DHANALAKSHMI COLLEGE OF ENGINEERING

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DEPARTMENT OF MECHANICAL ENGINEERING

III YEAR MECHANICAL - VI SEMESTER

ME6601 – DESIGN OF TRANSMISSION SYSTEMS

EVEN SEMESTER

UNIT II – STUDY NOTES

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UNIT II

SPUR GEARS AND PARALLEL AXIS HELICAL GEARS 9 hours

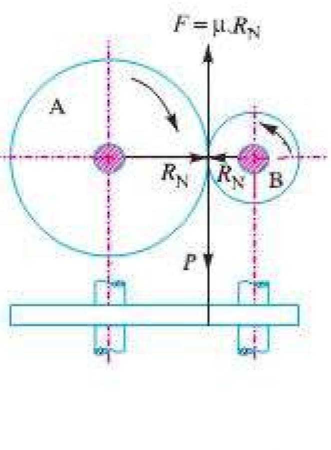
Gear Terminology - Speed ratios and number of teeth - Force analysis - Tooth stresses - Dynamic effects - Fatigue strength - Factor of safety - Gear materials – Module and Face width-power rating calculations based on strength and wear considerations - Parallel axis Helical Gears – Pressure angle in the normal and transverse plane - Equivalent number of teeth-forces and stresses. Estimating the size of the helical gears.

Introduction

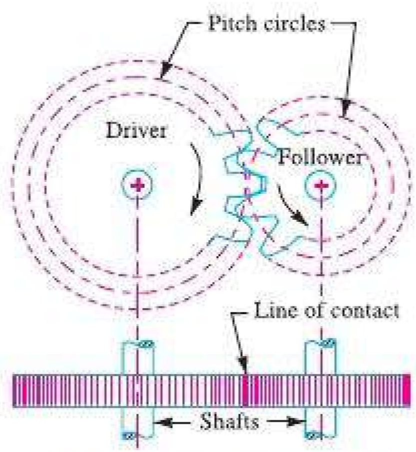
We have discussed earlier that the slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance(as in watch mechanism), the only positive drive is by gears or toothed wheels. A gear drive is also provided, when the distance between the driver and the follower is very small.

Friction Wheels

The motion and power transmitted by gears is kinematically equivalent to that transmitted by frictional wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels A and B mounted on shafts. The wheels have sufficient rough surfaces and press against each other as shown in Fig.



Friction wheels.



Gear or toothed wheel.

Let the wheel *A* is keyed to the rotating shaft and the wheel *B* to the shaft to be rotated. A little consideration will show that when the wheel *A* is rotated by a rotating shaft, it will rotate the wheel *B*in the opposite direction as shown in Fig. 28.1. The wheel *B* will be rotated by the wheel *A* so long as the tangential force exerted by the wheel *A* does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (*P*) exceeds the *frictional resistance (*F*), slipping will take place between the two wheels.

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig are provided on the periphery of the wheel *A* which will fit into the corresponding recesses on the periphery of the wheel *B*. A friction wheel with the teeth cut on it is known as *gear* or *toothed wheel*. The usual connection to show the toothed wheels is by their pitch circles.

Advantages and Disadvantages of Gear Drives

The following are the advantages and disadvantages of the gear drive as compared to other drives, i.e. belt, rope and chain drives:

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It may be used for small centre distances of shafts.
4. It has high efficiency.
5. It has reliable service.
6. It has compact layout.

Disadvantages

1. Since the manufacture of gears require special tools and equipment, therefore it is costlier than other drives.
2. The error in cutting teeth may cause vibrations and noise during operation.
3. It requires suitable lubricant and reliable method of applying it, for the proper operation of gear drives.

Classification of Gears

The gears or toothed wheels may be classified as follows :

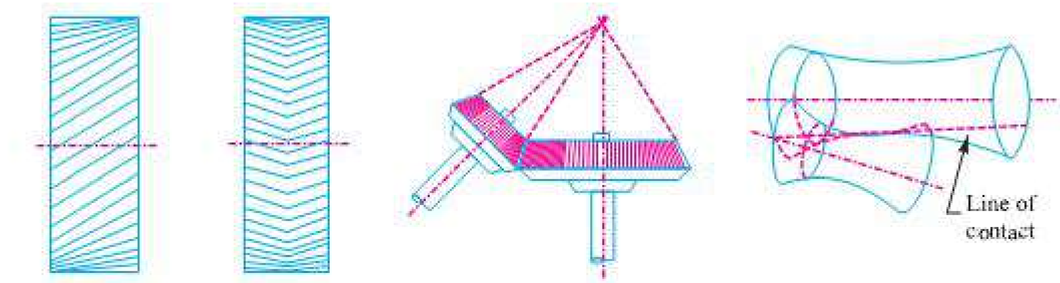
1. According to the position of axes of the shafts.

The axes of the two shafts between which the motion is to be transmitted, may be

(a) Parallel, (b) Intersecting, and (c) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. These gears are called spur gears and the arrangement is known as spur gearing. These gears have teeth parallel to the axis of the wheel as shown in Fig. Another name given to the spur gearing is helical gearing, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are shown in Fig. (a) and (b) respectively. The object of the double helical gear is to balance out the end thrusts that are induced in single helical gears when transmitting load. The double helical gears are known as herringbone gears. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to a parallel shaft having line contact.

The two non-parallel or intersecting, but coplaner shafts connected by gears is shown inFig. (c). These gears are called bevel gears and the arrangement is known as bevel gearing.The bevel gears, like spur gears may also have their teeth inclined to the face of the bevel, inwhich case they are known as helical bevel gears.



(a) Single helical gear. (b) Double helical gear. (c) Bevel gear. (d) Spiral gear.

The two non-intersecting and non-parallel i.e. non-coplanar shafts connected by gears is shown in Fig. These gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiral gearing. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as hyperboloids.

2. According to the peripheral velocity of the gears.

The gears, according to the peripheral velocity of the gears, may be classified as :

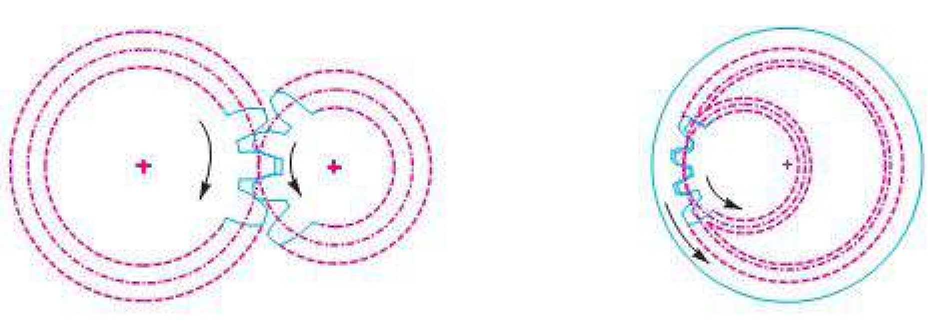
(a) Low velocity, (b) Medium velocity, and (c) High velocity.

The gears having velocity less than 3 m/s are termed as low velocity gears and gears having velocity between 3 and 15 m / s are known as medium velocity gears. If the velocity of gears is more than 15 m / s, then these are called high speed gears.

3. According to the type of gearing.

The gears, according to the type of gearing, may be classified as :

(a) External gearing, ***(b)*** Internal gearing, and ***(c)*** Rack and pinion.


(a) External gearing. (b) Internal gearing.

In external gearing, the gears of the two shafts mesh externally with each other as shown in Fig. The larger of these two wheels is called spur wheel or gear and the smaller wheel is called pinion. In an external gearing, the motion of the two wheels is always unlike, as shown in Fig.

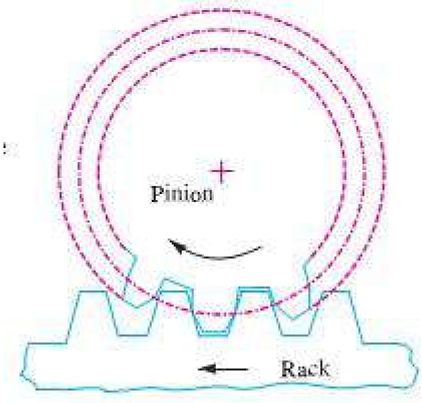
In internal gearing, the gears of the two shafts mesh internally with each other as shown in Fig. The larger of these two wheels is called annular wheel and the smaller wheel is called pinion. In an internal gearing, the motion of the wheels is always like as shown in Fig. Sometimes, the gear of a shaft meshes externally and internally with the gears in a *straight line, as shown in Fig. Such a type of gear is called rack and pinion. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and vice-versa as shown in Fig.

