DHANALAKSHMI COLLEGE OF ENGINEERING

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EVEN SEMESTER

UNIT - III
STUDY NOTES

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UNIT III

BEVEL, WORM AND CROSS HELICAL GEARS

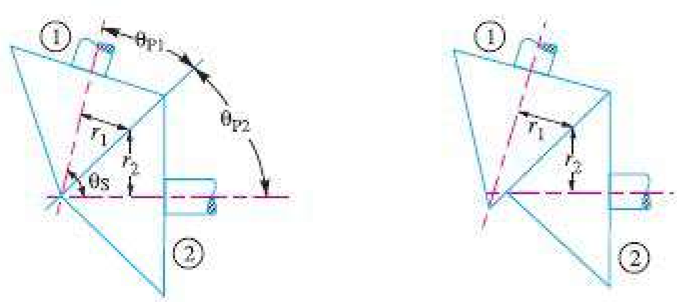
Straight bevel gear: Tooth terminology, tooth forces and stresses, equivalent number of teeth. Estimating the dimensions of pair of straight bevel gears. Worm Gear: Merits and demerits-terminology. Thermal capacity, materials-forces and stresses, efficiency, estimating the size of the worm gear pair.

Cross helical: Terminology-helix angles-Estimating the size of the pair of cross helical gears.

Bevel Gears

Introduction

The bevel gears are used for transmitting power at a constant velocity ratio between two shafts whose axes intersect at a certain angle. The pitch surfaces for the bevel gear are frustums of cones. The two pairs of cones in contact is shown in Fig. The elements of the cones, as shown in Fig (a). intersect at the point of intersection of the axis of rotation. Since the radii of both the gears are proportional to their distances from the apex, therefore the cones may roll together without sliding. In Fig(b). the elements of both cones do not intersect at the point of shaft intersection. Consequently, there may be pure rolling at only one point of contact and there must be tangential sliding at all other points of contact. Therefore, these cones cannot be used as pitch surfaces because it is impossible to have positive driving and sliding in the same direction at the same time. We, thus, conclude that the elements of bevel gear pitch cones and shaft axes must intersect at the same point.



(a) (b)

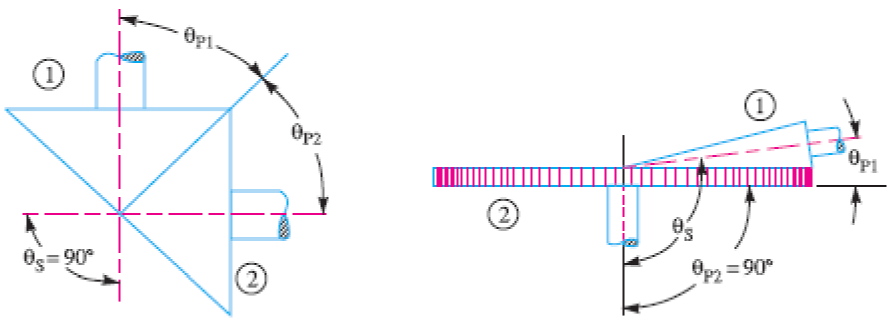
Classification of Bevel Gears

The bevel gears may be classified into the following types, depending upon the angles between the shafts and the pitch surfaces.

1**. Mitre gears**. When equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle, as shown in Fig. (*a*), then they are known as *mitre gears.*

2. **Angular bevel gears**. When the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as *angular bevel gears.*

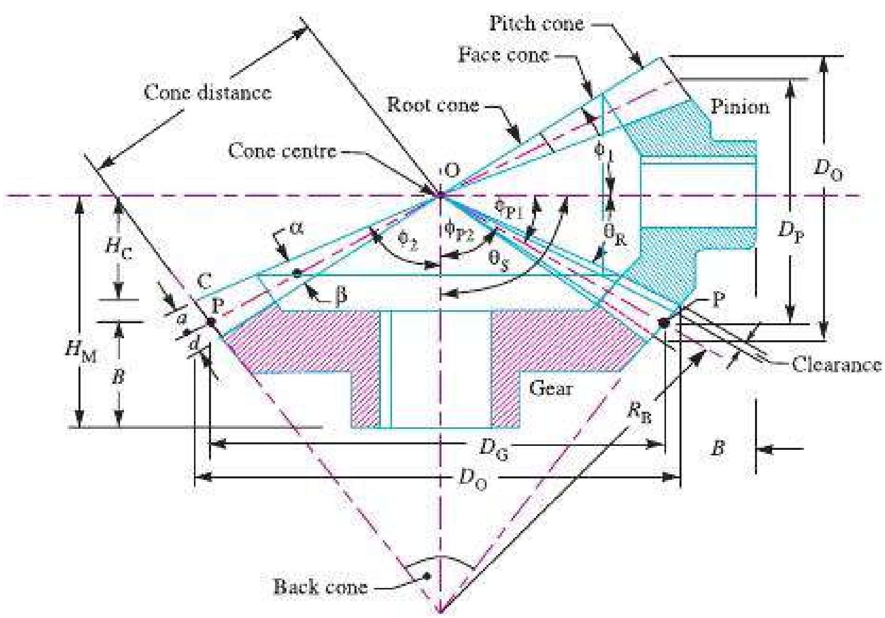
3. **Crown bevel gears**. When the bevel gears connect two shafts whose axes intersect at an angle greater than a right angle and one of the bevel gears has a pitch angle of 90º, then it is known as a crown gear. The crown gear corresponds to a rack in spur gearing, as shown in Fig. (*b*).



(*a*) Mitre gears. (*b*) Crown bevel gear.

4. Internal bevel gears. When the teeth on the bevel gear are cut on the inside of the pitch cone, then they are known as ***internal bevel gears.***

Terms used in Bevel Gear



A sectional view of two bevel gears in mesh is shown in Fig. The following terms in connection with bevel gears are important from the subject point of view:

1. *Pitch cone.* It is a cone containing the pitch elements of the teeth.

2. *Cone centre.* It is the apex of the pitch cone. It may be defined as that point where the axes of two mating gears intersect each other.

3. *Pitch angle.* It is the angle made by the pitch line with the axis of the shaft. It is denoted by 'θ_P'.

4. *Cone distance.* It is the length of the pitch cone element. It is also called as a ***pitch cone radius***. It is denoted by '*OP*'. Mathematically, cone distance or pitch cone radius,

$$OP = \frac{\text{Pitch radius}}{\sin \theta_P} = \frac{D_P/2}{\sin \theta_{P1}} = \frac{D_G/2}{\sin \theta_{P2}}$$

5. *Addendum angle.* It is the angle subtended by the addendum of the tooth at the cone centre. It is denoted by 'α' Mathematically, addendum angle,

$$\alpha = \tan^{-1}\left(\frac{a}{OP}\right)$$

where a = Addendum, and OP = Cone distance.

6. *Dedendum angle.* It is the angle subtended by the dedendum of the tooth at the cone centre. It is denoted by 'β'. Mathematically, dedendum angle,

$$\beta = \tan^{-1}\left(\frac{d}{OP}\right)$$

where d = Dedendum, and OP = Cone distance.

7. *Face angle.* It is the angle subtended by the face of the tooth at the cone centre. It is denoted by 'ϕ'. The face angle is equal to the pitch angle *plus* addendum angle.

8. *Root angle.* It is the angle subtended by the root of the tooth at the cone centre. It is denoted by 'θ_R'. It is equal to the pitch angle *minus* dedendum angle.

9. *Back (or normal) cone.* It is an imaginary cone, perpendicular to the pitch cone at the end of the tooth.

10. Back cone distance. It is the length of the back cone. It is denoted by 'R_B'. It is also called back cone radius.

11. *Backing.* It is the distance of the pitch point (*P*) from the back of the boss, parallel to the pitch point of the gear. It is denoted by '*B*'.

12. *Crown height.* It is the distance of the crown point (*C*) from the cone centre (*O*), parallel to the axis of the gear. It is denoted by 'H_C'.

13. *Mounting height.* It is the distance of the back of the boss from the cone centre. It is denoted by 'H_M'.

14. *Pitch diameter.* It is the diameter of the largest pitch circle.

15. *Outside or addendum cone diameter.* It is the maximum diameter of the teeth of the gear. It is equal to the diameter of the blank from which the gear can be cut. Mathematically, outside diameter,

$$D_O = D_P + 2\,a\cos\theta_P$$

where D_P = Pitch circle diameter,

a = Addendum, and

θ_P = Pitch angle.

16. *Inside or dedendum cone diameter.* The inside or the dedendum cone diameter is given by

$$D_d = D_P - 2d\cos\theta_P$$

where D_d = Inside diameter, and

d = Dedendum.

Proportions for Bevel Gear

The proportions for the bevel gears may be taken as follow :

1. Addendum, $a = 1\,m$

2. Dedendum d = 1.2 m

3. Clearance =0.2 m

4.working depth =2 m

5. Thickness of the tooth = 1.5708 m ……where m is the module.

PART – A

1. What is reference angle? How is it related to speed ratio of bevel gear ratio? **(M/J 2012)**

The pitch angle of the bevel gear is known as reference angle.

Speed ratio, i = tan δ_2. $\delta_1 = 90° - \delta_2$ for speed ratios as high as 300:1.

2. What is the effect of increasing the pressure angle in gears? **(N/D 2011)**

The increase of the pressure angle results in a stronger tooth, because the tooth acting as a beam is wider at the base.

3. What is working depth of a gear-tooth? **(M/J 2011)**

Working depth is the radial distance from the addendum circle to the clearance circle. It is equal to the some of the addendum of the two meshing gears.

4. Name few gear materials. **(M/J 2011)**

The materials used in the gear are

i. Steel,

ii. Cast iron and

iii. Bronze.

5. Mention the characteristics of hypoid gear. **(A/M 2010)**

Hypoid gears are similar in appearance to spiral-bevel gears. They differ from spiral gears in that the axis of pinion is offset from the axis of gear. The other difference is that their pitch surfaces are hyperboloids rather than cones.

In general, hypoid gears are most desirable for those application involving large speed reduction ratios. They operate more smoothly and quietly than spiral bevel gears.

6. What are the various forces acting on a bevel gear? **(N/D 2009)**

The various forces acting on bevel gears are

1. Tangential or useful component (F_t), and
2. Separating forces (F_s): It is resolved into two components. They are:
   i. Axial force (F_a), and

ii. Radial force (F*r*).

7. Calculate the angle between the shafts of a crossed helical gears made of two right handed helical gears of 15º helix angle each. (M/J 2009)

Shaft angle, $\theta = \beta_1 + \beta_2 = 2\beta = 2\,(15°) = 30°$

8. When is bevel gear preferred? (M/J 2009)

Bevel gears are used to transmit power between two intersecting shafts.

9. How can you specify a pair of worm gears? (A/M 2008)

A pair of worm gears is specified as: (z_1/ z_2 / q / m_x)

Where z_1=Number of starts on the worm,

z_2 = Number of teeth on the worm wheel,

q = Diameter factor = d_1/ m_x, and

m_x = Axial module.

10. Give the advantage of worm gear drive in weight lifting machines. (A/M 2008)

The worm gear drives are irreversible. It means that the motion cannot be transmitted from worm wheel to the worm. This property of irreversible is advantageous in load hoisting application like cranes and lifts.

