi} = \frac{l_n(\phi)}{l_n(std(\phi))} = \frac{l_n(1.415)}{l_n(1.12)} \approx 2$$

No. of speeds = from R 20 series  $N_1 = 8 \times 10 = 80$ , skip 9,10 and 1 from R 20 series calculate  $N_2 = 1.12 \times 100 = 112$ , skip next two speeds that is 1.25 and 1.40 and the  $N_3 = 1.6 \times 100 = 160$  and so on.

Problem.3: Selest the spindle speeds, 50 – 800 r.p.m., 12 speeds.

**Given Data:** N<sub>min</sub> = 50 r.p.m.; N<sub>max</sub> = 800 r.p.m.; n = 12.

*To find*: Spindle speeds.

$$\odot$$
 Solution: We know that Progression ratio  $(\phi) = \left(\frac{N_{\text{max}}}{N_{\text{min}}}\right)^{\frac{1}{n-1}} = \left(\frac{800}{50}\right)^{\frac{1}{12-1}} = 1.2866$ 

We find  $\phi = 1.2866$  is not a standard ratio. So let us find out whether multiples of standard ratio 1.12 or 1.06 come close to 1.286.

### Calculation of non standard speeds:

$N_1 = N_{min}$	$N_1 = 50$
$N_2 = N_{min} x \phi^1$	$N_2 = 50 \times 1.2866^1 = 64.3$
$N_3 = N_{min} x \phi^2$	$N_3 = 50 \text{ x } 1.2866^2 = 82.7$
$N_n = N_{min} x \phi^{n-1}$	$N_{12} = 50 \text{ x } 1.2866^{11} = 795.5 \text{ r.p.m.}$

#### **STRUCTURAL FORMULA:**

Let

n = Number of speeds available at the spindle.

 $p_1, p_2, p_3 \dots =$ Stage numbers in the gear box, and

 $X_1, X_2, X_3 \dots$  = Characteristic of the stage.

Then, the structural formula is given as

 $N= \quad p_1\left(X_1\right) \qquad . \qquad p_2\left(X_2\right) \qquad . \qquad p_3\left(X_3\right) \qquad . \qquad p_4\left(X_4\right)$ 

•••

1 <sup>st</sup> stage	2 <sup>nd</sup> stage	3 <sup>rd</sup> stage	$4^{\text{th}}$
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stage

Where  $X_1 = 1$ ;  $X_2 = p_1$ ;  $X_3 = p_1 x p_2$ ;  $X_4 = p_1 x p_2 x p_3$ 

### **Preferred structural formulas:**

- 1. Six speed gear box
  - i) Divide the given six speed by  $2 \ 2\overline{)6} = 3$ Here  $p_1 = 2, p_2 = 3$

 $X_1 = 1$ ,  $X_2 = p_1$  then structural formula is 2 (1). 3(2)

ii) Divide the given six speed by  $3 \ 3\overline{)6} = 2$ Here

$$p_1 = 3, p_2 = 2$$
  
 $X_1 = 1, X_2 = p_1$  then structural formula is 3 (1) . 2 (3)

2. Eights speed gear box

i) Divide the given eight speed by 2 we get 2 x 2 x 2 = 8 Here

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p_1 = 2, X_1 = 1
p_2 = 2, X_2 = 2
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 $p_3 = 2 X_3 = 2 x 2 = 4$  then structural formula is 2 (1) . 2 (2) . 2 (4)

### 3. Twelve speed gear box

- i) Divide the given Twelve speed by 2 and 3 we get  $2 \times 2 \times 3 = 12$
- ii) Divide the given Twelve speed by 2 and 3 we get  $3 \times 2 \times 2 = 12$
- iii) Divide the given Twelve speed by 2 and 3 we get  $2 \times 3 \times 2 = 12$ Here

Structural formula is i)  $2(1) \cdot 2(2) \cdot 3(4)$ 

ii)	3 (1) . 2 (3) . 2 (6)
iii)	2 (1) . 3 (2) . 2 (6)

### KINEMATIC LAYOUT (OR KINEMATIC ARRANGEMENT)

The kinematic arrangement of a multi – speed gear box is shown in figure.



From the figure, it is clear that the kinematic layout shows the arrangement of gears in a gear box. The kinematic layout provides the following informations required for gear box design.

• The number of speeds available at the spindle, *i.e.*, at the driven shaft.

- The number of stages used to achieve the required spindle speeds.
- The number of simple gear trains required to obtain the required spindle speeds and their arrangement.
- The overall working principle of the gear box.
- The information required for structural formula and ray diagram.

**Illustration:** in figure, the power is transmitted from driving shaft to driven shaft through a intermediate shaft. In this conventional gear box, speed changing is obtained using sliding gear mechanism. It can be seen that the number of speeds from driving shaft to intermediate shaft is 3, and that from intermediate shaft is 3. Then the number of spindle speeds is equal to  $3 \ge 3 = 9$ .