4. *According to the position of teeth on the gear surface*.

The teeth on the gear surface may be

(*a*) Straight, **(*b*)** Inclined, and **(*c*)** Curved.

We have discussed earlier that the spur gears have straight teeth whereas helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.



Terms used in Gears

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig.

1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called as pitch diameter.

3. Pitch point. It is a common point of contact between two pitch circles.

4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point

of contact and the common tangent at the pitch point. It is usually denoted by). The standard pressure angles are 14 /1 2° and 20°.

6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter × cos ⎞, where ⎞ is the pressure angle.

10. Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by pc. Mathematically,

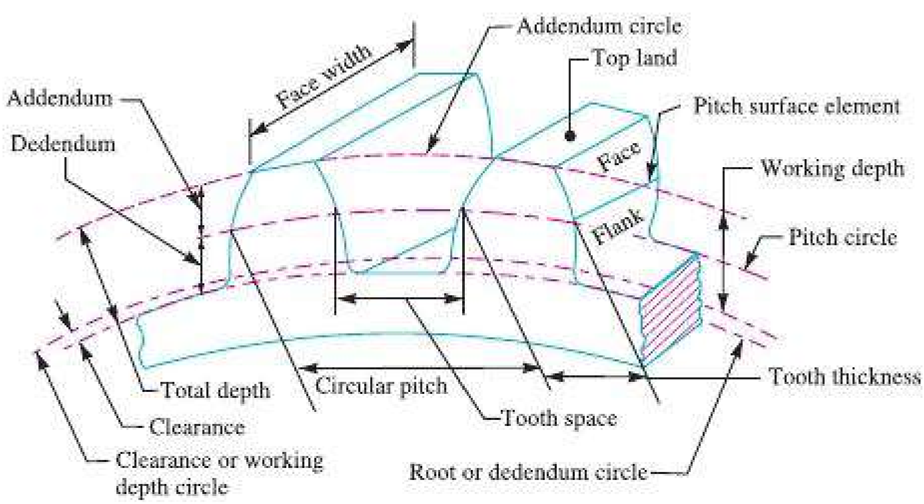
Circular pitch, pc = □ D/T

Where ,

D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.



11. Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It denoted by pd.

12. Module. It is the ratio of the pitch circle diameter in millimetres to the number of teeth. It is usually denoted by m. Mathematically,

Module, m = D / T

13. Clearance. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as clearance circle.

14. Total depth. It is the radial distance between the addendum and the dedendum circle of a gear. It is

equal to the sum of the addendum and dedendum.

15. Working depth. It is radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. Tooth thickness. It is the width of the tooth measured along the pitch circle.

17. Tooth space. It is the width of space between the two adjacent teeth measured along the pitch circle.

18. Backlash. It is the difference between the tooth space and the tooth thickness, as measured on the pitch circle.

19. Face of the tooth. It is surface of the tooth above the pitch surface.

20. Top land. It is the surface of the top of the tooth.

21. Flank of the tooth. It is the surface of the tooth below the pitch surface.

22. Face width. It is the width of the gear tooth measured parallel to its axis.

23. Profile. It is the curve formed by the face and flank of the tooth.

24. Fillet radius. It is the radius that connects the root circle to the profile of the tooth.

25. Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. Length of the path of contact. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. Arc of contact. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, i.e.

(a) Arc of approach. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(b) Arc of recess. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Gear Materials

The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The non- metallic materials like wood, rawhide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise.

The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important. The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness.

Design Considerations for a Gear Drive

In the design of a gear drive, the following data is usually given :

1. The power to be transmitted.
2. The speed of the driving gear,
3. The speed of the driven gear or the velocity ratio, and
4. The centre distance.

The following requirements must be met in the design of a gear drive :

(a) The gear teeth should have sufficient strength so that they will not fail under static loading or dynamic loading during normal running conditions.

(b) The gear teeth should have wear characteristics so that their life is satisfactory.

(c) The use of space and material should be economical.

(d) The alignment of the gears and deflections of the shafts must be considered because they effect on the performance of the gears.

(e) The lubrication of the gears must be satisfactory.

Causes of Gear Tooth Failure

The different modes of failure of gear teeth and their possible remedies to avoid the failure, are as follows :

1. *Bending failure*. Every gear tooth acts as a cantilever. If the total repetitive dynamic load acting on the gear tooth is greater than the beam strength of the gear tooth, then the gear tooth will fail in bending, *i.e.* the gear tooth will break.

In order to avoid such failure, the module and face width of the gear is adjusted so that the beam strength is greater than the dynamic load.

2. Pitting. It is the surface fatigue failure which occurs due to many repetition of Hertz contact stresses. The failure occurs when the surface contact stresses are higher than the endurance limit of the material. The failure starts with the formation of pits which continue to grow resulting in the rupture of the tooth surface.

In order to avoid the pitting, the dynamic load between the gear tooth should be less than the wear strength of the gear tooth.

3. Scoring. The excessive heat is generated when there is an excessive surface pressure, high speed or supply of lubricant fails. It is a stick-slip phenomenon in which alternate shearing and welding takes place rapidly at high spots.

This type of failure can be avoided by properly designing the parameters such as speed, pressure and proper flow of the lubricant, so that the temperature at the rubbing faces is within the permissible limits.

4. Abrasive wear. The foreign particles in the lubricants such as dirt, dust or burr enter between the tooth and damage the form of tooth. This type of failure can be avoided by providing filters for the lubricating oil or by using high viscosity lubricant oil which enables the formation of thicker oil film and hence permits easy passage of such particles without damaging the gear surface.

5. Corrosive wear. The corrosion of the tooth surfaces is mainly caused due to the presence of corrosive elements such as additives present in the lubricating oils. In order to avoid this type of wear, proper anti-corrosive additives should be used.

Helical Gears

Introduction

A helical gear has teeth in form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears. The helixes may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission.

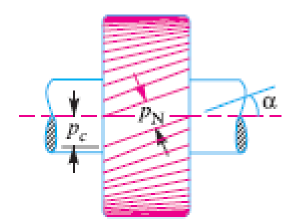
Terms used in Helical Gears

The following terms in connection with helical gears, as shown in Fig., are important from the subject point of view.

1. **Helix angle:** It is a constant angle made by the helices with the axis of rotation.

2. **Axial pitch**: It is the distance, parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by pc. The axial pitch may also be defined as the circular pitch in the plane of rotation or the diametral plane.

3. Normal pitch: It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by pN. The normal pitch may also be defined as the circular pitch in the normal plane which is a plane perpendicular to the teeth. Mathematically, normal pitch,pN = pc cos□α



Helical gear

PART – A

1 What is a herringbone gear? Where it is used? (N/D 2009), (M/J 2012)

Herring Bone Gear :

The double helical gears connecting two parallel shafts are known as herringbone gears.They are used in heavy machinery and gear boxes.

2 What is backlash in gears? (A/M 2008)

Backlash is the difference between the tooth space and the tooth thickness along the pitch circle.

3 What is the advantage of helical gear over spur gear? (A/M 2008)

- Helical gears produce less noise than spur gears.
- Helical gears have a greater load capacity than equivalent spur gears.

4 What are the common forms of gear tooth profile? (A/M 2010)

i. Involute tooth profile, and

ii. Cycloidal tooth profile.

5 Define – module. (A/M 2011)

Module:

It is the ratio of pitch circle diameter to the number of teeth.

6 How does failure pitting happen in gears? (N/D 2011)

Pitting is the process during which small pits are formed on the activate surfaces of gear tooth. It is a surface fatigue failure which occurs when the load on the gear tooth exceeds the surface endurance strength of the material.

7 What is the effect of increasing the pressure angle in gears? (N/D 2011)

The increase of the pressure angle results in a stronger tooth, because the tooth acting as a beam is wider at the base.

8 What condition must be satisfied in order that a pair of spur gears may have a constant velocity ratio? (M/J 2012)

The law of gearing states that for obtaining a constant velocity ratio, at any instant of teeth the common normal at each point of contact should always pass through a pitch point (fixed point), situated on the line joining the centre's of rotation of the pair mating gears.

9 Define – pitch circle with reference to spur gears. (M/J 2011)

Pitch circle:

Pitch circle is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

10 What are the materials used for gear manufacturing? (M/J 2011)

Metallic gears: Steel, cast iron, and bronze.