11. State the advantages of herringbone gear. (A/M 2015)

Herringbone gears eliminate the existence of axial thrust load in the helical gears. Because, in herringbone gears, the thrust force of the right hand is balanced by that of the left hand helix.

12. What is a zerol bevel gear? (A/M 2015)

Spiral bevel gear with curved teeth but with a zero degree spiral angle is known as zerol bevel gear.

13. What is virtual number of teeth in bevel gears? (M/J,N/D 2014)

An imaginary spur gear considered in a plane perpendicular to the tooth of the bevel gear at the larger end is known as virtual spur gear.

The number of teeth z_v on this imaginary spur gear is called virtual number of teeth in bevel gear.

$z_v = z/\cos\delta$

where z = actual number of teeth on the bevel gear and δ = pitch angle.

14. Define the following terms: a. Cone distance, b. Face angle. (M/J 2014)

Back cone distance is the length of the back cone. Back cone is an imaginary cone, perpendicular to the pitch cone at the end of the tooth.

Face angle is the angle subtended by the face of the tooth at the cone centre.

15. What is the difference between an angular gear and a miter gear? (N/D 2013)

When the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as angular bevel gears.

When equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle, then they are known as mitre gears.

BEVEL GEAR DESIGN BASED ON GEAR LIFE

(Bevel Gear Design using Basic Equations)

1. Calculation of gear ratio (i):

Calculation of gear ratio and pitch angle

2. Selection of materials:

From PSGDB – 1.40 or 1.9, knowing the gear ratio i, choose the suitable combination of materials for pinion and wheel.

3. If not given, assume gear life (say 20,000 hrs)

4. Calculation of initial design torque (M_t):

$(M_t) = M_t \times K \times K_d$

Initially assume $K \times K_d = 1.3$

(M_t) = Transmitted torque = $\dfrac{60 \times P}{2\pi N}$

K = Load concentration factor, from PSGDB – 8.15

K_O = Dynamic load factor, from PSGDB – 8.16

Since datas are inadequate to select the values of K and K_d, initially assume $K \times K_d = 1.3$

5. **Calculation of E_{eq}, (σ_b) and (σ_c):**

   - From PSGDB – 8.14, calculate the equivalent Young's modulus (Eeq)
   - To Find (σ_b): calculate the designing bending stress $[\sigma_b]$

     $[\sigma b] = \frac{1.4 \times K_{bl}}{n \times K_\sigma}$ x σ_{-1} for rotation in one direction only.

- Where
- $K_{bl} = \sqrt[9]{\frac{10^7}{N}}$, for C.I,

  from PSGDB page no. 8.20

- n value taken from PSGDB page number 8.19 according to the material
- K_σ value taken from PSGDB page no. 8.19, according to the material and
- $\sigma_{-1} = 0.45\ \sigma_u$
- But σ_u value taken from PSGDB 1.40 according to the material
- **To find $[\sigma_c]$**: Calculate the design contact stress $[\sigma_c]$ is given by $[\sigma_c] = C_B$ x HB x K_{cl}
- C_R value, from PSGDB page no. 8.16
- HB , from PSGDB page no. 8.16
- $K_{cl} = \sqrt[6]{\frac{10^7}{N}}$ value taken from PSG DB page no. 8.17 according to the material

6. Calculation of cone distance (R):

Calculate the cone distance

$$R \geq \psi_y \sqrt{i^2+1} \sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$$

Where $\psi_y = R / b = 3$ initially assumed

7. Selection of number of teeth:

- **Number of teeth on pinion, z_1:**

  Assume $z_1 \geq 17$, say 18.
- **Number of teeth on gear, z_2:**

  $z_2 = i \times z_1$
- **Virtual number of teeth : Z_{v1} =** $\frac{Z_1}{\cos\delta_1}$

Z_{v2} = $\frac{Z_2}{\cos\delta_2}$

8. Calculate the transverse module (m_t):

- We know that,

$$m_t = \frac{R}{0.5\sqrt{z_1{}^2 + z_2{}^2}}$$

- From PSGDB page no. 8.2 , the nearest higher standard transverse module,

9. Revise the cone distance (R) using the relation

$$R = 0.5\, m_t \sqrt{z_1{}^2 + z_2{}^2}$$

10. Calculation of b, m_{av}, $d_{1\,av}$, v and ψ_y:

- Face width (b) : $b = \frac{R}{\psi_y}$
- Average module (m_{av}) : $m_{av} = m_t - \frac{b \sin\delta_1}{z_1}$
- Average pcd of pinion (d_{1av}) : $d_{1\,av} = m_{av} \times z_1$
- Pitch line velocity (v): $v = \frac{\pi\, d_{1av}\, N_1}{60}$
- $\psi_y = \frac{b}{d_{1\,av}}$

11. **Selection of quality of gears:**

From PSGDB page no. 8.3, IS quality 6 bevel gear is assumed.

12. **Revision of design torque (Mt):**

We know that $(M_t) = Mt \times K \times K_d$

- Revise K using ψ_y from PSGDB 8.15, and
- Revise K_d, from PSGDB 8.16
- Revise $[M_t]$, using the revised values of K and K_d.

13. **Check for bending:**

Calculate the induced bending stress using the equation

$$\sigma_b = \frac{R\sqrt{i^2+1}\,[M_t]}{(R-0.5\,b)^2 \times b \times M_t \times y_{v1}}$$

Compare the induced bending stress σ_b and the design bending stress (σ_b). For the value of (σ_b),refer step 5. If $\sigma_b \leq (\sigma_b)$, then the design is satisfactory.

14. If $\sigma_b > [\sigma_b]$, then the design is not satisfactory. Then increase the transverse module or face width, or change the gear material. The above procedure is repeated until the design is satisfactory. i.e., $\sigma_b \leq [\sigma_b]$.

15. Check for wear strength:

- Calculation of induced contact stress, σ_c:

$$\sigma_c = \frac{0.74}{(R-0.5\,b)} \left(\frac{\sqrt{\left(i^2+1\right)^3}}{i \times b} \times E_{eq}\,[M_t]\right)^{\frac{1}{2}}$$

Then we find $(\sigma_c < [\sigma_c])$. Therefore the design is safe and satisfactory.

16. **Calculation of basic dimensions of the gear pair :**

Calculate all the basic dimensions of the pinion and gear using the equations listed in PSGDB 8.38

PART B :-

1. Design a cast iron bevel gear drive for a pillar drilling machine to transmit 1875 W at 800 rpm to a spindle at 400 rpm. The gear is to work for 40hours per week for 3 years. Pressure angle is 20º.

Given Data:

P = 1875 W; N_1 =800 rpm; N_2 =400 rpm; α= 20º.

To find: Design the bevel gear drive.

Solution:

Since the pinion and gear are made of same material, therefore we have to do the design of pinion alone.

STEP 1: Gear Ratio:

Gear ratio: i = $\frac{N_1}{N_2} = \frac{800}{400} = 2$

Pitch angles: for right angel bevel gears, tan δ_2 = i = 2

Or $\delta_2 = \tan^{-1}(2)$ = 63.43º

and δ_1 = 90º - δ_2 = 90º - 63.43º = 26.57º

STEP 2: Material for pinion and gear: cast iron, grade 35 heat treated.

σ_u = 350 N/mm^2, from PSGDB - 1.5

STEP 3: Gear Life:

Gear Life in hours = (40 hrs / week) × (52 weeks / year x 3 years) = 6240 hours

Gear life in cycles, N = 6240 x 800 x 60 = 29.952 x 10^7 cycles

STEP 4: Calculation of initial design torque [Mt]:

We know that, Design torque, $[M_t] = M_t \times K \times K_d$

Where $M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 1875}{2 \times \pi \times 800}$= 22.38 N-m, and

$K \times K_d$ = 1.3 (initially assumed)

$[M_t] = 22.38 \times 1.3 = 29.095$ N-m

STEP 5: Calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:

(i) **To find E_{eq}** :$E_{eq} = 1.4 \times 10^5$ N/mm^2 for cast iron, $\sigma_c > 280$ N/mm^2, from PSDB 8.14, T-9

(ii) **To find $[\sigma_b]$**: The design bending stress $[\sigma_b]$ is given by

$[\sigma_b] = \frac{1.4 \times K_{bl}}{n \times K_\sigma} \times \sigma_{-1}$ for rotation in one direction only. From PSGDB 8.18,

Where $K_{bl} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{29.952 \times 10^7}} = 0.6854$, for C.I, from PSGDB - 8.20, T-22.

$n = 2$, for steel tempered, from PSGDB - 8.19, T-20

$K_\sigma = 1.2$, for steel, from PSGDB - 8.19, T-21, and

$\sigma_{-1} = 0.45\, \sigma_u$ From PSGDB – 8.19, T-19.

But $\sigma_u = 350$ N/mm^2, For C.I., from PSGDB - 1.40

$\sigma_{-1} = 0.45 \times 350$

$= 157.5$ N/mm^2

Then, $[\sigma_b] = \frac{1.4 \times 0.6854}{2 \times 1.2} \times 157.5$

$= 62.93$ N/mm^2

(iii) **To find $[\sigma_c]$**: The design contact stress $[\sigma_c]$ is given by

$[\sigma_c] = C_B \times HB \times K_{cl}$ From PSGDB 8.16

Where

$C_R = 2.3$, from PSGDB - 8.16, T-16

HB = 200 to 260, from PSGDB - 8.16, T-16

$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{29.952 \times 10^7}} = 0.567$, for C.I, from PSG DB - 8.17, T-17

$[\sigma_c] = 2.3 \times 260 \times 0.567 = 339.07$ N/mm^2

STEP 6: Calculation of Cone distance (R):

We know that, $R \geq \psi_y \sqrt{i^2+1} \sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$ From PSGDB 8.13, T-8

Where ψ_y = R / b = 3 initially assumed

$$R \geq 3\sqrt[1]{2^2+1} \sqrt[3]{\left[\frac{0.72}{(3-0.5)\times 339.07}\right]^2 \times \frac{1.4\times10^5 \times 29.095\times10^3}{2}}$$

$\geq$ 76.26 mm or R = 76 mm.

STEP 7: Selection of Z_1 and Z_2:

(i) Assume, $Z_1 = 20$
(ii) $Z_2 = i \times Z_1 = 2 \times 20 = 40$

Virtual number of teeth: $Z_{v1} = \frac{z_1}{\cos\delta_1} = \frac{20}{\cos 26.57°} \approx 23$; and

$$Z_{v2} = \frac{z_2}{\cos\delta_2} = \frac{40}{\cos 63.43°} \approx 90$$

STEP 8: Calculation of transverse module (m_t):

We know that, $m_t = \frac{R}{0.5\sqrt{z_1^2+z_2^2}} = \frac{68}{0.5\sqrt{20^2+40^2}}$ = 3.39 mm , From PSGDB 8.38, T-31

From PSGDB 8.2, the nearest higher standard transverse module,

m_t= 4 mm.