The structural formula for the kinematic arrangement of gear box shown in figure, is given by

 $n = p_1 (X_1) \cdot p_2 (X_2)$ Where  $p_1 = 3 (i.e., \text{ in stage 1, there are 3 speeds available})$   $p_2 = 3 (i.e., \text{ in stage 2, there are 3 speeds available})$   $X_1 = 1; \text{ and } X_2 = p_1 = 3$   $\therefore \text{ structural formula, } z = 3 (1) \cdot 3 (3).$ Where n = Number of speeds available at the driven shaft  $= p_1 \cdot p_2 = 3 \times 3 = 9$ 

### Calculation of No. of shaft to draw kinetic arrangement:

No. of shafts = No. of stages + 1 that is

If 3 stages are there in gear box design, then No. of shaft will be (3+1 = 4 shafts)

### Calculation of No. of gears to draw kinematic arrangement:

Shaft  $1 = p_1$  gears

Shaft  $2 = p_1$  and  $p_2$  gears

Shaft  $3 = p_2$  and  $p_3$  gears

Shaft  $n = p_n$  and  $p_{n+1}$  gears

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## Example: Draw kinematic arrangement for 12 speed gear box

**Case.1**  $z = 2(1) \cdot 2(2) \cdot 3(4)$ 

Calculation of No. of shaft to draw kinetic arrangement:

No. of shafts = No. stages + 1

$$= 3 + 1 = 4$$

Calculation of No. of gears to draw kinematic arrangement:

Shaft $1 = p_1$ gears	shaft 1	= 2 gears
Shaft $2 = p_1 + p_2$ gears	shaft $2 = 2 + 2$	2 = 4 gears
Shaft $3 = p_2 + p_3$ gears	shaft $3 = 2 + 3$	3 = 5 gears

Shaft  $n = p_n$ 



Total No. of gear = 14 gears



**Case.2**  $z = 2(1) \cdot 3(2) \cdot 2(6)$ 

Calculation of No. of shaft to draw kinetic arrangement:

No. of shafts = No. stages + 1  
= 
$$3 + 1 = 4$$

Calculation of No. of gears to draw kinematic arrangement:

Shaft $1 = p_1$ gears	shaft 1	= 2 gears
Shaft $2 = p_1 + p_2$ gears	shaft $2 = 2$	+3 = 5 gears
Shaft $3 = p_2 + p_3$ gears	shaft $3 = 3$	+2=5 gears





**Case.3**  $z = 3(1) \cdot 2(3) \cdot 2(6)$ 

Calculation of No. of shaft to draw kinetic arrangement:

No. of shafts = No. stages + 1

$$= 3 + 1 = 4$$

Calculation of No. of gears to draw kinematic arrangement:

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Shaft $1 = p_1$ gears	shaft 1	= 3 gears
Shaft $2 = p_1 + p_2$ gear	s shaft $2 = 3$	+2=5 gears
Shaft $3 = p_2 + p_3$ gean	shaft $3 = 2$	+2=4 gears
Shaft $n = p_n$	shaft4	= 2 gears
	Total No. of gear	= 14 gears
_h	_	



### **RAY DIAGRAM (OR SPEED DIAGRAM)**

The ray diagram is graphical representation of the drive arrangement in general from. In other words, the ray diagram is a graphical representation of the structural formula, as shown in figure. P1(X1) P2(X2)



It provides the following data on the drive:

- The number of stages (a stage is a set of gear trains arranged on two consecutive shafts)
- The number of speeds in each stage.
- The order of kinematic arrangement of the stages.
- The specific values of all the transmission ratios in the drive
- The total number of speeds available at the spindle.

### Procedure

- In this diagram, shafts are shown by vertical equidistant and parallel lines.
- The speeds are plotted vertical on a logarithmic scale with log  $\phi$  as a unit.
- Transmission engaged at definite speeds of the driving and driven shafts are shown on the diagram by rays connecting the points on the shaft lines representing these speeds.
- Figure shows the ray diagram for a 9 speed gear box, having the structural formula,  $z = 3(1) \cdot 3(3)$

### BASIC RULES FOR OPTIMUM GEAR BOX DESIGN

The basic rules to be followed while designing the gear boxes are as follows:

1. The transmission ratio (i) in a gear box is limited by



2. For stable operation, the speed ratio at any stage should not be greater than 8. In other words,  $\frac{N_{\text{max}}}{N} \le 8...$ 

$$\frac{1}{N_{\min}} \leq 0$$

- 3. In all stages except in the first stage,  $N_{max} \ge N_{input} \ge N_{min}$
- 4. The sum of teeth of mating gears in a given stage must be the same for same module in a sliding gear set.
- 5. The minimum number of teeth on smallest gear in drives should be greater than or equal to 17.
- 6. The minimum difference between the number of teeth of adjacent gears must be 4.