Non-metallic gears: Wood, rowhide, compressed paper and synthetic resins.

DESIGN OF SPUR GEAR BASED ON MANUFACTURER'S DATA

GEARS ARE MADE OF SAME MATERIALS:

1) In a spur gear drive for a stone crusher, the gears are made of C40 steel. The pinion is transmitting 30KW at 1200 rpm. The gear ratio is 3. Gear is to work 8 hours per day, six days a week and 3 years. Design the drive [N/D 2014]

Given:-

Pinion and gear are made of same material = C40 steel

$P = 30KW = 30 \times 10^3 W$

$N_1 = 1200$ rpm

$i = 3$

To find:-

Design the spur gear drive.

Solution:-

Step 1: Gear ratio

$i = 3$

Step 2: material selection

Pinion & gear are made of C40 steel.

Assume surface hardness >350

Step 3: Gear life

Given that the gear is to work 8 hours per day, six days a week and for 3 years.

$$\text{Gear life} = 8 \times (52 \times 6) \times 3 = 7488 \text{ hours}$$
$$= 449280 \text{ min}$$

Life in numbers of cycles, $N = 449280 \times N_1$
$$= 449280 \times 1200$$
$$= 53.9 \times 10^7 \text{ cycles}$$

Step 4: calculation of initial design torque $[M_t]$

Design torque, $[M_t] = M_t \times (K \times K_d)$

$$M_t = \frac{60 \times P}{2 \pi N_1} = \frac{60 \times 30 \times 10^3}{2 \times \pi \times 1200}$$

$$\boxed{M_t = 238.73 \text{ N-m}}$$

Assume $K \cdot K_d = 1.3$

$$[M_t] = 238.73 \times 1.3$$

$$\boxed{[M_t] = 310.35 \text{ N-m}}$$

Step 5: calculation of E_{eq}, $[\sigma_b]$ & $[\sigma_c]$

(i) E_{eq} [From PSGDB : 8.14, T-9]

$E_{eq} = 2.15 \times 10^5$ N/mm²

(ii) $[\sigma_b]$ = [From PSGDB : 8.18, below, T-18]

$$[\sigma_b] = \frac{1.4 \times K_{bl}}{n \times K_\sigma} \times \sigma_{-1}$$ [Assuming rotation in one direction only]

From PSGDB 8.20, T-22

$K_{bl} = 0.7$ for HB > 350 & $N \geq 25 \times 10^7$

From PSGDB 8.19, T-20

For steel tempered, $n = 2$

From PSGDB 8.19, T-21

For $0 \leq x \leq 0.1$, steel, $K_\sigma = 1.5$

From PSGDB 8.19, T-19

For C40 Alloy steel, $\sigma_{-1} = 0.35\,\sigma_u + 120$

From PSGDB 1.9 $\sigma_u = 630$ N/mm² [Assume]
C40 steel, (580 - 680 N/mm²)

$\sigma_{-1} = 0.35 \times 630 + 120$

$= 340.5$ N/mm²

$$[\sigma_b] = \frac{1.4 \times 0.7}{2 \times 1.5} \times 340.5$$

$$\boxed{[\sigma_b] = 111.23\ N/mm^2}$$

(iii) $[\sigma_c]$ [From PSGDB 8.16, below T-15]

$$[\sigma_c] = C_R \times HRC \times K_{cl}$$

From PSGDB 8.16, T-16

For C40 alloy steel, Hardened & Tempered

$C_R = 26.5$

From PSGDB 8.16, T-16

For HRC = 40 to 55 [surface hardness]

From PSGDB 8.17, T-17

For $HB > 350$, $N \geq 25 \times 10^7$, $K_{cl} = 0.585$

$$[\sigma_c] = 26.5 \times 55 \times 0.585$$

$$\boxed{[\sigma_c] = 852.64\ N/mm^2}$$

step 6: calculation of centre distance (a)

From PSGDB 8.13, T-8

$$a \geq (i+1) \sqrt[3]{\left[\frac{0.74}{[\sigma_c]}\right]^2 \times \frac{E_{eq}[M_t]}{i \times \psi}}$$

Assume $\psi = b/a = 0.3$

$$a \geq (3+1)\sqrt[3]{\left[\frac{0.74}{852.64}\right]^2 \times \frac{2.15\times10^5 \times 310.35 \times 10^3}{3\times0.3}}$$

$a \geq 152.89mm$

$\boxed{a = 155mm}$

Step 7: selection of Z_1 and Z_2

(i) Assume $Z_1 = 17$, for $20°$ full depth

(ii) $Z_2 = i \times Z_1 = 3\times17 = 51$

$\boxed{Z_2 = 51}$

Step 8:- calculation of module(m)

$$m = \frac{2a}{Z_1+Z_2} = \frac{2\times155}{(17+51)}$$

$\boxed{m = 4.56mm}$

From PSGDB 8.2, T-1

nearest higher std. module $\boxed{m = 5\ mm}$

Step 9:- Revision of centre distance

$$a = \frac{m(Z_1+Z_2)}{2} = \frac{5(17+51)}{2} = 170mm$$

$\boxed{a = 170mm}$

Step 10: calculation of b, d_1, v & ψ_p

1) $b = \psi \times a = 0.3 \times 170 = 51 mm$

2) $d_1 = m \times z_1 = 5 \times 17 = 85 mm$

3) $v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 85 \times 10^{-3} \times 1200}{60}$

$$\boxed{v = 5.34 m/s}$$

4) $\psi_p = \frac{b}{d_1} = \frac{51}{85} = 0.6$

Step 11: selection of quality of gear

From PSGDB 8.3, T-2

For velocity 5.34 m/s, → cylindrical gears
↓
Above 1 & upto 8
↓
8 gears are selected.

Step 12: Revision of design torque of gear $[M_t]$

From PSGDB 8.15, T-14

For $\psi_p = 0.6 \rightarrow K = 1.03$

From PSGDB 8.16, T-15

For ~~...~~ 8 gears, > 350, $v = 5.34 m/s$ → near 8 m/s

$K_d = 1.4$

Design torque $[M_t] = M_t \times k \times k_d$

$= 238.73 \times 1.03 \times 1.4$

$$\boxed{[M_t] = 344.25\ N\text{-}m}$$

$= 344.25 \times 10^3\ N\text{-}mm$

Step 13: checking for bending

From PSGDB 8.13 A

$$\sigma_b = \frac{i \pm 1}{a \times m \times b \times Y} \times [M_t]$$

From PSGDB 8.18, T-18

$z_1 = 17$, $z = 16$ $z = 18$

$$Y = \frac{0.355 + 0.377}{2} = 0.366$$

$$\boxed{Y = 0.366}$$

$$\sigma_b = \frac{(3+1)}{170 \times 5 \times 51 \times 0.366} \times 344.25 \times 10^3$$

$$\boxed{\sigma_b = 86.79\ N/mm^2}$$

$\sigma_b < [\sigma_b]$

$86.79 < 111.23$

$\therefore$ Design is safe & satisfactory

Step 14: check for wear strength

From PSGDB 8.13

$$\sigma_c = 0.74 \times \frac{i+1}{a}\sqrt{\frac{i+1}{ib} \times E_{eq}[M_t]}$$

$$= 0.74 \times \frac{(3+1)}{170}\sqrt{\frac{(3+1)}{3 \times 51} \times 2.15 \times 10^5 \times 344.25 \times 10^3}$$

$$\boxed{\sigma_c = 765.9 \text{ N/mm}^2}$$

$$\sigma_c < [\sigma_c]$$

$$765.9 \text{ N/mm}^2 < 852.64 \text{ N/mm}^2$$

∴ Design is safe and satisfactory

Step 15:- calculation of basic dimensions of pinion & gear

(i) module $m = 5$ mm

ii) Face width $b = 51$ mm

iii) Height factor $f_0 = 1$, for full depth teeth

iv) Bottom clearance $= c = 0.25m = 0.25 \times 5 = 1.25$ mm

v) Tooth depth $h = 2.25m = 2.25 \times 5 = 11.25$ mm

vi) PCD $d_1 = mz_1 = 5 \times 17 = 85$ mm

$d_2 = mz_2 = 5 \times 51 = 255$ mm

vii) Tip diameter $d_{a1} = (z_1 + 2f_0)m$

$= (17 + 2\times1)\times5$

$\boxed{d_{a1} = 95mm}$

$d_{a2} = (z_2 + 2f_0)m$

$= (51 + 2\times1)\times5 = 265mm$

$\boxed{d_{a2} = 265mm}$

viii) Root diameter

$d_{f1} = (z_1 - 2f_0)m - 2c$

$= (17 - 2\times1)\times5 - 2\times1.25$

$\boxed{d_{f1} = 72.5mm}$

$d_{f2} = (z_2 - 2f_0)m - 2c$

$= (51 - 2\times1)5 - 2\times1.25$

$\boxed{d_{f2} = 242.5mm}$

9 Design a spur gear drive to transmit 22.5 kW at 900 rpm. speed reduction is 2.5. materials for pinion and wheel are C15 steel and cast iron grade 30 respectively. Take pressure angle of 20° and working life of the gears as 10000 hrs.

given data:-

$P = 22.5\,kW,\ N_1 = 900\ rpm$

$i = 2.5,\ \phi = 20°,\ N = 10000\,hrs.$

To find:-

Design a spur gear.

solution:-

Since the materials for pinion and wheel are different, therefore we have to design the pinion first and check both pinion and wheel.