STEP 9: Revision of Cone distance: (R)

We know that, $R = 0.5\, m_t \sqrt{z_1^2 + z_2^2} = 0.5 \times 4\sqrt{20^2+40^2}$ = 89.44 mm

STEP 10: Calculation of b, m_{av}, d_{1av}, v and ψ_y:

- ✓ Face width (b) : $b = \frac{R}{\psi_y} = \frac{89.44}{3}$ = 29.81 mm
- ✓ Average module (m_{av}) : $m_{av} = m_t - \frac{b \sin\delta_1}{z_1} = 4 - \frac{29.81\times\sin 26.57°}{20}$ = 3.33 mm
- ✓ Average pcd of pinion (d_{1av}) : $d_{1av} = m_{av} \times z_1 = 3.33 \times 20$ =66.6 mm

- ✓ Pitch line velocity (v): v = $\frac{\pi\, d_{1\,av}\, N_1}{60}$ = $\frac{\pi \times 66.6 \times 10^{-3} \times 800}{60}$ = 2.789 m/s
- ✓ $\psi_y = \frac{b}{d_{1\,av}} = \frac{29.81}{66.6}$ = 0.447

STEP 11: Selection of quality of gear:

From PSGDB - 8.3, IS quality 8 bevel gear is assumed.

STEP 12: Revision of design torque [M$_t$]:

We know that, $[M_t] = M_t \times K \times K_d$

- ✓ Revise K : 1.1, for $b/d_{1\,av} \leq 1$, from PSGDB - 8.15, T-14, and
- ✓ Revise K_d: 1.45, for Is quality 8 and v upto 3 m/s, from PSGDB 8.16, T-15.
- ✓ Revise $[M_t]$: $[M_t]$ = 22.38 × 1.1 × 1.45 = 35.70 N-m

STEP 13: Checking for bending: We know that the induced bending stress,

- ✓ Calculation of induced bending stress, b :

$$\sigma_b = \frac{R\sqrt{i^2+1}\,[M_t]}{(R-0.5\,b)^2 \times b \times mt \times y_{v1}}$$

Where $y_{v1} \approx 0.408$, for z_{v1} = 23, PSGDB 8.18

$$\sigma_b = \frac{89.44\sqrt{2^2+1} \times 35.70 \times 10^3}{(89.44-0.5 \times 29.81)^2 \times 29.81 \times 4 \times 0.408}$$

= 26.40 N/mm^2

We find $\sigma_b < [\sigma_b]$, Thus the design is safe and satisfactory.

STEP 14: Check for wear strength:

- ✓ Calculation of induced contact stress, σ_c:

$$\sigma_c = \frac{0.74}{(R-0.5\,b)}\left(\frac{\sqrt{(i^2+1)^3}}{i \times b} \times E_{eq}\,[M_t]\right)^{\frac{1}{2}}$$

$$= \frac{0.74}{(89.44-0.5 \times 29.81)}\left[\frac{\sqrt{(2^2+1)^3}}{2 \times 29.81} \times 1.4 \times 10^5 \times 29.095 \times 10^3\right]^{\frac{1}{2}}$$

= 132.74 N/mm^2

- ✓ We find ($\sigma_c < [\sigma_c]$). Therefore the design is safe and satisfactory.

STEP 15: Calculation of basic dimensions of pinion and gear:

Refer PSGDB 8.38

- ✓ Transverse Module : $m_t = 4$ mm
- ✓ Number of teeth : $z_1 = 20$ and $z_2 = 40$.
- ✓ Pitch circle diameter : $d_1 = m_t z_1 = 4 \times 20 = 80$ mm; and $d_2 = m_t z_2 = 4 \times 40 = 160$ mm.
- ✓ Cone distance : R = 87.44 mm
- ✓ Face width : b = 29.81 mm
- ✓ Pitch angle : $\delta_1 = 26.57^o$ and $\delta_2 = 63.43^o$
- ✓ Tip diameter : $da_1 = (z_1 + 2\cos\delta_1)\, m_t = (20 + 2\cos 26.57^o)\, 4 = 87.15$ mm; and $da_2 = (z_2 + 2\cos\delta_2)\, m_t = (40 + 2\cos 63.43^o)\, 4 = 163.57$ mm.
- ✓ Height factor : $f_0 = 1$
- ✓ Clearance : c = 0.2
- ✓ addendum angle : $\tan\theta_{a1} = \tan\theta_{a2} = \frac{M_t \times f_0}{R} = \frac{4 \times 1}{89.44} = 0.0447$

  Or $\theta_{a1} = \theta_{a2} = 2.56^o$
- ✓ Dedendum angle : $\tan\theta_{f1} = \tan\theta_{f2} = \frac{M_t \times (f_0 + c)}{R} = \frac{4 \times (1+0.2)}{89.44} = 0.0537$

  Or $\theta_{f1} = \theta_{f2} = 3.07^o$
- ✓ Tip angle : $\delta_{a1} = \delta_1 + \theta_{a1} = 26.57^o + 2.56^o = 29.13^o$; and

  $\delta_{a2} = \delta_2 + \theta_{a2} = 63.43^o + 2.56^o = 65.99^o$
- ✓ Root angle : $\delta_{f1} = \delta_1 - \theta_{f1} = 26.57^o - 3.07^o = 23.5^o$; and

  $\delta_{f2} = \delta_2 - \theta_{f2} = 63.43^o - 3.07^o = 60.36^o$
- ✓ Virtual number of teeth: $z_{v1} = 23$; and $z_{v2} = 90$.

Both gears are made of same material. Hence design of pinion alone is sufficient. So, we need not to check for gear.

2. Design a straight bevel gear drive between two shafts at right angles to each other. Speed of the pinion shaft is 360 rpm and the speed of the gear wheel shafts is 120 rpm. Pinion is of steel and wheel of cast iron. Each gear is expected to work 2 hours/day for 10 years. The drive transmits 9.37 kW.

Given data: $\theta = 90^o$; $N_1 = 360$ rpm; $N_2 = 120$ rpm; P= 9.37 kW.

To find: Design the bevel gear

Solution: since the materials of pinion and gear are different, we have to **design the pinion first and check the gear.**

STEP: 1

Gear ratio: $i = \frac{N_1}{N_2} = \frac{360}{120} = 3$

Pitch angles: tan δ_2 = i = 3 or $\delta_2 = \tan^{-1}(3) = 71.56^\circ$

Then, $\delta_1 = 90^\circ - \delta_2 = 90^\circ - 71.56^\circ = 18.44^\circ$

STEP: 2

Material selection: pinion – C45 steel, $\sigma_u = 700$ N/mm^2 and $\sigma_y = 360$ N/mm^2 PSGDB 1.40 OR 1.9

Gear – CI grade 35, $\sigma_u = 350$ N/mm^2, from1.5

STEP: 3

Gear life:

Gear Life in hours = (2hrs / day) × (365 days / year × 10 years) = 7300 hours

Gear life in cycles, N = 7300 × 360 × 60 = 15.768 × 10^7 cycles

STEP: 4

Calculation of initial design torque [M_t]:

We know that, $[M_t] = M_t \times K \times K_d$

Where $M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 9.37 \times 10^3}{2 \times \pi \times 360}$= 248.5 N-m, and

$K \times K_d = 1.3$ (initially assumed)

$[M_t] = 248.5 \times 1.3 = 323.05$ N-m

STEP: 5

Calculation of E_{eq}, [σ_b] and [σ_c]:

To find E_{eq} : $E_{eq} = 2.15 \times 10^5$ N/mm^2, from PSDB 8.14, T-9

To find $[\sigma_b]$: The design bending stress $[\sigma_b]$ for pinion,

$$[\sigma_b] = \frac{1.4 \times K_{bl}}{n \times K_\sigma} \times \sigma_{-1} \text{ for rotation in one direction only.}$$

Where K_{bl} = 1, for HB $\lesssim$ 350 and N $\gtrsim 10^7$, from PSGDB 8.20, T-22

K_σ = 1.2, for steel pinion, from PSGDB - 8.19, T-21

n = 2.5, steel hardened, from PSGDB 8.19, T-20

σ_{-1} = 0.25(σ_u + σ_y) + 50, for forged steel, from PSGDB 8.19, T-19

= 0.25(700 + 300) + 50

= 315 N/mm^2

$$[\sigma_b] = \frac{1.4 \times 1}{2.5 \times 1.5} \times 315 = 117.6 \text{ N/mm}^2$$

To find $[\sigma_c]$: The design contact stress $[\sigma_c]$ for pinion,

$[\sigma_c]$ = C_R x HRC x K_{cl}

C_R = 23, from PSGDB 8.16, T-16

HB = 40 to 55, from PSGDB - 8.16, T-16

K_{cl} = 1, for steel pinion, for HB $\lesssim$ 350 and N $\gtrsim 10^7$, from PSGDB 8.17, T-17

$[\sigma_c]$ = 23 x 55 x 1 = 1265 N/mm^2

STEP: 6

Calculation of Cone distance (R):

We know that, $R \geq \psi_y \sqrt{i^2 + 1} \sqrt[3]{\left[\frac{0.72}{(\psi_y - 0.5)[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i}}$

Where, ψ_y = R / b = 3 initially assumed

$$R \geq 3\sqrt{3^2 + 1} \sqrt[3]{\left[\frac{0.72}{(3 - 0.5) \times 1265}\right]^2 \times \frac{2.15 \times 10^5 \times 323.05 \times 10^3}{3}}$$

$\geq$ 100.81

R = 100 mm.

STEP: 7

Selection of Z_1 and Z_2:

Assume, $Z_1 = 20$

$Z_2 = i \times Z_1 = 3 \times 20 = 60$

Virtual number of teeth : $Z_{v1} = \frac{z_1}{\cos \delta_1} = \frac{20}{\cos 18.44°} \approx 22$; and

$$Z_{v2} = \frac{z_2}{\cos \delta_2} = \frac{60}{\cos 71.56°} \approx 190$$

STEP: 8

We know that, $m_t = \frac{R}{0.5\sqrt{z_1^2 + z_2^2}} = \frac{100}{0.5\sqrt{20^2 + 60^2}} = 3.162$ mm

From PSGDB 8.2, the nearest higher standard transverse module, m_t= 4 mm.

STEP: 9

Revision of Cone distance: (R)

We know that, R = $0.5\, m_t\sqrt{z_1^2 + z_2^2} = 0.5 \times 4\sqrt{20^2 + 60^2}$ = 126.49 mm

STEP: 10

Calculation of b, m_{av}, d_{1av}, v and ψ_y:

- ✓ Face width (b) : $b = \frac{R}{\psi_y} = \frac{126.49}{3} = 42.16$ mm
- ✓ Average module (m_{av}) : $m_{av} = m_t - \frac{b \sin \delta_1}{z_1} = 4 - \frac{42.16 \times \sin 18.44°}{20} = 3.333$ mm
- ✓ Average pcd of pinion (d_{1av}) : $d_{1av} = m_{av} \times z_1 = 3.333 \times 20 = 66.66$ mm
- ✓ Pitch line velocity (v) : $v = \frac{\pi d_{1av} N_1}{60} = \frac{\pi \times 66.66 \times 10^{-3} \times 360}{60} = 1.256$ m/s
- ✓ $\psi_y = \frac{b}{d_{1\,av}} = \frac{42.16}{66.66} = 0.632$

STEP: 11

From PSGDB page no. 8.3, IS quality 6 bevel gear is assumed.