7. Gear box should be of minimum possible size. Both radial as well as axial dimensions should be as small as possible.

**Problem 4**: A gear box is to be designed to provide 12 output speeds ranging from 160 to 2000 r.p.m. The input speed of motor is 1600 r.p.m. Choosing a standard speed ratio, construct the speed diagram and the kinematic arrangement.

Given data: n = 12;  $N_{min} = 160$  r.p.m.;  $N_{max} = 2000$  r.p.m.;  $N_{input} = 1600$  r.p.m.

To find: Construction of the speed diagram and the kinematic arrangement.

### $\odot$ Solution:

Selection of spindle speeds:

We know that,  $\frac{N_{\text{max}}}{N_{\text{min}}} = \phi^{n-1}$ 

or

$$\frac{2000}{160} = \phi^{12-1} \text{ or } \phi = 1.258$$

We can write, 1.12 x 1.12 = 1.254 ...

So  $\phi = 1.12$  satisfies the requirement. Therefore the spindle speeds from R 20 series, skipping one speed, are given by

160, 200, 250, 315, 400, 500, 630, 800, 1000, 1250, 1600 and 2000 r.p.m.

Structural formula: For 12 speeds, the preferred structural formula

= 3 (1)	2 (3)	2 (6)
1 <sup>st</sup> stage	2 <sup>nd</sup> stage	3 <sup>rd</sup> stage

Speed diagram (or Ray diagram):

**Procedure:** 

- Since there are 4shafts, draw 4 vertical equidistant lines to represent shafts.
- Since there are 12 spindle speeds, draw 12 horizontal equidistant lines.
- From the structural formula, it is clear that there are stages. In the third stage, i.e., in 2 (6), 2 represents the number of speeds available in that stage and (6) represents the steps or intervals between these two speeds.
- Locate the first point A on the lowest speed i.e., at 160 r.p.m. on the last shaft. After 6 steps above, locate the second point B at 630 r.p.m. These are the two output speeds.
- Locate the input speed at any point on the preceding shaft (i.e., shaft 2), meeting the ratio requirements. We find, the input speed 400 r.p.m. at point C satisfies the ratio requirements.

- In the second stage, there are two speeds. Lowest is at C, which is already located. Now locate point D on the 3<sup>rd</sup> shaft, above point C, in a three step interval. For these two output speeds in the second stage, the input should be from shaft 2. We find, the input speed 630 r.p.m. at point E on shaft 2 satisfies the ratio requirements.
- In the first stage, there speeds. Lowest speed is at E, which is already located. Now locate points F and G on the shaft 2, above point E, in a single step interval.
- Input speed can be located anywhere on shaft 1 meeting the ratio requirements. But in this problem, given that, input speed is at 1600 r.p.m.
- In stage 2, we find input speed at E gives two output speeds at C and D. Similarly, input speeds at F and G, should give two output speeds. This can be achieved by drawing lines parallel to EC and ED, from points F and G, as shown in figure
- Now for stage 3, to get the output speeds to all the input speeds in shaft 3, draw lines parallel to CA and CB. Thus we have located all the input and the output speeds. The completed ray diagram is shown in figure



:. Ratio requirements are satisfied.

*Kinematic arrangement:* The kinematic arrangement for 12 speed gear box is drawn, as shown in figure.



### <u>PART - B</u>

- Design a 9 speed gear box to give output speeds between 280 and 1800 rpm. The input power is 5.5 kW at 1440 rpm. Draw the kinematic layout diagram and the speed diagram. Determine the number of teeth on all gears. (M/J 2013)
- Design the layout of a 12 speed gear box for a lathe. The minimum and maximum speeds are 100 and 1200 rpm. Power is 5 kW from 1440 rpm. Draw the speed and kinematic diagram. Also calculate the number of teeth on all gears. (M/J 2013)
- Design a sliding mesh nine speed gear box for a machine tool with speed ranging from 36 rpm to 550 rpm. Draw the speed diagram and kinematic arrangement showing number of teeth in all gears. (M/J 2012)
- 4. An all geared headstock of a lathe requires a 12 speed gear box with minimum and maximum speeds of 110 rpm and 1440 rpm respectively. Draw speed diagram and show the details of number of teeth in all the gears in a kinematic layout. (M/J 2012)
- 5. A gear box is to be designed with the following specification:
  Power = 14.72 kW. Number of speeds = 18. Minimum speed = 16 rpm. Step ratio = 1.25. Motor speed = 1400 rpm. The 18 speeds are obtained as 2×3×3.
  - (i) Sketch the layout of the gear box.
  - (ii) Draw the speed diagram.

(N/D 2011)

- 6. The minimum and maximum speed of a six speed box are to be 160 and 500 rpm. Construct the kinematic arrangement and the ray diagram of the gear box. Also find the number of teeth on all gears.
   (A/M 2011)
- 7. Design a 12 speed gear box for an all geared headshock of a lathe. Maximum and minimum speeds are 600 rpm and 25 rpm respectively. The drive is from an electric motor giving 2.25 kW at 1440 rpm. (A/M 2011)

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