1) Gear ratio:

$i = 2.5$

2) Material selection:-

Pinion: C15 steel, case hardened to 55RC and core hardness < 350 wheel : C.I. grade 30.

3) Gear life :

$N = 10000 hrs$

Gear life in terms of number of cycles is given by

$N = 10000 \times 60 \times 900 = 54 \times 10^7$ cycles

4) Design torque $[M_t]$:

$$[M_t] = M_t \cdot K \cdot K_d$$

$$M_t = \frac{60 \times P}{2\pi N_1} = \frac{60 \times 22.5 \times 10^3}{2\pi \times 900} = 238.73 \text{ N-m}$$

$K \cdot K_d = 1.3$

$\therefore$ Design torque, $[M_t] = 238.73 \times 1.3$

$= 310.35$ N-m

5) Calculation of E_{eq}, $[\sigma_b]$, $[\sigma_c]$:

(i) To find E_{eq} :

From PSGDB - 8.14, T - 9,

for pinion (15) steel, Gear : Cast iron grade 30, equivalent Young's modulus, $E_{eq} = \frac{2E_1 E_2}{E_1 + E_2} = \frac{2 \times 2.08 \times 10^5 \times 1 \times 10^5}{2.08 \times 10^5 + 1 \times 10^5}$

$= \frac{4.16 \times 10^5 \times 10^5}{3.08 \times 10^5} = 1.35 \times 10^5$

(ii) To find $[\sigma_b]$:

The design bending stress $[\sigma_b]$ is given by

$$[\sigma_b] = \frac{1.4 \times k_{bl}}{n \cdot k_\sigma} \times \sigma_{-1}$$, assuming rotation in one direction only.

⇒ From PSGDB – 8.20, T–22, for steel [HB ≤ 350] and $N \geq 10^7$, $k_{bl} = 1$

⇒ From PSGDB – 8.19, T–20, for steel case hardened, factor of safety $n = 2$

⇒ From PSGDB – 8.19, T–21, for steel case hardened, stress concentration factor, $k_\sigma = 1.2$.

⇒ From PSGDB – 8.19, T–19, for forged steel, $\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50$.

But from PSGDB – 1.9, for C15,

$\sigma_u = 490\ N/mm^2$, & $\sigma_y = 240\ N/mm^2$.

$$\sigma_{-1} = 0.25(490 + 240) + 50$$
$$= 232.5\ N/mm^2.$$

$$[\sigma_b] = \frac{1.4 \times 1}{2 \times 1.2} \times 232.5 = 135.625\ N/mm^2$$

iii) To find $[\sigma_c]$: The design contact stress $[\sigma_c]$ is given by

$$[\sigma_c] = C_R \cdot HRC \cdot k_{cl}$$

$C_R = 22$, for C15, case hardened steel, from PSGDB-8.16, T-16

HRC = 55 to 63, for C15 steel, From PSGDB-8.16, T-16.

$K_{cl} = 1$, for $HB \leq 350$, $N \geq 10^7$, from PSGDB-8.17, T-17

$[\sigma_c] = 22 \times 63 \times 1 = 1386\ N/mm^2$

6) calculation of centre distance (a):

$$a \geq (i+1)\sqrt[3]{\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \frac{E_{eq}\,[M_t]}{i \times \psi}}$$

where $\psi = b/a = 0.3$ (assume initially)

$$a \geq (2.5+1)\sqrt[3]{\left(\frac{0.74}{1386}\right)^2 \times \frac{1.35 \times 10^5 \times 310.35 \times 10^3}{2.5 \times 0.3}}$$

$a \geq 88$

$\boxed{a = 88mm}$

7) To find z_1 and z_2:

(i) For 20° full depth system, select $z_1 = 18$

(ii) $z_2 = i \times z_1 = 2.5 \times 18 = 45$

8) Calculation of module (m):

W.K.T, $m = \frac{2a}{z_1+z_2} = \frac{2 \times 88}{18+45}$

$m = 2.79$ mm

Next std. module m=3mm, design is not safe

From PSGDB - 8.2, the next higher std. module, $m = 4$ mm.

9) Revision of centre distance:

New centre distance $a = \frac{m(z_1+z_2)}{2}$

$= \frac{4(18+45)}{2}$

$\boxed{a = 126 \text{ mm}}$

10) Calculation of b, d_1, ν and ψ_p

⇒ Face width (b): $b = \psi \cdot a = 0.3 \times 126$

$b = 37.8$ mm

⇒ pitch diameter of pinion (d_1):

$d_1 = m \cdot z_1$

$= 4 \times 18$

$\boxed{d_1 = 72 \text{ mm}}$

⇒ pitch line velocity (ν):

$\nu = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 72 \times 10^{-3} \times 900}{60} = 3.39$ m/s

11) selection of quality of gear:

From PSGDB 8.3, T-2

$v = 4.24\ m/s$, 8 gears are selected.

12) Revision of design torque $[M_t]$

From PSGDB 8.15, T-14

For $\psi_p = 0.525 \rightarrow k = 1.03$

From PSGDB 8.16, T-15

For 8 gears, $v = 4.24\ m/s \rightarrow k_d = 1.55$

$$[M_t] = M_t \times k \times k_d$$

$$= 238.73 \times 1.03 \times 1.55 = 381.13\ N\text{-}m$$

13) Checking for bending:-

$$\sigma_b = \frac{(i+1)}{a \times m \times b \times y}[M_t]$$

From PSGDB 8.18, T-18

For $z_1 = 18 \quad y = 0.377$

$$\sigma_b = \frac{(2.5+1)}{157.5 \times 5 \times 47.25 \times 0.377} \times 381.13 \times 10^3$$

$$\sigma_b = 95.092\ N/mm^2$$

$$\sigma_b < [\sigma_b]$$

$$95.092\ N/mm^2 < 135.625\ N/mm^2$$

Design is safe and satisfactory

14) Check for wear strength

From PSGDB 8.13 / T-8

$$\sigma_c = 0.74\ \frac{i+1}{a}\sqrt{\frac{i+1}{ib} \times E_{eq}[M_t]}$$

$$= 0.74\left(\frac{2.5+1}{157.5}\right)\sqrt{\left(\frac{2.5+1}{2.5 \times 47.25}\right) \times 1.35 \times 10^5 \times 381.13 \times 10^3}$$

$$\boxed{\sigma_c = 642.075\ N/mm^2}$$

$$\sigma_c < [\sigma_c]$$

$$642.075\ N/mm^2 < 1386\ N/mm^2$$

Design is safe and satisfactory.

15) Check for wheel:

i) cal. of $[\sigma_b]$ wheel and $[\sigma_c]$ wheel

Wheel material: CI grade 30

$$N_2 = \frac{N_1}{i} = \frac{900}{2.5} = 360 rpm$$

Life of wheel $= 10000$ hrs

$= 10000 \times 60 \times 360$

$= 21.6 \times 10^7$ cycles.