STEP: 12

Revision of design torque [M_t]:

We know that, [M_t] = $M_t \times K \times K_d$

- ✓ Revise K : 1.1, for $b/d_{1\,av} \leq 1$, from PSGDB - 8.15, T-14, and
- ✓ Revise K_d: 1.35, for Is quality 6 and v up to 6 m/s, from PSGDB 8.16, T-15
- ✓ Revise [M_t]: [M_t] = 248.6 × 1.1 × 1.35 = 369.171 N-m

STEP: 13

Checking for bending of pinion:

We know that the induced bending stress,

- ✓ Calculation of induced bending stress, :

$$\sigma_b = \frac{R\sqrt{i^2+1}\,[M_t]}{(R-0.5\,b)^2 \times b \times M_t \times y_{v1}}$$

Where $y_{v1} = 0.402$, for $z_{v1} = 22$, from PSGDB 8.18, T-18

$$\sigma_b = \frac{126.49\sqrt{3^2+1} \times 369.28 \times 10^3}{(126.49-0.5 \times 42.16)^2 \times 42.16 \times 4 \times 0.402}$$

$$= 196.09 \text{ N/mm}^2$$

We find $\sigma_b > [\sigma_b]$, Thus the design is not satisfactory.

Trial 2: Now we will try with increased transverse module 5 mm. repeating from step 9 again we get

$$R = 0.5 \times m_t \times \sqrt{{z_1}^2 + {z_2}^2} = 0.5 \times 5 \times \sqrt{20^2 + 60^2} = 158.11 \text{ mm}$$

$$b = \frac{R}{\psi_y} = \frac{158.11}{3} = 52.7 \text{ mm}$$

$$m_{av} = m_t - \frac{b \sin \delta_1}{z_1} = 5 - \frac{52.7 \times \sin 18.44^\circ}{20} = 4.166 \text{ mm}$$

$$d_{1av} = m_{av} \text{ x } z_1 = 4.166 \text{ x } 20 = 83.33 \text{ mm}$$

$$v = \frac{\pi d_{1av} N_1}{60} = \frac{\pi \times 83.33 \times 10^{-3} \times 360}{60} = 1.57 \text{ m/s}$$

$\psi_y = \frac{b}{d_{1\,av}} = \frac{52.7}{83.33} = 0.632$

Is quality 6 bevel gear is assumed.

K = 1.1, from PSGDB 8.15, T-14

K_d = 1.35, from PSGDB 8.16, T-15

$[M_t] = M_t \times K \times K_d = 248.6 \times 1.1 \times 1.35$

= 369.171 N-m

$$\sigma_b = \frac{158.11\sqrt{3^2+1}\times 369.171\times 10^3}{(158.11-0.5\times 52.7)^2\times 52.7\times 5\times 0.402}$$

= 100.4 N/ mm^2

Now we find $\sigma_b < [\sigma_b]$. Thus the design is satisfactory.

STEP: 14

Check for wearing of pinion:

- ✓ Calculation of induced contact stress, σ_c:

$$\sigma_c = \frac{0.72}{(R-0.5\,b)}\left(\frac{\sqrt{(i^2+1)^3}}{i\times b}\times E_{eq}\,[M_t]\right)^{\frac{1}{2}}$$

= 688.28 N/mm^2.

- ✓ We find ($\sigma_c < [\sigma_c]$). Therefore the design is safe and satisfactory.

STEP: 15

Check for Gear (i.e., wheel): Gear material: CI grade 35

First we have to calculate $[\sigma_{b2}]$ and $[\sigma_{c2}]$.

Gear life of wheel, N =7300 ×120×60= 5.256 x 10^7 cycles.

- ✓ **To find $[\sigma_{b2}]$:** we know that the design bending stress for gear,

$$[\sigma_{b2}] = \frac{1.4\times K_{bl}}{n\times K_\sigma} \times \sigma_{-1}$$

Where

$K_{bl} = \sqrt[9]{\frac{10^7}{N}} = \sqrt[9]{\frac{10^7}{5.256\times10^7}}$ = 0.832, from PSGDB 8.20, T-22

K_σ = 1.2, from PSGDB 8.19, T-21

n = 2, from PSGDB - 8.19, T-20

$\sigma_{-1} = 0.45\sigma_u$ = 0.45 X 350 = 157.5 N/mm^2, From PSGDB 8.19, T-19

$[\sigma_{b2}] = \frac{1.4\times0.832}{2\times1.2}$ x 157.5 = 76.44 N/mm^2

To find (σ_{c2}): We know that the design contact stress for gear,

$(\sigma_{c2}) = C_B\times HB\times K_{cl}$

C_B = 2.3, from PSGDB 8.16, T-16

HB = 200 to 260, from PSGDB 8.16, T-16

$K_{cl} = \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{5.256\times10^7}}$ = 0.758

(σ_{c2}) = 2.3×260×0.758 = 453.284 N/mm^2

(a) **Check for bending of gear:** The induced bending stress for gear can be calculated using the relation

$\sigma_{b1}\times y_{v1} = \sigma_{b2}\times y_{v2}$

y_{v1} = 0.402, for $z_{v1} = 22$, from PSGDB 8.18, T-18

y_{v2} = 0.521, for z_{v2} = 190, from PSGDB 8.18, T-18

100.4 × 0.402 = σ_{b2} × 0.520

σ_{b2} = 77.46 N/mm^2

We find σ_{b2} is almost equal to (σ_{b2}). Thus the design is okay and it can be accepted.

(b) **Check for wearing of gear:**

$\sigma_{c2} = \sigma_{c1}$ = 688.28 N/mm^2

We fine σ_{c2} >(σ_{c2}). It means the gear does not have adequate beam strength. In order to increase the wear strength of the gear, surface hardness may be raised to 400 BHN. Then we get

$(\sigma_{c2}) = 2.3 \times 400 \times 0.758 = 697.36$ N/mm^2.

Now we find $\sigma_{c2} < (\sigma_{c2})$ thus the design is safe and satisfactory.

BEVEL GEAR DESIGN USING LEWIS AND BUCKINGHAM'S EQUATION

DESIGN PROCEDURE:

The design procedure for bevel gears is the same as for spur gears.

1. Select the material.
2. Calculate z_1 and z_2. If not given, assume z1 $\geq$ 17.
3. Calculate the pitch angles (i.e., δ_1 and δ_2) and the virtual number of teeth (i.e., Z_{v1} and Z_{v2}) using the following relations.
   - ✓ Pitch angles : $\tan \delta_2 = i$ and $\delta_1 = 90° - \delta_2$, for right angle bevel gears
   - ✓ $Z_{v1} = \frac{Z_1}{\cos \delta_1}$ and $Z_{v2} = \frac{Z_2}{\cos \delta_2}$
4. Calculate the tangential load on tooth using the relation $F_t = \frac{P}{v} \times K_o$
5. Calculate the preliminary value of dynamic load using the relation $F_d = \frac{F_t}{C_v}$
6. Calculate the beam strength F_s in terms of transverse module using the relation

$$F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\frac{R-b}{R}\right)$$

7. Calculate the transverse module m_t by equating F_s and F_d.
8. Calculate the values of b, d_1 and v using the following relation :
   - ✓ Face width : $b = 10\ m_t$
   - ✓ Pitch circle diameter: $d_1 = m_t \times z_1$.
   - ✓ Pitch line velocity : $v = \frac{\pi\ d_1 N_1}{60}$
9. Recalculate the beam strength using the relation

$$F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\frac{R-b}{R}\right)$$

10. Calculate the dynamic load more accurately using buckingham's equation,

$$F_d = F_t + \frac{21\ v\ (bc+F_t)}{21\ v + \sqrt{bc+F_t}}.$$

11. Check for beam strength (or tooth breakage). If $F_d \leq F_s$, the gear tooth has adequate beam strength and will not fail by breakage. Thus the design satisfactory.
12. Calculate the maximum wear load using the relation

$$F_w = \frac{0.75 \times d_1 \times b \times Q' \times K_w}{\cos\delta_1}.$$

13. Check for wear strength, if $F_d < F_w$, the gear tooth has adequate wear capacity and will not wear out. Thus the design is safe and satisfactory.
14. Calculate the basic dimensions of pinion and gear using the PSGDB 8.38.

3. Design a pair of bevel gears to transmit 10 kW at a pinion speed of 1440 rpm. Required transmission ratio is 4. Material for gears is 15 Ni 2Cr 1 Mo 15/steel. The tooth profile of the gears is of 20° composite form.

Given data:

P = 10 kW; N_1 = 1440 rpm; i = 4; α = 20°

To find:

Design the pair of bevel gears.

Solution:

Since the same material is used for both pinion and gear, the pinion is weaker than the gear. Therefore, we have to design only pinion.

STEP 1: Material for gears

15 Ni 2Cr 1 Mo 15

STEP 2: Calculation of z_1 and z_2

Assume $z_1 = 20$, then $z_2 = i \times z_1 = 4 \times 20 = 80$

STEP 3: Calculation of pitch angle and virtual number of teeth

Pitch angles: we know that, $\tan\delta_2$ = i or $\delta_2 = \tan^{-1}(4)$ = 75.96°

Then, δ_1 = 90° - δ_2 = 90° - 75.96° = 14.04°

To find **z_{v1} and z_{v2}:**

$z_{v1} = \frac{z_1}{\cos(\delta_1)} = \frac{20}{\cos 14.04°} = 20.61 \approx 21$ and

$z_{v2} = \frac{z_2}{\cos(\delta_2)} = \frac{80}{\cos 75.96°} = 329.76 \approx 330.$

STEP 4: Calculation of tangential load on tooth (Ft)

We know that, $F_t = \frac{P}{v}$ x Ko

Where $v = \frac{\pi\, d_1 N_1}{60} = \frac{\pi\, N_1}{60}\left(\frac{m_t \times z_1}{1000}\right)$ [i.e $d_1 = m_t \times z_1$ and mt is in 'mm']

$= \frac{\pi \times 1440}{60}\left(\frac{m_t \times 20}{1000}\right)$

$= 1.508 m_t$ m/s

Ko = 1.25, assuming light shock, from Table 1

$F_t = \frac{10 \times 10^3}{1.508\, m_t} \times 1.25 = \frac{8289.12}{m_t}$

STEP 5: Calculation of initial dynamic load (F_d)

We know that, $F_d = \frac{F_t}{C_v}$

Where $C_v = \frac{5.6}{5.6 + \sqrt{v}}$, for general teeth

$= \frac{5.6}{5.6 + \sqrt{5}} = 0.714$, assuming $v = 5$ m/s

$F_d = \left(\frac{8289.12}{m_t}\right) \times \frac{1}{0.714} = \frac{11609.41}{m_t}$

STEP 6: Calculation of beam strength (Fs)

We know that, $F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\frac{R-b}{R}\right)$

Where $b = 10\, m_t$ (initially assumed)

$[\sigma_b] = 450$ N/mm^2, for alloy steel, from Table 2

y' = Form factor based on virtual number of teeth

$= 0.154 - \frac{0.912}{z_{v1}}$, for 20° full depth (PSGDB 8.50)

$$= 0.154 - \frac{0.912}{21} = 0.1106$$

$$R = \text{cone distance} = 0.5\, m_t \sqrt{z_1^2 + z_2^2}$$

$$= 0.5\, m_t \sqrt{20^2 + 80^2} = 41.23 m_t$$

$$F_s = \pi \times m_t \times 10\, m_t \times 450 \times 0.1106 \times \left(\frac{41.23\, m_t - 10\, m_t}{41.23\, m_t}\right)$$

$$= 1184.34\, m_t^2$$

<u>STEP 7:</u> Calculation of transverse module (m_t)

We know that, $F_s \geq F_d$

$$1184.38\, m_t^2 \geq \frac{11609.4}{m_t}$$

$$m_t \geq 2.14$$

from PSGDB 8.2 , the nearest higher standard transverse module is 3 mm.