From PSGDB: 8.18, below T-18

$[\sigma_b]$ wheel $= \dfrac{1.4 \times K_{bl}}{n \times K_\sigma} \times \sigma_{-1}$ [Assuming rotatio in one direction only]

From PSGDB 8.20, T-22

For cast Iron wheel, $K_{bl} = \sqrt[9]{\dfrac{10^7}{N}}$

$= \sqrt[9]{\dfrac{10^7}{21.6 \times 10^7}}$

$\boxed{K_{bl} = 0.710}$

From PSGDB 8.19, T-20

For cast iron, Tempered (or) normalized

$n = 2$

From PSGDB 8.19, T-21

For cast iron, $0 \leq x\ 0.1$

$K_\sigma = 1.2$

From PSGDB 8.19, T-19

for cast iron, $\sigma_{-1} = 0.45\sigma_u$

Assume $\sigma_u = 290 N/mm^2$ [From PSGDB 1.5]

$\therefore \sigma_{-1} = 0.45 \times 290 = 130.5 N/mm^2$

$[\sigma_b]$ wheel $= \dfrac{1.4 \times 0.710}{2 \times 1.2} \times 130.5$

$\boxed{[\sigma_b]_{wheel} = 54.04 N/mm^2}$

To find $[\sigma_c]$ wheel

From PSGDB 8.16, below T-15

$$[\sigma_c]_{wheel} = C_B \cdot HB \cdot K_{cl}$$

$\Rightarrow$ From PSGDB 8.16, T-16
For cast iron 30
$C_B = 2.3$

$\Rightarrow$ From PSGDB 8.16, T-16
$HB = 200$ to 260, Take 260
$\therefore HB = 260$

$\Rightarrow$ From PSGDB 8.17, T-17
For cast iron

$$K_{cl} = \sqrt[6]{\frac{10^7}{N}}$$

$$= \sqrt[6]{\frac{10^7}{21.6\times10^7}} = 0.599$$

$$[\sigma_c]\,\text{wheel} = 2.3\times260\times0.879 = 358.202\ N/mm^2$$

(ii) check for bending

$$\sigma_{b_1}\times y_1 = \sigma_{b_2}\times y_2$$

$z_2 = 45 \rightarrow y_2 = 0.471$

$z_1 = 18 \rightarrow y_1 = 0.377$, $\sigma_{b_1} = 85.89\ N/mm^2$

$$95.09\times0.377 = \sigma_{b_2}\times0.471$$

$\boxed{\sigma_{b2} = 76.11 \ N/mm^2}$

Compare σ_{b2} with $[\sigma_b]$ wheel

$\sigma_{b2} > [\sigma_b]_{wheel}$

$76.11 \ N/mm^2 > 54.04 \ N/mm^2$

Design is not safe.

we are going to Redesign,

Change the material, choose C40 steel for wheel.

From 8.18, below T-18

$$[\sigma_b]_{wheel} = \frac{1.4 \ K_{bl}}{K_\sigma \cdot n} \times \sigma_{-1}$$

From 8.20, T-22

$K_{bl} = 1$

[for <350, >10^7]

$n = 2.5$

[surface hardened]

$K_\sigma = 1.5$

$\sigma_{-1} = 0.25(\sigma_u + \sigma_y) + 50$

From PSGDB: 1.9

$\sigma_u = 680 N/mm^2$, $\sigma_y = 330 \ N/mm^2$

$\sigma_{-1} = 0.25(680 + 330) + 50$

$= 302.5 \ N/mm^2$

$$[\sigma_b]_{wheel} = \frac{1.4 \times 1}{2.5 \times 1.5} \times 302.5$$

$\boxed{[\sigma_b]_{wheel} = 112.94 N/mm^2}$

$\sigma_{b2} = 76.11\ N/mm^2$ $[\sigma_b]_{wheel} = 112.94\ N/mm^2$

$\sigma_{b2} < [\sigma_b]_{wheel}$

$\therefore$ Design is safe and satisfactory.

Check for wearing

σ_c wheel $= \sigma_c$ pinion $= 642.075\ N/mm^2$

σ_c wheel $> [\sigma_c]$ wheel

$\therefore$ Design is not safe.

Redesign $[\sigma_c]$ wheel.

For C40 steel, From 8.16

$C_R = 23$ HRC $= 40$ to 55, take HRC $= 55$

$K_{cl} = 1$ [from 8.17, For < 350, $\geq 10^7$]

$[\sigma_c]$ wheel $= C_R \times HRC \times K_{cl}$

$= 23 \times 55 \times 1$

$[\sigma_c]$ wheel $= 1265\ N/mm^2$

σ_c wheel $< [\sigma_c]$ wheel

$642.075 < 1265\ N/mm^2$

$\therefore$ Design is safe and satisfactory.

16) Calculation of basic dimensions of Pinion and Wheel

(i) module $m = 5mm$

(ii) Face width $b = 47.25mm$

(iii) Height factor $f_0 = 1$ for Full depth teeth.

(iv) Bottom clearance : $c = 0.25m = 0.25 \times 5$

$\boxed{c = 1.25mm}$

(v) Tooth depth : $h = 2.25m = 2.25 \times 5$

$\boxed{h = 11.25mm}$

(vi) pitch circle diameter $d_1 = m \cdot z_1 = 5 \times 18$

$\boxed{d_1 = 90mm}$

$d_2 = m \cdot z_2 = 5 \times 45$

$\boxed{d_2 = 225mm}$

(vii) Tip diameter $d_{a1} = (z_1 + 2f_0)m$

$= (18 + 2 \times 1)5 = 100mm$

$\boxed{d_{a1} = 100mm}$

$d_{a2} = (z_2 + 2f_0)m$

$= (45 + 2 \times 1)5$

$\boxed{d_{a2} = 235mm}$

(viii) Root diameter $d_{f1} = (z_1 - 2f_0)m - 2c$

$= (18 - 2 \times 1)5 - 2 \times 1.25$

$\boxed{d_{f1} = 77.5mm}$

$d_{f2} = (z_2 - 2f_0)m - 2c$

$(45 - 2 \times 1)5 - 2 \times 1.25 \Rightarrow \boxed{d_{f2} = 212.5mm}$

Design Procedure of Spur Gear using Lewis - Buckingham Equations

Step 1: Section of materials (Same as life method) (PSGDB 1.40, 1.9)

Step 2: Calculation of No. of teeth Assume Z_1 = 16 to 24 (if not given) and calculate $Z_2 = iZ_1$

Step 3: Calculation of tangential load $F_t = \frac{\text{Power}}{\text{Velocity}} \times K_0$

where K_0 = Service / Shock factor

Type of Load	K_o
Steady	1.00
Light shock	1.25
Medium Shock	1.50
Heavy Shock	2.00

Step 4: Calculate of initial dynamic load $F_d = F_t \times C_v$

Step 5: Calculate the beam strength $F_s = (\sigma_b)byP_c$ (PSGDB: 8.50)

$F_s = \pi\, mb(\sigma_b)y$

$F_s \geq F_d$

Step 6: Calculation of Module.

Module is calculate by equating F_s & F_d

Step 7: Calculate the face plate, PCD& Pitch line velocity of the pinion.

Step 8: Recalculate F_s

Step 9: Calculate the Buckingham dynamic load formula available in 8.51

Step 10: Check for beam strength compare F_s and F_d, $F_s \geq F_d$ then the Gear tooth has adequate beam strength and their not fail by percentage that is design is satisfactory.

$F_s \leq F_d$ the design is not satisfactory then increase the face plate (or) module (or) both until $F_s \geq F_d$

Step 11: Calculation of wear load (F_w)

Step 12: Checking for wear compare dynamic load and wear load if $F_d < F_w$ (or) $F_w > F_d$, then the gear tooth have adequate wear capacity and it will not wear out thus the design is satisfactory if not, increase the face plate (N) until the condition satisfies.

Step 13: Calculation of basic dimension of gears P.No. 8.22

Write the specification and draw a neat sketch

Note :

In Lewis Buckingham equation method the design should be based on weather element, if the pinion and wheel are made up of same material then the pinion is weaker. If pinion and wheel is made up of different materials then (σ_b) decides the weaker element. If (σ_b) allowable of pinion then the wheel is weather element and the design should be based on wheel otherwise and life.

Design Procedure of Spur Gear using Lewis - Buckingham Equation

1. Design a spur gear drive to transmit 45 kW at a pinion of 800 rpm the velocity ratio is 3.5 both the pinion and wheel is made up of steel use beam strength to check the validity of your design.

Given

Power P = 45 kW

Pinion speed N = 800 rpm

Velocity ratio i = 3.5

Step 1: Section of materials

(Assume C_{45} Steel for both pinion and wheel)

Assuming module up to 6 mm

$\sigma_b = 1400$ kgf/cm^2

Step 2: Calculation of Z_1 & Z_2

Assume $Z_1 = 18$

$Z_2 = 3.5 \times 18$

$Z_2 = 63$

Step 3: Calculation of tangential load

$$F_t = \frac{\text{Power}}{\text{Velocity}} \times K_0 \qquad (d_1 = mZ_1)$$

$$\text{Velocity} = \frac{\pi d_1 n_1}{60} = \frac{\pi \times m \times 18 \times 10^{-3} \times 800}{60} = 0.754m \text{ m/s}$$

$V_m = 0.754m$ m/sec

Assume Medium shock condition

$K_0 = 1.5$

$$F_t = \frac{45 \times 10^3}{0.754\ m} \times 1.5$$

$$F_t = \frac{89.522 \times 10^3}{m}\ N$$

Step 4: Calculate of initial dynamic load ($F_d = F_t \times C_v$)

Assuming Velocity V_m = 5 to 20 m/Sec, V_m = 12 m/Sec (From PSGDB 8.51) $C_V = (6 + V_m) / 6$

$= 6 + 12 / 6 = 3$

$F_d = F_t \times C_v$

$$F_d = \frac{89.522 \text{ x } 10^3}{m} \times 3$$

$$F_d = \frac{268.566 \text{ x } 10^3}{m}$$

Step 5: Calculate the beam strength

$F_s = (\sigma_b) byP_c$

P_c = Circular Pitch = $\pi d / z = P_c = \pi m$ ($m = d/z$)

Assume $b / m = 10$, $b = 10m$

From PSGDB-8.50 $Y = 0.154 - (0.912 / Z)$ (for angle 20^o involute)

$Y = 0.154 - (0.912 / 18)$

$Y = 0.1033$

$F_s = 1400 \times 10 \times 10^{-2} \times 10m \times 0.1033 \times \pi m$

$F_s = 454.34\ m^2 N$

Step 6: Calculation of Module

Equating F_s & F_d

$F_s = F_d$

$$454.34m^2 = (268.56 \times 10^3)/ m$$

$$m^3 = (268.56 \times 10^3)/ 454.34$$

$$m = 8.392mm$$

get std m = 10mm

Step 7: Calculation of b, d_1, d_2 and v

$b = 10\ m$

$b = 10 \times 10$

$= 100\ mm$

$b = 100\ mm$

$d_1 = mZ_1$

$= 10 \times 18$

$d_1 = 180\ mm$

$d_2 = mZ_2$

$= 10 \times 63$

$d_2 = 630\ mm$

$V_m = 0.754\ m$

$= 0.754 \times 10$

$V_m = 7.54\ m/Sec$

Step 8: Recalculate F_s

$F_s = 454.34 \times 10^2$

$F_s = 45.434 \times 10^3 N$

$F_s = 45.43\ KN$

Step 9: Buckingham dynamic

$$F_d = F_t + \left(\frac{0.164\ V_m\ (Cb + F_t)}{0.164\ V_m + 1.485\ \sqrt{c \times b + F_t})} \right)$$

$$F_t = \frac{89.52 \times 10^3}{10} = 8.952 \times 10^3 N$$

C = 11860 e (from PSGDB -8.53, T-41 & T-42)

For carefully cut gear and module m = 10 mm

e = 0.044

C = 521.84 mm

$V1 = V_m = 7.54 \times 60$

$= 452.4 \text{ m / min}$

$$F_d = 8.952x10^3 + \left(\frac{0.164x452.4(521.8x10^{-3} \text{ x } 100 \text{ x } 10^{-3} + 8.952x10^3}{0.164 \text{ x}452.4+1.485\sqrt{521.8x10^{-3}x100 \text{ x } 10^{-3} + 8.952x10^{3-}}} \right)$$

$F_d = 12045.84$

$F_d = 12.045 \text{ x } 10^3 N = 12.045 \text{ kN}$

Step 10: Check for beam strength

$F_s = 45.43$ KN $F_d = 12.045$ KN

The Condition to be satisfied $Fs \geq F_d$

Here $F_s \geq F_d$ so design is satisfied

Step 11: Calculation of wear load (F_w)

$$F_W = \frac{d_1 Qkb}{}$$

(From PSGDB-8.51)

$$Q = \frac{2i}{i \pm 1} = \frac{2 \times 3.5}{3.5 + 1}$$

$= 1.56$

$$K = \frac{\sigma_c^2 \text{ Sin } \alpha \text{ ((1/E1) + (1/E2))}}{1.4}$$

Assume $\sigma_c = 5000 \text{ kgf/cm}^2$

$E1 = E2 = 2.15 \times 10^6 \text{ kgf/cm}^2$

$$K = \frac{(5000)^2 \text{ Sin } 20 \text{ ((1/2.15}\times10^6) + (1/2.15\times10^6))}{1.4}$$

$K = 5.68 \text{ kgf/cm}^2$

$F_w = 180 \times 1.56 \times 5.68 \times 10 \times 10^{-2} \times 100$

$= 15.949 \text{ x } 10^3 N$

$F_w = 15.949 \text{ kN}$

Step 12: Checking for wear

compare $F_d < F_w$

F_d = 12.045 KN

F_w = 15.949 KN

For Safe Design $F_w \geq F_d$

Here $F_w \geq F_d$ So design is safe

Step 13: **Specification** 8.22

Module (m) = 10 mm

Centre distance (a) = m $(z_1+z_2)/2$ = 10 × (18 + 63)/2 = 405

Height Factor (f_o) = 1

Bottom Clearance (C) = 0.25 m

= 0.25×10

C = 2.5 mm

Tooth Depth (b) = 2.25 m = 2.25×10 = 22.5 mm

Pitch Dia (d) = $d_1 = mz_1$ = 10×18 = 180 mm

$d_2 = m\ z_2$ = 10 × 63 = 630 mm

Tip Dia $da_1 = (Z_1 + 2f_o)$

= 18 + 2 × 1 = 20 mm

$da_2 = (Z_2 + 2f_o)$

= 63 + 2 ×1 = 65 mm

UNIT-II

HELICAL GEAR

10/2/15

DESIGN PROCEDURE FOR HELICAL GEAR BASED ON MANUFACTURER'S DATA [(OR) GEAR LIFE]

Design procedure is same as spur gear design.

1) calculation of gear ratio(i):

$$i = \frac{N_1}{N_2} = \frac{z_2}{z_1}$$

2) select the suitable combination of materials:

For pinion & wheel [1.40]

3) GEAR LIFE:-

If gear life is not given (assume gear life = 20000 hrs)

4) calculation of initial design torque $[M_t]$:

$$[M_t] = M_t \times k \times k_d \qquad M_t = \frac{60 \times P}{2\pi N}$$

Initially assume $k \cdot k_d = 1.3$

5) calculation of E_{eq}, $[\sigma_b]$ and $[\sigma_c]$:-

(i) From PSGDB - 8.14, T-9

(ii) From PSGDB - 8.18 $[\sigma_b] = \frac{1.4}{n} \frac{k_{bl}}{k_\sigma} \sigma_{-1}$

(iii) $[\sigma_c]$, PSGDB 8.13, T-8

k_{bl} - 8.20, T-22

k_σ = 8.19, T-20

σ_{-1} = 8.19, T-19

n = 8.19, T-20

6) calculation of centre distance (a).

From 8.13

$\psi = b/a = 0.3$ (initial)

7) selection of number of teeth (z_1 and z_2):

If it is not given, assume $z_1 \geq 17$

$z_2 = i z_1$

8) calculation of normal module (m_n):

$$m_n = \frac{2a}{(z_1+z_2)} \times \cos\beta \quad [8.22]$$

From PSGDB: 8.2

9) Revision of centre distance (a):

$$a = \left(\frac{m_n}{\cos\beta}\right) \times \left(\frac{z_1+z_2}{2}\right) \quad [8.22]$$

10) calculation of b, d_1, v and ψp:

(i) $b = \psi \times a$

(ii) Axial pitch $= \dfrac{\pi d_1}{z_1 \times \tan\beta}$

(iii) $d_1 = \dfrac{m_n}{\cos\beta} \times z_1$ [8.22] T-26

(iv) $v = \dfrac{\pi d_1 N_1}{60}$

(v) $\psi = b/d_1$

11) ~~selection~~ select the suitable quality of gear

From [8.3, T-2]

12) Revision of design torque [M_t]

(i) Revise k: 8.15, T-14

(ii) Revise k_d: 8.16, T-15

(iii) Revise [M_t]: $[M_t] = M_t \times k \times k_d$

13) Check for bending:

From 8.13A, T-8 $\Rightarrow \sigma_b$

y_v - 8.18, T-18 | $z_{v_1} = \frac{z_1}{\cos^3\beta}$ [8.22, T-26]

$\sigma_b < [\sigma_b]$. Thus the design is satisfactory

14) Check for wear strength:

From 8.13 $\quad \sigma_c < [\sigma_c]$. Thus the design is satisfactory.