<u>STEP 8:</u> Calculation of b, d_1 and v

- ✓ Face width : $b = 10\, m_t = 10 \times 3 = 30$ mm
- ✓ Pitch circle diameter: $d_1 = m_t \times z_1 = 3 \times 20 = 60$ mm
- ✓ Pitch line velocity : $v = \frac{\pi \times d_1 \times N_1}{60} = \frac{\pi \times 60 \times 10^{-3} \times 1440}{60} = 4.52$ m/s

<u>STEP 9:</u> Recalculation of beam strength

We know that, $F_s = 1184.38\, m_t^2$

$$= 1184.38 \times 3^2 = 10659 \text{ N}$$

<u>STEP 10:</u> Calculation of accurate dynamic load (F_d)

We know that, $F_d = F_t + \frac{21\, v\, (bc + F_t)}{21\, v + \sqrt{bc + F_t}}$

Where, $F_t = \frac{P}{v} = \frac{10 \times 10^3}{4.52} = 2212.4$ N c = deformation factor, from PSGDB 8.53, T-41

= 11860 e, for steel and steel 20º FD, from PSGDB 8.53, T-41 and

e = 0.0125, for precision gears and m_t up to 4, from PSGDB 8.53,T-42.

c = 11860×0.0125 = 148.25 N/mm

then, $F_d = 2212.4 + \dfrac{21 \times 4.52 \times 10^3\,(30 \times 148.25 + 2212.4)}{21 \times 4.52 \times 10^3 + \sqrt{30 \times 148.25 + 2212.4}} = 8866.58$ N

STEP 11: Check for beam strength (or tooth breakage)

We find $F_s > F_d$. it means the gear tooth has adequate beam strength and will not fail by breakage. Thus the design is satisfactory.

STEP 12: Calculation of maximum wear load (F_w)

We know that, $F_w = \dfrac{0.75 \times d_1 \times b \times Q' \times K_w}{\cos\delta_1}$

Where Q' = ratio factor = $\dfrac{2 \times Z_{v2}}{Z_{v1} + Z_{v2}} = \dfrac{2 \times 330}{21 + 330} = 1.88$

$K_w = 2.553$ N/mm^2, for steel gears hardened to 400 BHN, from Table-3(ref. Spur gear Lewis eqn.)

$F_w = \dfrac{0.75 \times 60 \times 30 \times 1.88 \times 2.553}{\cos 14.04^\circ} = 6679$ N

STEP 13: Check for wear

Since $F_w < F_d$, the design is unsatisfactory.

Trial 2: Now we have to increase the transverse module to 5 mm. Repeating step 8 again, we get

$b = 10 \times m_t = 10 \times 5 = 50$ mm

$d_1 = m_t \times z_1 = 5 \times 20 = 100$ mm

$v_1 = \dfrac{\pi \times 0.1 \times 1440}{60} = 7.54$ m/s

$F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \left(\dfrac{R-b}{R}\right)$

$$= \pi \times 5 \times 50 \times 450 \times 0.1106 \times \left(\frac{41.23 \times 5 - 10 \times 5}{41.23 \times 5}\right) = 29608.5 \text{ N}$$

$$F_t = \frac{P}{v} = \frac{10 \times 10^3}{7.54} = 1326.26 \text{ N}$$

$$F_d = 1326.26 + \frac{21 \times 7.54 \times 10^3 \times (50 \times 148.25 + 1326.26)}{21 \times 7.54 \times 10^3 + \sqrt{50 \times 148.25 + 1326.26}}$$

$$= 10059.9 \text{ N}$$

We find $F_s > F_d$, it means the gear tooth has adequate wear capacity and will not wear out. Thus the design is safe against wear failure also.

$F_w = 0.75 \times 100 \times 1.88 \times 2.553 / \cos 14.04^o = 18552.89$ N

<u>STEP 14:</u> Calculation of basic dimensions of pinion and gear from PSGDB 8.38

- ✓ Transverse module : $m_t = 5$ mm
- ✓ Number of teeth : $Z_1 = 20$; and $Z_2 = 80$
- ✓ Pitch circle diameter: $d_1 = 100$ mm ; and
  $d_2 = m_t \times z_2 = 5 \times 80 = 400$mm
- ✓ Cone distance : $R = 0.5\, m_t \sqrt{Z_1^2 + Z_2^2}$
  $= 0.5 \times 5 \sqrt{20^2 + 80^2}$
  $= 206.15$ mm
- ✓ Face width : $b = 10\, m_t = 10 \times 5 = 50$ mm
- ✓ Pitch angles : $\delta_1 = 14.04^o$; and $\delta_2 = 75.96^o$
- ✓ Tip diameter : $d_{a1} = m_t (z_1 + 2 \cos \delta_1) = 5 (20 + 2 \times \cos 14.04^o) = 109.7$ mm
  and $d_{a2} = m_t (z_2 + 2 \cos \delta_2) = 5 (80 + 2 \times \cos 75.96^o) = 402.43$mm
- ✓ Height factor : $f_o = 1$
- ✓ Clearance : $c = 0.2$
- ✓ Addendum angle : $\tan \theta_{a1} = \tan \theta_{a2} = \frac{m_t \times f_o}{R} = \frac{5 \times 1}{206.15} = 0.02425$ Or
  $\theta_{a1} = \theta_{a2} = 1.4^o$
- ✓ Dedendum angle : $\tan \theta_{f1} = \tan \theta_{f2} = \frac{m_t \times (f_o + c)}{R} = \frac{5 \times (1 + 0.2)}{206.15} = 0.0291$
  Or $\theta_{f1} = \theta_{f2} = 1.67^o$

- Tip angle : $\delta_{a1} = \delta_1 + \theta_{a1}$ = 14.04º + 1.4º = 15.44º ; and

  $\delta_{a2} = \delta_2 + \theta_{a2}$ = 75.96º + 1.4º = 77.36º

- Root angle : $\delta_{f1} = \delta_1 - \theta_{f1}$ = 14.04º - 1.67º = 12.37º ; and

  $\delta_{f2} = \delta_2 - \theta_{f2}$ = 75.96º - 1.67º = 74.29º

- Virtual number of teeth: Z_{v1} = 21 ; and Z_{v2} = 330

Practice:

3. **Design a bevel gear drive to transmit 7.36 kW at 1440 rpm for following data. Gear ratio = 3. Material for pinion and gear C45 surface hardened. (Assume life as 10,000 hours)**

4. **Design a bevel gear drive to transmit 3.5 kW. Speed ratio. Driving shaft speed = 200 rpm. The drive is non- reversible. Pinion is of steel and wheel of CI. Assume a life of 25000 hours.**

Worm Gears

Introduction

The worm gears are widely used for transmitting power at high velocity ratios between non-intersecting shafts that are generally, but not necessarily, at right angles. It can give velocity ratios as high as 300 : 1 or more in a single step in a minimum of space, but it has a lower efficiency. The worm gearing is mostly used as a speed reducer, which consists of worm and a worm wheel or gear. The worm (which is the driving member) is usually of a cylindrical form having threads of the same shape as that of an involute rack. The threads of the worm may be left handed or right handed and single or multiple threads. The worm wheel or gear (which is the driven member) is similar to a helical gear with a face curved to conform to the shape of the worm. The worm is generally made of steel while the worm gear is made of bronze or cast iron for light service. The worm gearing is classified as non-interchangeable, because a worm wheel cut with a hob of one diameter will not operate satisfactorily with a worm of different diameter, even if the thread pitch is same.

Types of Worms

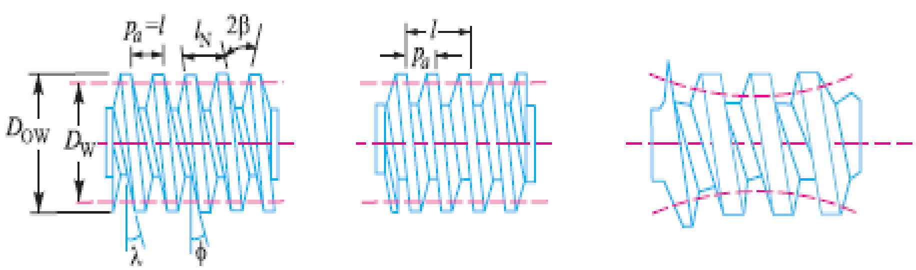
The following are the two types of worms :

1. Cylindrical or straight worm, and

2. Cone or double enveloping worm.

The cylindrical or straight worm, as shown in Fig. (a), is most commonly used. The shape of the thread is involute helicoid of pressure angle 14 ½° for single and double threaded worms and 20° for triple and quadruple threaded worms. The worm threads are cut by a straight sided milling cutter having its diameter not less than the outside diameter of worm or greater than 1.25 times the outside diameter of worm.

The cone or double enveloping worm, as shown in Fig. (b), is used to some extent, but it requires extremely accurate alignment.



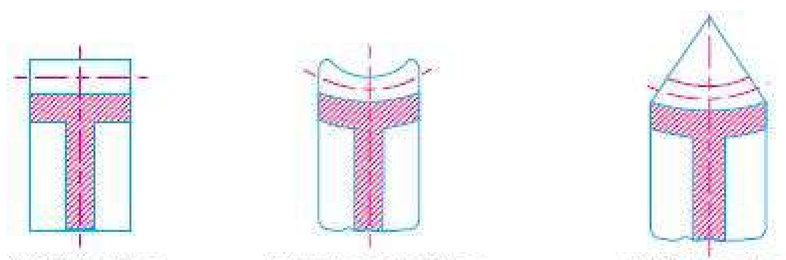
Single threaded. Double threaded.

(*a*) Cylindrical or straight worm. (*b*) Cone or double enveloping worm.

Types of Worm Gears

The following three types of worm gears are important from the subject point of view :

1. Straight face worm gear, as shown in Fig. (*a*),
2. Hobbed straight face worm gear, as shown in Fig. (*b*), and
3. Concave face worm gear, as shown in Fig. (*c*).



(*a*) Straight face. (*b*) Hobbed straight face. (*c*) Concave face.

The **straight face worm gear** is like a helical gear in which the straight teeth are cut with a form cutter. Since it has only point contact with the worm thread, therefore it is used for light service.

The **hobbed straight face worm gear** is also used for light service but its teeth are cut with a hob, after which the outer surface is turned.