15) Calculation of basic dimensions of pinion and gear

From 8.22, T-26

For intermittent duty of an elevator, two cylindrical gears have to transmit 12.5 kW at a pinion speed of 1200 rpm. Design the gear pair for the following specifications: gear ratio 3.5, pressure angle 20°, involute full depth, helix angle 15°. Gear are expected to work 6 hours a day for 10 years.

Given:-

$P = 12.5\,kW \quad N_1 = 1200\,rpm, \; i = 3.5, \; \phi = 20°\,FD$

$\beta = 15°$

To find:-

Design the helical gear pair.

Solution:-

1) Gear ratio: $i = 3.5$ [given]

2) Selection of material: 40 Ni2 Cr1 Mo28 [Direct hardening Alloy steel]

3) Gear life:-

Gear life = 6 hrs/day × 365 days/year × 10

= 21,900 hrs

Gear life in terms of number of cycles,

$N = 21,900 \times 1200 \times 60$

$= 157.7 \times 10^7$ cycles.

4) Calculation of initial design torque $[M_t]$

$[M_t] = M_t \times k \times k_d$

Assume $k \times k_d = 1.3$

$$[M_t = \frac{60 \times P}{2\pi N} = \frac{60 \times 12.5 \times 10^3}{2 \times \pi \times 1200}$$

$$M_t = 99.47 \text{ N-m}$$

$$[M_t] = 99.47 \times 1.3 = 129.31 \text{ N-m}$$

$$\boxed{[M_t] = 129.31 \text{ N-m}}$$

5) calculation of Eeq, [σ_b] and [σ_c]

(i) $E_{eq} = 2.15 \times 10^5 \ N/mm^2$

(ii) $[\sigma_b] = \dfrac{1.4 \ K_{bl}}{n \cdot K_\sigma} \times \sigma_{-1}$, [For rotation in one directio]

8.18

From 8.20, T-22

$K_{bl} = 0.7$ [≥ 350, Life in No. of cycles $\geq 25 \times 10^7$) [8.20, T-22]

$K_\sigma = 1.5$, [For steel hardened] [8.18, T-21]

$n = 2.5$, [For steel hardened] [8.19, T-20]

[Surface hardened]

$\sigma_{-1} = 0.35 \ \sigma_u + 120$, For alloy steel [8.19, T-19]

$\sigma_u = 1550 \ N/mm^2$ [1.40]

$\sigma_{-1} = 0.35 \times 1550 + 120 = 662.5 \ N/mm^2$

$[\sigma_b] = \dfrac{1.4 \times 0.7}{2.5 \times 1.5} \times 662.5 = 173.133 \ N/mm^2$

$$\boxed{[\sigma_b] = 173.133 \ N/mm^2}$$

(iii) To find [σ_c]

$[\sigma_c] = C_R \times HRc \times K_{cl}$

$C_R = 26.5$, [8.16, T-16, For alloy steel hardened]

$HRc = 40$ to 55 [8.16, T-16]

$K_{cl} = 0.585$ [8.17, T-17, For HB > 350 & N $\geq 25 \times 10^7$]

$[\sigma_c] = 26.5 \times 55 \times 0.585 = 852.6 \ N/mm^2$

$[\sigma_c] = 852.6 \ N/mm^2$

6) calculation of centre distance (a):

W.K.T,

$$a \geq (i+1)\sqrt[3]{\left(\frac{0.7}{[\sigma_c]}\right)^2 \frac{Eeq[M_t]}{i\psi}}$$

$$a \geq (3.5+1)\sqrt[3]{\left(\frac{0.7}{852.6}\right)^2 \times \frac{2.15\times10^5\times129.31\times10^3}{3.5\times0.3}}$$

$a \geq 117.6\ mm \approx \boxed{a=120mm}$

7) Assume $z_1=20$

$z_2 = i \times z_1 = 3.5\times20=70$

$\boxed{z_1=20}$ $\boxed{z_2=70}$

8) calculation of normal module (m_n):

W.K.T, $m_n = \frac{2a}{(z_1+z_2)} \times \cos\beta = \frac{2\times120}{(20+70)} \times \cos15^\circ$

$\boxed{m_n = 2.576\ mm}$

From 8.2, T-1 [For choice(1)]

std. normal module $\boxed{m_n=3}$

9) Revision of centre distance:

$$a = \left(\frac{m_n}{\cos\beta}\right)\times\left(\frac{z_1+z_2}{2}\right)$$

$$= \left(\frac{3}{\cos15}\right)\times\left(\frac{20+70}{2}\right) = 139.76mm$$

$\boxed{a=139.76mm}$

10) calculation of b, d_1, v, and ψ_p:

(i) Face width $b = \psi \times a = 0.3 \times 139.76 = 41.93$

$\approx 42mm$

$\boxed{b = 42mm}$

Axial pitch $p_a = \frac{\pi \times m_n}{\sin\beta} = \frac{\pi \times 3}{\sin 15^\circ} = 36.4mm$

$\boxed{p_a = 36.4mm}$

(ii) pitch diameter of pinion (d_1): $d_1 = \frac{m_n}{\cos\beta} \times z_1$

$d_1 = \frac{3}{\sin 15^\circ} \times 20$

$\boxed{d_1 = 62.12mm}$

(iii) pitch line velocity (v): $v = \frac{\pi d_1 N_1}{60}$

$v = \frac{\pi \times 62.12 \times 10^{-3} \times 1200}{60}$

$\boxed{v = 3.903 m/s}$

(iv) $\psi_p = \frac{b}{d_1} = \frac{42}{62.12} = 0.676 \Rightarrow \boxed{\psi_p = 0.676}$

11) selection of quality of gear:

From 8.3, T-2, For $v = 3.903 m/s$.

IS quality 8 is selected.

12) Revision of design torque $[M_t]$:

W.K.T, $[M_t] = M_t \times K \times K_d$

From, 8.15, T-14,

$K = 1.045$ [For $\psi p = 0.676$]

From, 8.16, T-15,

$K_d = 1.2$ [For $v = 3$ m/s, 7350]

$[M_t] = 99.47 \times 1.045 \times 1.2$

$\boxed{[M_t] = 124.74 \text{ N-m}}$

13) Check for bending:-

From 8.13A, For checking

$$\sigma_b = \frac{0.7(i+1)[M_t]}{a b m_n y_v}$$

[8.22, T-16]

From 8.18, T-18

$y_v = 0.402$ [For $z_{v_1} = 22$]

$$z_v = \frac{z_1}{\cos^3\beta} = \frac{20}{\cos^3 15°}$$

$z_v = 22$

$$\sigma_b = \frac{0.7(3.5+1)[124.74 \times 10^3]}{139.76 \times 42 \times 3 \times 0.402}$$

$\boxed{\sigma_b = 55.5 \text{ N/mm}^2}$

$\sigma_b < [\sigma_b]$

$\because 55.5 < 173.33$

Thus the design is satisfactory.

14) Check for wear strength:-

From 8.13, For checking

$$\sigma_c = 0.7 \frac{i+1}{a} \sqrt{\frac{i+1}{ib} \times E_{eq} [M_t]}$$

$$= 0.7 \left(\frac{3.5+1}{139.76}\right) \sqrt{\frac{3.5+1}{3.5 \times 42} \times 2.15 \times 10^5 \times 124.74 \times 10^3}$$

$$\boxed{\sigma_c = 645.8 \, N/mm^2}$$

$$645.8 < 852.6$$

$$\therefore \sigma_c < [\sigma_c]$$

Thus the design is safe and satisfactory.