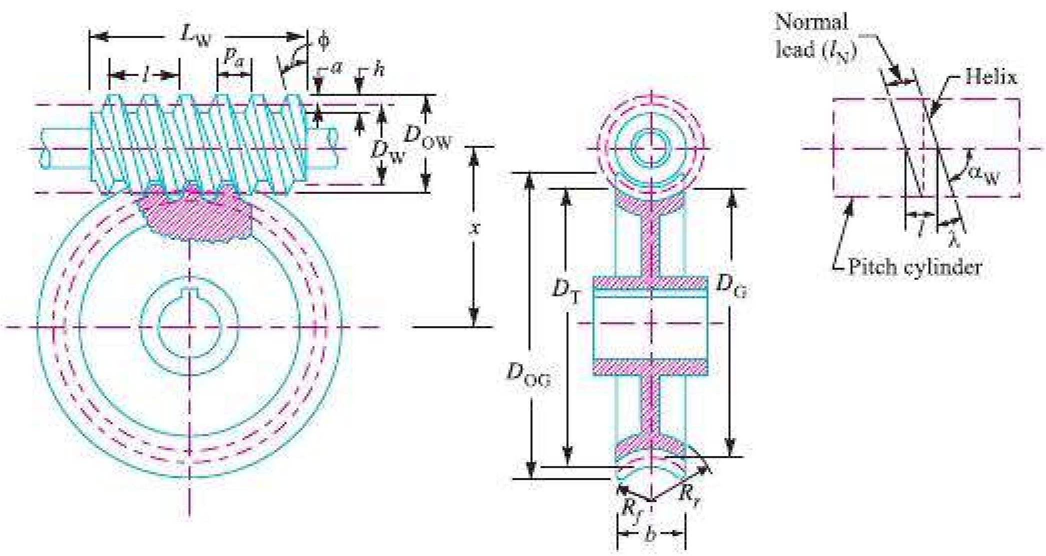
The **concave face worm gear** is the accepted standard form and is used for all heavy service and general industrial uses. The teeth of this gear are cut with a hob of the same pitch diameter as the mating worm to increase the contact area.

Terms used in Worm Gearing

The worm and worm gear in mesh is shown in Fig. The following terms, in connection with the worm gearing, are important from the subject point of view :

1. *Axial pitch.* It is also known as ***linear pitch*** of a worm. It is the distance measured axially (*i.e.* parallel to the axis of worm) from a point on one thread to the corresponding point on the adjacent thread on the worm, as shown in Fig. 31.3. It may be noted that the axial pitch (*pa*) of a worm

is equal to the circular pitch (*pc*) of the mating worm gear, when the shafts are at right angles.



2. Lead. It is the linear distance through which a point on a thread moves ahead in one revolution of the worm. For single start threads, lead is equal to the axial pitch,but for multiple start threads, lead is equal to the product of axial pitch and number of starts. Mathematically,

Lead angle (λ) in degrees	0 – 16	16 – 25	25 – 35	35 – 45
Pressure angle(φ) in degrees	14½	20	25	30

Lead, $l = p_a \cdot n$

where p_a = Axial pitch ; and n = Number of starts.

3. *Lead angle.* It is the angle between the tangent to the thread helix on the pitch cylinder and the plane normal to the axis of the worm. It is denoted by λ.

A little consideration will show that if one complete turn of a worm thread be imagined to be unwound from the body of the worm, it will form an inclined plane whose base is equal to the pitch circumference of the worm and altitude equal to lead of the worm, as shown in Fig. 31.4.

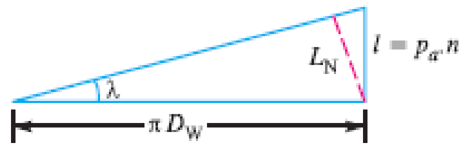


Fig. 31.4. Development of a helix thread.

From the geometry of the figure, we find that

$$\tan\lambda = \frac{\text{Lead of the worm}}{\text{Pitch circumference of the worm}}$$

$$= \frac{l}{\pi D_W} = \frac{p_a \cdot n}{\pi D_W} \qquad \ldots(\because\ l = p_a \cdot n)$$

$$= \frac{p_c \cdot n}{\pi D_W} = \frac{\pi\, m \cdot n}{\pi D_W} = \frac{m \cdot n}{D_W} \qquad \ldots(\because\ p_a = p_c\ ;\ \text{and } p_c = \pi\, m)$$

where m = Module, and

D_W = Pitch circle diameter of worm.

The lead angle (λ) may vary from 9° to 45°. It has been shown by F.A. Halsey that a lead angle less than 9° results in rapid wear and the safe value of λ is 12½°.

For a compact design, the lead angle may be determined by the following relation, *i.e.*

$$\tan\lambda = \left(\frac{N_G}{N_W}\right)^{1/3},$$

where *N*G is the speed of the worm gear and *N*W is the speed of the worm.

4. *Tooth pressure angle.* It is measured in a plane containing the axis of the worm and is equal to one-half the thread profile angle as shown in Fig.The following table shows the recommended values of lead angle (λ) and tooth pressureangle (φ).For automotive applications, the pressure angle of 30° is recommended to obtain a high efficiency and to permit overhauling.

5. Normal pitch. It is the distance measured along the normal to the threads between two corresponding points on two adjacent threads of the worm.
Mathematically,

Normal pitch, pN = pa.cosλ

6. Helix angle. It is the angle between the tangent to the thread helix on the pitch cylinder and the axis of the worm. It is denoted by α_W, in Fig. The worm helix angle is the complement of worm lead angle, i.e.

$$\alpha_W + \lambda = 90°$$

It may be noted that the helix angle on the worm is generally quite large and that on the worm gear is very small. Thus, it is usual to specify the lead angle (λ) on the worm and helix angle (α_G) on the worm gear. These two angles are equal for a 90° shaft angle.

7. *Velocity ratio.* It is the ratio of the speed of worm (N_W) in r.p.m. to the speed of the worm gear (N_G) in r.p.m. Mathematically, velocity ratio,

$$V.R. = \frac{N_W}{N_G}$$

Let l = Lead of the worm, and

D_G = Pitch circle diameter of the worm gear.

We know that linear velocity of the worm,

$$v_W = \frac{l.N_W}{60}$$

and linear velocity of the worm gear,

$$v_G = \frac{\pi\, D_G\, N_G}{60}$$

Since the linear velocity of the worm and worm gear are equal, therefore

$$\frac{l\,.N_W}{60} = \frac{\pi\, D_G\,.N_G}{60} \quad \text{or} \quad \frac{N_W}{N_G} = \frac{\pi\, D_G}{l}$$

We know that pitch circle diameter of the worm gear,

$$D_G = m\,.\,T_G$$

where m is the module and T_G is the number of teeth on the worm gear.

$$\therefore \quad V.R. = \frac{N_W}{N_G} = \frac{\pi\, D_G}{l} = \frac{\pi\, m.T_G}{l}$$

$$= \frac{p_c\,.T_G}{l} = \frac{p_a\,.T_G}{p_a\,.n} = \frac{T_G}{n} \qquad \ldots (\because p_c = \pi\, m = p_a \text{; and } l = p_a\,.\,n)$$

where n = Number of starts of the worm.

From above, we see that velocity ratio may also be defined as the ratio of number of teeth on the worm gear to the number of starts of the worm.

WORM GEAR DESIGN USING LEWIS AND BUCKINGHAM'S EQUATIONS

DESIGN PROCEDURE:

1. **Selection of the materials:**
   Select materials for worm and worm wheel.
2. **Calculation of z_1 and z_2:**
   Depending upon the efficiency requirement, select the number of the starts (z_1) in the worm, referring PSGDB 8.46
   Then, $z_2 = i \times z_1$
3. **Calculation of the diameter factor (q) and lead angle (γ) :**
   Diameter factor, $q = d_1/ m_x$, If not given,
   assume q = 11.
   Lead angle, $\gamma = \tan^{-1} (z_1/q)$
4. **Calculation of tangential load (F_t) acting on the wheel in terms of axial module:**
   Tangential load, $F_t = P/v \times K_0$
5. **Calculation of dynamic load (F_d):**
   Dynamic load, $F_d = F_t/ C_v$, assuming initial pitch line velocity.

$$C_v = \frac{6}{6+v}$$

Where v = Pitch line velocity of the worm gear in m/s.

6. **Calculation of beam strength (F_s) in terms of axial module:**

Beam strength, $F_s = \pi \times m_x \times b \times [\sigma_b] \times y'$

M_x = Axial module

b = Face width from PSGDB 8.48, T-38

(σ_b) = Permissible static stress, from PSGDB 8.45

y' = Form factor for worm wheel, PSGDB 8.52

7. **Calculation of axial module (m_x) :**

Calculate axial module by equating F_s and F_d.

8. **Calculation of b, d_2 and v.**
9. **Recalculation of beam strength :**

Use $F_s = \pi \times m_x \times b \times [\sigma_b] \times y'$

10. **Recalculation of dynamic load:**

Using the calculated pitch line velocity of the wheel, recalculate the dynamic load,

$F_d = F_t / c_v$.

11. **Check for the beam strength:**

If $F_d \leq F_s$, the gear tooth has adequate beam strength and will not fail by breakage. Thus the design is satisfactory.

12. **Calculation of maximum wear load (F_w) :**

$F_w = d_2 \times b \times K_w$

d_2 = Pitch diameter of worm wheel

b = Face width

K_w = Wear factor, depends on the materials of worm and worm wheel from PSGDB 8.54, T-43

13. **Check for wear strength :**

If $F_d < F_w$, the gear tooth has adequate wear capacity and will not wear out. Thus the design is safe and satisfactory.

14. **Check for efficiency:**

If $\eta_{actual} > \eta_{desired}$, then the design is satisfactory. If it is less than the desired, increase lead angle (γ).

$$\eta_{actual} = 0.95 \times (\tan\gamma / \tan(\gamma+\rho)$$

$$\rho = \tan^{-1}\mu$$

15. **Calculate the power loss** and the area required to dissipate the heat.
16. **Calculate the basic dimensions** of the worm and worm wheel using the PSGDB 8.43.

1. **A hardened steel worm rotates at 1440 rpm and transmits 12kW to a phosphor bronze gear. The speed of the worm wheel should be 60 rpm. Design the worm gear drive if an efficiency of atleast 82% is desired.**

Given Data:

N_1 = 1440 rpm; P = 12 kW; N_2 = 60 rpm; $\eta_{desired}$ = 82%

To Find: Design the worm gear drive.

Solution: Gear ratio required, i = 1440/60 =24

1. **Material Selection:**
   Worm – Hardened steel and Worm wheel – Phosphor Bronze
2. **Selection of z_1 and z_2:**
   For η = 82%, z_1 = 3, From PSG DB 8.46, T-37
   Then $z_2 = i \times z_1 = 24 \times 3 = 72$
3. **Calculation of q and γ:**
   Diameter factor: $q = d_1/m_x = 11$ (assumed)
   Lead angle: $\gamma = \tan^{-1}(z_1/q) = \tan^{-1}(3/11) = 15.25^\circ$
4. **Calculation of F_t in terms of m_x:**
   Tangential load, $F_t = \frac{P}{v} K_0$

   Where,
   $v = (\pi d_2 N_2)/(60 \times 1000) = (\pi (z_2 \times m_x) \times N_2)/(60 \times 1000)$ [since d_2 and m_x are in 'mm']
   $= (\pi \times 72 \times m_x \times 60)/(60 \times 1000) = 0.226\, m_x$ m/s
   K_0 = 1.25, assuming Light shock
   $F_t = [(12 \times 10^3)/(0.226 \times m_x)] \times 1.25 = 66371.68/m_x$
5. **Calculation of dynamic load (F_d):**
   Dynamic Load, $(F_d) = F_t/c_v$
   Where, $c_v = 6/(6+v)$, Where v = 5m/s is assumed

   $= 6/(6+5) = 0.545$

   $F_d = (66371.68/m_x) \times (1/0.545) = 121782.8/m_x$
6. **Calculation of Beam Strength (F_s) in terms of axial module:**
   Beam strength, $F_s = \pi \times m_x \times b \times [\sigma_b] \times y'$
   Where, $b = 0.75 d_1$, from PSBDB 8.48, T-38
   $= 0.75 \times q m_x$
   $= 0.75 \times 11 m_x$
   $= 8.25 m_x$
   $[\sigma_b] = 78$ N/mm^2 from PSGDB 8.45
   y' = 0.392, assuming
   α = 20°, from PSGDB 8.52
   $F_s = \pi \times m_x \times 8.25 m_x \times 78 \times 0.392 = 792.47\, m_x^2$

7. **Calculation of axial module (m_x):**
   We know that, $F_s \geq F_d$ or $792.47m_x^2 \geq 121782.8/m_x$ or $m_x \geq 5.35$ mm
   From PSG DB 8.2, the nearest higher standard axial pitch is 6 mm.
8. **Calculation of b, d_2 and v:**
   Face width (b): $b = 8.25\ m_x = 8.25 \times 6 = 49.5$ mm
   Pitch diameter of worm wheel (d_2): $d_2 = z_2 \times m_x = 72 \times 6 = 432$ mm
   Pitch line velocity of worm wheel (v): $v = 0.226m_x = 0.226 \times 6 = 1.356$ m/s
9. **Recalculation of Beam strength (F_s):**

   Beam strength $(F_s) = 792.47\ m_x^2 = 792.47 \times (6)^2 = 28528.92$ N

10. **Recalculation of dynamic load (F_d):**
    Dynamic Load, $(F_d) = F_t/c_v$
    Where, $c_v = 6/(6+v)$
    $= 6/(6+1.356)$
    $= 0.816$, and

    $F_t = 66371.68/m_x$

    $= 66371.68/6$
    $= 11061.95$ N

    $F_d = 11061.95/0.816$

    $= 13556.31$ N
11. **Check for beam strength:**
    We find $F_d < F_s$. It means gear tooth has adequate beam strength and will not fail by breakage. Thus the design is satisfactory.
12. **Calculation of maximum wear load (F_w) :**
    Wear load, $F_w = d_2 \times b \times K_w$
    Where, $K_w = 0.56$ N/mm^2 from PSG DB 8.54,T-43, $\alpha = 20°$

    $F_w = 49.5 \times 432 \times 0.56 = 11975.04$ N

13. **Check for wear:**
    We find $F_d > F_w$. It means gear tooth has in adequate wear capacity and will wear out. Thus the design is not safe and unsatisfactory.
    Trial 2 : Increase m_x =8
14. **Calculation of b, d_2 and v:**
    Face width (b): $b = 8.25\ m_x$
    $= 8.25 \times 8$
    $= 66$ mm
    Pitch diameter of worm wheel (d_2): $d_2 = z_2 \times m_x$
    $= 72 \times 8$
    $= 576$ mm
    Pitch line velocity of worm wheel (v): $v = 0.226m_x$
    $= 0.226 \times 8$
    $= 1.808$ m/s

15. Recalculation of Beam strength (F_s):

Beam strength (F_s) = $792.47\ m_x^2$

$= 792.47 \times (8)^2$

$= 50718.08$ N

16. Recalculation of dynamic load (F_d):

Dynamic Load, (F_d) = F_t/c_v

Where, $c_v = 6/(6+v)$

$= 6/(6+1.808)$

$= 0.768$, and

$F_t = 66371.68/m_x$

$= 66371.68/8$

$= 8296.46$ N

$F_d = 8296.46/0.768$

$= 10802.68$ N

17. Check for beam strength:

We find $F_d < F_s$. It means gear tooth has adequate beam strength and will not fail by breakage. Thus the design is satisfactory.

18. Calculation of maximum wear load (F_w) :

Wear load, $F_w = d_2 \times b \times K_w$

Where, $K_w = 0.56\ N/mm^2$ from PSG DB 8.54

$F_w = 576 \times 66 \times 0.56 = 21288.96$ N

19. Check for wear:

We find $F_d < F_w$. It means gear tooth has adequate wear capacity and will not wear out. Thus the design is safe and satisfactory.

20. Check for efficiency:

We know that, $\eta_{actual} = 0.95\ (\tan\gamma/\tan(\gamma+\rho))$

where ρ = Friction angle

$= \tan^{-1}\mu$ [since $\mu = \tan\rho$)

$= \tan^{-1}(0.03) = 1.7^o$ [since $\mu = 0.03$, assumed]

$\eta = 0.95 \times (\tan 15.25^o/(\tan 15.25^o + 1.7^o))$

$= 0.8498$ or 84.98%

we find that the actual efficiency is greater than the desired efficiency. Thus the design is satisfactory

21. Calculation of basic dimensions of worm and worm gears:

Refer PSG DB 8.43

Axial Module	: $m_x = 8$mm
Number of starts	: $z_1 = 3$
Number of teeth on worm wheel	: $z_2 = 72$

Face width of worm wheel: b =66mm

Length of the worm :$L \geq (12.5 + 0.09z_2)\, m_x$ from 8.48 $\geq (12.5 + 0.09 \times 72)\, 8$
$= 151.84$mm

Center distance : $a = 0.5m_x (q + z_2) = 0.5 \times 8\, (11+72)$
$= 332$mm

Height Factor : $f_0 = 1$

Bottom clearance : $c = 0.25m_x$
$= 0.25 \times 8$
$= 2$mm

Pitch Diameter :$d_1 = q \times m_x$
$= 11 \times 8$
$= 88$mm and
$d_2 = z_2 \times m_x$
$= 72 \times 8$
$= 576$mm

Tip diameter : $d_{a1} = d_1 + 2f_0 \times m_x$
$= 88 + 2 \times 1 \times 8$
$= 104$mm and
$d_{a2} = (z_2 + 2\, f_0)\, m_x$
$= (72+2 \times 1)8$
$= 592$mm

Root Diameter, $d_{f1} = d_1 - 2f_0 \times m_x - 2.c$
$= 88 - 2 \times 1 \times 8 - 2 \times 2$
$= 68$mm and
$d_{f2} = (z_2 - 2\, f_0)\, m_x - 2.c$
$= (72-2 \times 1)8 - 2 \times 2$
$= 556$mm

DESIGN PROCEDURE:

1. Select the suitable combination of materials for worm and worm wheel, consulting PSGDB 8.45 (or) following table

S.No.	Condition	Material	
		Worm	Worm Wheel
1.	Light loads and low speed	Steel	Cast iron
2.	Medium service conditions	Case hardened steel of BHN 250	Phosphor bronze
3.	High speeds, heavy loads with shock conditions.	Hardened molybdenum steel or chrome vanadium steel	Phosphor bronze (chilled)

2. **Calculate the initial design torque** (Mt). Use $(M_t) = M_t \times K \times K_d$. Initially assume Assume $K.K_d = 1$.

3. **Selection of z_1 and z_2:**
   - Selection the number of starts of worm depending on the efficiency requirement, consulting PSGDB 8.46.
   - Then, $z_2 = i \times z_1$.
4. **Selection of (σ_b) and (σ_c):**
   Select the design bending stress and design contact stress of the worm wheel from PSGDB 8.45. To select (σ_c), initially take $v_s = 3$ m/s.

5. **Calculate the centre distance (a)** using the equation

   $$a = [(z_2/q) + 1] \sqrt[3]{\left[\frac{540}{\left(\frac{z_2}{q}\right)[\sigma_c]}\right]^2 \frac{[M_t]}{10}}$$

   Choose initially diameter factor, q = 11. q can vary from 8 to 13.

6. **Calculate the axial module (m_x)** using the relation $m_x = 2a/(q+z_2)$. Then, choose the nearest higher standard axial module from PSGDB 8.2.
7. **Revise centre distance (a)** using the relation $a = 0.5m_x(q+z_2)$.
8. **Calculate d,v,γ and v_s :**
   - **Pitch diameter (d):** $d_1 = q \times m_x$ and $d_2 = z_2 \times m_x$, PSGDB 8.43
   - **Pitch line velocity (v):** $v_1 = \pi d_1 N_1/60$ and $v_2 = \pi d_2 N_2/60$
   - Lead angle $(\gamma) = \tan^{-1}(z_1/q)$, PSGDB 8.43
   - Sliding velocity: $v_s = v_1/\cos\gamma$, PSGDB 8.44

9. **Recalculate the design contact stress (σ_c)** for the actual v_s , using PSGDB 8.45.
10. **Revise K, K_d and (M_t)** for the actual velocity of the worm wheel (v_2).
11. **Check for bending:**
    - Calculate the induced bending stress using the equation

      $\sigma_b = \frac{1.9\,(Mt)}{m_x^3 \times q \times z_2 \times y_v}$ PSGDB 8.44

      m_x = Axial module,
      q = Diameter factor (or diameter quotient),
      z_2 = Number of teeth on worm wheel,
      y_v = Form factor based on virtual number of teeth, from PSGDB 8.18.
    - Compare the induced bending stress with the design bending stress.
    - If $\sigma_b \leq (\sigma_b)$, then the design is satisfactory.

12. If $\sigma_b > (\sigma_b)$, then the design is not satisfactory. Then increase the axial module.

13. Check for wear strength:

- Calculate the induced contact stress using the equation

$$\sigma_c = \frac{540}{(z_2/q)}\sqrt{\left[\frac{\left(\frac{z_2}{q}\right)+1}{a}\right]^3 \times \frac{[M_t]}{10}}$$ PSGDB 8.44

- Compare the induced contact stress with the design contact stress.
- If $\sigma_c \leq (\sigma_c)$, then the design is safe and satisfactory.

14. Check for efficiency:

- If $\eta_{calculated} \geq \eta_{desired}$, then the design is satisfactory.
- Otherwise increase the lead angle γ.

15. Calculate the power loss and the area required to dissipate the heat.
16. **Calculate all the basic dimensions** of the worm and worm wheel using the equations listed in PSGDB 8.43.

2. A steel worm running at 240 rpm receives 1.5 kW from its shaft. The speed reduction is 10:1. Design the drive so as to have an efficiency of 80%. Also determine the cooling area required, if the temperature rise is restricted to 45°C. Take overall heat transfer coefficient as 10 W/m^2°C.

Given data: N_1= 240 rpm; P=1.5 kW ; i=10 ; $\eta_{desired}$ = 80% ;

$t_o - t_a$=45°C ; K_t=10 W/m^2 °C.

To find: 1. Design the worm gear drive, and

2. The cooling area required (A).

Solution: $N_2 = N_1 / i = 240/10 = 24$ rpm

STEP 1: Selection of material

Worm - steel

Worm Wheel - bronze (sand cast), selected from PSGDB 8.45

STEP 2: Calculation of initial design wheel torque $[M_t]$

We know that, $[M_t] = M_t \times K \times K_d$

Where M_t = wheel torque = $\frac{60 \times P}{2\pi N_2} = \frac{60 \times 1.5 \times 10^3}{2\pi \times 24}$ = 596.83 N-m

$K \times K_d = 1$, initially assumed.

So now design wheel torque, $[M_t]$ = 596.83 x 1 = 596.83 N-m

STEP 3: Selection of z_1 and z_2

For $\eta_{desired}$ = 80%, z_1=3 or 4, from PSGDB 8.46. Here z_1=3 is selected.

Then $z_2 = i \times z_1 = 10 \times 3 = 30$

STEP 4: Selection of $[\sigma_b]$ and $[\sigma_c]$

For bronze wheel, σ_u **< 390** N/mm^2, $[\sigma_b]$ **= 50** N/mm^2 is selected, for rotation in one direction, from PSGDB 8.45.

From PSGDB 8.45 $[\sigma_c]$ **= 159** N/mm^2 is selected, assuming v_s = 3 m/s

STEP 5: Calculation of centre distance (a):

We know that, $a = [(z_2/q) + 1]\sqrt[3]{\left[\frac{540}{\left(\frac{z_2}{q}\right)[\sigma_c]}\right]^2 \frac{[M_t]}{10}}$

Where q = 11, initially chosen.

$$a = [(30/11) + 1]\sqrt[3]{\left[\frac{540}{\left(\frac{30}{11}\right)159}\right]^2 \frac{596.83\times10^3}{10}} = 168.6 \text{ mm}$$

STEP 6: Calculation of axial module (m_x):

$$m_x = \frac{2a}{(q+z_2)} = \frac{2\times168.6}{(11+30)} = 8.22\ mm$$

from PSGDB 8.2, the nearest higher standard axial module is 10 mm

STEP 7: Revision of centre distance (a)

$a = 0.5m_x(q+z_2) = 0.5 \times 10\ (11 + 30) = 205$ mm

STEP 8: Calculation of d,v,γ and v_s:

Pitch diameters: $d_1 = q \times m_x = 11\times10 = 110$mm; and

$d_2 = z_2 \times m_x = 30 \times 10 = 300$mm.

pitch line velocity: $v_1 = \frac{\pi\, d_1 N_1}{60} = \frac{\pi \times 110 \times 10^{-3} \times 240}{60}$ = 1.382 m/s; and

$v_2 = \frac{\pi\, d_2 N_2}{60} = \frac{\pi \times 300 \times 10^{-3} \times 24}{60}$ = 0.377m/s.

lead angle: $\gamma = \tan^{-1}\left(\frac{z_1}{q}\right) = \tan^{-1}\left(\frac{3}{11}\right)$ = 15.25º

sliding velocity: $v_s = \frac{v_1}{\cos\gamma} = \frac{1.382}{\cos 15.25°}$ = 1.432 m/s

STEP 9: Recalculation of design contact stress $[\sigma_c]$

For v_s = 1.432 m/s, **$[\sigma_c]$ = 172** N/mm^2, from PSGDB 8.45

STEP 10: Revision of $[M_t]$

For v_2< 3 m/s, $K \times K_d = 1$

$$[M_t] = M_t \times K \times K_d = 596.83 \times 1 = 596.83 \text{ N-m}$$

STEP 11: Check for bending

We know that the induced bending stress,

$$\sigma_b = \frac{1.9\,[M_t]}{m_x^3 \times q \times z_2 \times y_v}$$

Where y_v = form factor based on virtual no. of teeth, from PSGDB 8.18

$$Z_V = \frac{Z}{cos^3\gamma} = \frac{30}{cos^3 15.25°} = 34$$

y_V= 0.452, for z_v = 34,not given goto 35 , from PSGDB 8.18

Then, $\sigma_b = \frac{1.9 \times 596.83 \times 10^3}{10^3 \times 11 \times 30 \times 0.452}$ = 7.6 N/mm^2

We find $\sigma_b < [\sigma_b]$, thus the design is satisfactory against bending.

STEP 12: Check for wear

We know that the induced contact stress,

$$\sigma_c = \frac{540}{(z_2/q)}\sqrt{\left[\frac{\left(\frac{z_2}{q}\right)+1}{a}\right]^3 \times \frac{[M_t]}{10}}$$

$$= \frac{540}{(30/11)}\sqrt{\left[\frac{\left(\frac{30}{11}\right)+1}{205}\right]^3 \times \frac{596.83 \times 10^3}{10}} = 118.59 \text{ N/mm}^2$$

We find $\sigma_c < [\sigma_c]$, thus the design is satisfactory against wear.

STEP 13: Check for efficiency

$$\eta_{actual} = 0.95 \text{ x } \frac{\tan\gamma}{\tan(\gamma+\rho)}$$

Where μ = 0.05, for v_s = 1.432 m/s and bronze wheel, from graph PSGDB 8.49.

$$\rho = \tan^{-1}(\mu) = \tan^{-1}(0.05) = 2.862^\circ$$

Then, $\eta_{actual} = 0.95 \text{ x } \frac{\tan 15.25^\circ}{\tan(15.25^\circ + 2.862^\circ)} = 80\%$

We find $\eta_{actual} = \eta_{desired}$, thus the design is satisfactory.

STEP 14: Calculation of cooling area required (A):

We know that, (1- η) x input power = $K_t \times A\,(t_o - t_a)$

$$(1 - 0.8) \text{ x } 1.5 \text{ x } 10^3 = 10 \text{ x A x } 45$$

Or cooling area required, A = .666 m^2

STEP 15: Calculation of basic dimensions of the worm and worm gear: From PSGDB 8.43

- Axial module: m_x = 10 mm
- Number of starts:worm z_1 = 3
- Number teeth on worm wheel: z_2 = 30

- Length of worm : $L \geq (12.5 + 0.09 \times z_2)m_x$, from PSGDB 8.48,T-39

  $\geq (12.5 + 0.09 \times 30)\ 10 = 152$ mm
- Centre distance : $a = 205$ mm
- Face width : $b = 0.75\ d_1 = 0.75 \times 110 = 82.5$mm ,PSGDB 8.48,T-38
- Height factor : $f_o = 1$
- Bottom clearance : $c = 0.25\ m_x = 0.25 \times 10 = 2.5$ mm
- Pitch diameter : $d_1 = 110$ mm; and $d_2 = 300$ mm
- Tip diameter : $d_{a1} = d_1 + 2\ f_o.\ m_x$

  $= 110 + 2 \times 1 \times 10 = 130$mm; and

  $d_{a2} = (z_2 + 2\ f_o)\ m_x$

  $= (30 + 2 \times 1)\ 10 = 320$ mm
- Root diameter : $d_{f1} = d_1 - 2\ f_o.\ m_x - 2 \times c$

  $= 110 - 2 \times 1 \times 10 - 2 \times 2.5 = 85$ mm; and

  $d_{f2} = (z_2 - 2.f_o)\ m_x - 2 \times c$

  $= (30 - 2 \times 1)\ 10 - 2 \times 2.5 = 275$ mm

Practice:

1. The input to worm gear shaft is 18 kW and 600 rpm. Speed ratio is 20. The worm is to be of hardened steel and the wheel is made of chilled phosphor bronze. Considering wear and strength, design worm and worm wheel.
2. Design worm and gear speed reducer to transmit 22 kW at a speed of 1440 rpm. The desired velocity ratio is 24:1. An efficiency of atleast 85% is desired. Assume that the worm is made of hardened steel and the gear of phosphor bronze.

PART – B

16. Design a bevel gear drive to transmit 7.5 KW at 1500 rpm. Gear ratio is 3.5. Material for pinion and gear is C45 steel. Minimum number of teeth is to be 25. **(16) (M/J 2012)**

17. Design a worm gear and determine the power loss by heat generation. The hardened steel worm rotates at 1500 rpm and transmits 10 KW to a phosphor bronze gear with gear ratio of 16. **(16) (M/J 2012)**

18. Design a bevel gear drive to transmit 3.5 KW. Speed ratio = 4. Driving shaft speed = 200 rpm. The drive is non-reversible. Pinion is of steel and wheel of C1. Assume a life of 25,000 hours. **(16) (N/D 2011)**

19. Design a worm and gear speed reducer to transmit 22 KW at a speed of 1440 rpm. The desired velocity ratio is 24:1. An efficiency of at least 85% is desired. Assume that the worm is made of hardened steel and the gear of phosphor bronze. Take the Centre distance as 100mm. **(16) (N/D 2009)**

20. (a) A pair of straight bevel gears has a velocity ratio of 2:1. The pitch circle diameter of the pinion is 80 mm at the large end of the tooth. A 5 KW power is supplied to the pinion, which rotates at 800 rpm. The face width is 40 mm and the pressure angle is 20°. Calculate the tangential, radial and axial components of the resultant tooth force acting on the pinion. **(8)**

(b) A pair of bevel gears is to be used to transmit 10 KW from a pinion rotating at420 rpm to a gear mounted on a shaft which intersects the pinion shaft at an angle of 70°. Assume that the pinion is to have on outside pitch diameter of 180 mm, a pressure angle of 20°, a face width of 45 mm, and the gear shaft is to rotate at 140 rpm, Determine

i. The pitch angle for the gears,
ii. The forces on the pinion and gear and
iii. The torque produced about the shaft axis. **(8) (M/J 2009)**

21. (a) A pair of worm gears is designated as 2/54/10/4. 10 KW power at 720 rpm is supplied to the worm shaft. The coefficient of friction is 0.04 and the pressure angle is 20°. Calculate the tangential, axial and radial components of the resultant gear tooth force acting on the worm and the worm wheel. **(8)**

(b) A double threaded worm drive is required for power transmission between two shafts having their axes at right angle to each other. The worm has 14 1/2° involute teeth. The Centre distance is approximately 200 mm. If the axial pitch of the worm is 30mm and lead angle is 23°, Find

i. Lead
ii. Pitch circle diameter of worm and worm gear
iii. Helix angle of the worm and
iv. Efficiency of the drive if the coefficient of friction is 0.05. **(8) (A/M 2009)**

22. Design a pair of cast iron bevel gears for a special purpose machine tool to transmit 3.5 KW from shaft at 500 rpm to another at 800 rpm. The gears overhang in their shafts. Life required is 8000 hours. **(16) (A/M 2010)**

23. Design a worm gear drive with a standard Centre distance to transmit 7.5 KW from a worm rotating at 1440 rpm to a worm wheel at 20 rpm. **(16) (A/M 2010)**

24. A pair of cast iron bevel gears connects two shafts at right angles. The pitch diameters of the pinion and gear are 80 mm and 100 mm respectively. The tooth profile of the gears is 14 ½ ° composite forms. The allowable static stress for both the gears is 55 MPa. If the pinion transmits 2.75 KW at 1100 rpm. Find the module and number of teeth on each gear and check the design. Take surface endurance limit as 630 MPa and check the design. Take surface endurance limit as 630 MPa and modulus of elasticity for cast iron as 84 KN/mm^2.

(16) (N/D 2009)

25. A 2 KW power is applied to a worm shaft at 720 rpm. The worm is of quadruple start type with 50 mm as pitch circle diameter. The worm gear has 40 teeth with 5 mm module. The pressure angle in the diametrical plane is 20°. Determine

i. The lead angle of the worm
ii. Velocity ratio and
iii. Centre distance.
iv. Also, calculate efficiency of the worm gear drive, and power lost in friction.

(16) (A/M 2008)

<u>Reference books:</u>

1. Machine design (volume –II), Design of Transmission Systems, S.Md.Jalaludeen
2. Machine design – R.S. Khurmi & J.K. Gupta
3. Design of transmission systems – T.J. Prabhu
4. Design of transmission systems – V. Jayakumar