15) Calculation of basic dimensions of pinion and gear:

From 8.22, T-26

(i) Normal module $m_n = 3mm$

(ii) No. of teeth $z_1 = 20, z_2 = 70$

(iii) pitch circle diameter $(d_1) = 62.12mm$

$$d_2 = \frac{m_n}{\cos\beta} \times z_2$$

$$= \frac{3}{\cos 15} \times 70 = 217.4mm$$

(iv) centre distance $a = 139.76mm$

(v) Height factor $f_o = 1$

(vi) Bottom clearance $C = 0.25 m_n = 0.25 \times 3$

$$\boxed{C = 0.75\,mm}$$

(vii) Tooth depth $h = 2.25 m_n = 2.25 \times 3 = 6.75\,mm$

$$\boxed{h = 6.75\,mm}$$

(viii) Tip diameter $d_{a1} = \left(\frac{z_1}{\cos\beta} + 2f_0\right) m_n$

$$d_{a1} = \left(\frac{20}{\cos 15^\circ} + 2\times1\right)3$$

$$\boxed{d_{a1} = 68.12\,mm}$$

$$d_{a2} = \left(\frac{z_2}{\cos\beta} + 2f_0\right) m_n$$

$$= \left(\frac{70}{\cos 15^\circ} + 2\times1\right)\times3$$

$$\boxed{d_{a2} = 223.41\,mm}$$

(ix) Root diameter $d_{f1} = \left(\frac{z_1}{\cos\beta} - 2f_0\right) m_n - 2C$

$$d_{f1} = \left(\frac{20}{\cos 15^\circ} - 2\times1\right)3 - 2\times0.75$$

$$\boxed{d_{f1} = 54.6\,mm}$$

$$d_{f2} = \left(\frac{z_2}{\cos\beta} - 2f_0\right) m_n - 2C = \left(\frac{70}{\cos 15^\circ} - 2\times1\right)3$$

$$\boxed{d_{f2} = 209.91\,mm}$$

(x) Virtual no. of teeth $\boxed{z_{v1} = 22}$, $z_{v2} = \frac{z_2}{\cos^3\beta} = \frac{70}{\cos^3 15^\circ} = 78$

HELICAL GEAR DESIGN USING LEWIS AND BUCKINGHAM'S EQUATIONS

1) Selection of material:-

If not given, select a suitable pinion and gear materials.

2) calculation of z_1 and z_2:

Assume $z_1 \geq 17$

$$z_2 = i z_1$$

3) calculation of tangential load (F_t):

$$F_t = \frac{P}{v} \times K_o$$

P = power (given)

$v = \frac{\pi d N}{60}$ m/s

K_o = service / shock factor [Same as spur gear]

4) calculation of initial dynamic load (F_d):

$$F_d = \frac{F_t}{C_v}$$

C_v = same as spur gear [8.51]

5) calculation of beam strength (F_s)

From 8.51

$$F_s = [\sigma_b]\, b\, Y_v P_c$$
$$= [\sigma_b]\, b\, Y_v \pi m_n$$

[W.K.T $P_c = \pi m$
helical gear $P_c = \pi m_n$]

where m_n = normal module in mm

b = Face width in mm, Assume $b = 10m_n$

$[\sigma_b]$ = Design bending stress.

Y_v = Form factor based on virtual number of teeth

6) calculation of normal module (m_n):

equating F_s and F_d. Find m_n

From 8.2, T-1, std. module is selected.

7) calculation of b, d and v:

(i) $b = 10m_n$

(ii) Pcd (d_1): $d_1 = \frac{m_n}{\cos\beta} \times z_1$, [From 8.22]

(iii) PLV (v) : $v = \frac{\pi d_1 N_1}{60}$

8) Recalculation of the beam strength (F_s):

$F_s = [\sigma_b] b Y_v \pi m_n$

9) calculation of accurate dynamic load:- (F_d)

From 8.51, For helical gear

$$F_d = F_t + \left[\frac{0.164 V_m (C b \cos^2\beta + F_t) \cos\beta}{0.164 V_m + 1.485 \sqrt{C b \cos^2\beta + F_t}} \right]$$

(i) $F_t = P/v$

ii) $V_m = v \times 60$ m/min

(iii) c [From 8.53, T-41]

(iv) e [From 8.53, T-42]

10) check for beam strength (or tooth breakage)

compare F_s & F_d

$F_s > F_d$ Design is safe and satisfactory

IF $F_d > F_s$, Design is not safe, redesign

change the e value. (i.e select Precision gear)

11) calculation of max. wear load (F_w):

From 8.51, For helical gears

$$F_w = \frac{b\, d_1\, Q\, k}{\cos^2\beta}$$

(i) b (calculated)

(ii) d_1 (calculated)

(iii) $Q = \frac{2i}{i+1}$ [same as spur gear, 8.51]

(iv) $k = [\sigma_c^2] \frac{\sin\alpha_n}{1.4} \left[\frac{1}{E_1} + \frac{1}{E_2}\right]$

[From 8.51, For helical gear]

12) Check for wear :-

compare Fw and Fd

$F_w > F_d$ ∴ Design is safe and satisfactory

13) calculation of basic dimensions of pinion and gear

From 8.22, T-26

HELICAL GEAR DESIGN USING LEWIS AND BUCKINGHAM's EQUATIONS

Design a helical gear to transmit 15KW at 1400 rpm to the following specifications: speed reduction is 3, Pressure angle is 20°, Helix angle is 15°, The material of both the gears is C45 steel. Allowable static stress 180 N/mm², surface endurance limit is 800 N/mm², Young's modulus of material $= 2\times10^5 N/mm^2$

Given:-

$P = 15KW = 15\times10^3 W$	$[\sigma_b] = 180 N/mm^2$
$N_1 = 1400$ rpm	$[\sigma_c] = b_{es} = 800 N/mm^2$
$i = 3$	$E_1 = E_2 = 2\times10^5 N/mm^2$
$\alpha_n = 20°$	
$\beta = 15°$	

To find:-

Design a helical gear.

solution:-

1) material selection:

Pinion and gear - C45 steel is selected.

2) calculation of z_1 & z_2:

(i) Assume $z_1 = 20$

(ii) $z_2 = i\times z_1 = 3\times20 = 60$

3) calculation of tangential load on tooth (F_t):

From 8.22

W.K.T $d_1 = \frac{m_n}{\cos\beta} \times z_1$

(i) $F_t = \frac{P}{v} \times k_o$

(ii) $v = \frac{\pi d_1 N_1}{60} = \frac{\pi N_1}{60}\left[\frac{m_n}{\cos\beta} \times \frac{z_1}{1000}\right]$

$= \frac{\pi \times 1400}{60}\left[\frac{m_n \times 20}{\cos 15 \times 1000}\right]$

$$\boxed{v = 1.518\, m_n \ \ m/s}$$

(iii) Assume $k_o = 1.25$ [Light shock]

(iv) $F_t = \frac{15 \times 10^3}{1.518\, m_n} \times 1.25$

$$\boxed{F_t = \frac{12351.77}{m_n}}$$

4) calculation of initial dynamic load (F_d):

(i) $F_d = \frac{F_t}{C_v}$

(ii) $C_v = \frac{6}{6+v} = \frac{6}{6+15} = 0.286$ $\quad v = 15\, m/s$ [Assume]

$$\boxed{C_v = 0.286}$$

$$F_d = \frac{12351.77}{m_n} \times \frac{1}{0.286}$$

$$\boxed{F_d = \frac{43188.00}{m_n}}$$

5) calculation of beam strength (F_s):

From 8.51, Helical gears

$F_s = [\sigma_b] \, b \, Y_v \, P_{cn}$ W.K.T $P_c = \pi m$

For helical gear $P_{cn} = \pi m_n$

(i) $[\sigma_b] = 180 \; N/mm^2$ [Given]

(ii) $b = 10 m_n$ [Assume]

iii) $Y_v = 0.154 - \frac{0.912}{Z_v}$, for 20° involute

$$\Rightarrow Z_v = \frac{Z_1}{\cos^3\beta} = \frac{20}{\cos 15 \times \cos 15 \times \cos 15} \quad [8.22]$$

$$Z_v = 22.192 \approx 23$$

$$\boxed{Z_v = 23}$$

$$Y_v = 0.154 - \frac{0.912}{23}$$

$$\boxed{Y_v = 0.1143}$$

(iv) $P_{cn} = \pi m_n$

$$F_s = 180 \times 10 m_n \times 0.1143 \times \pi \times m_n$$

$$\boxed{F_s = 646.35\, m_n^2}$$

6) calculation of normal module (m_n)

equating $F_s = F_d$

$$646.35\, m_n^2 = \frac{43188}{m_n}$$

$$m_n^3 = \frac{43188}{646.35}$$

$$m_n = \sqrt[3]{\frac{43188}{646.35}}$$

$$m_n = 4.05\ mm$$

From 8.2, T-1, std. normal module $m_n = 5\,mm$

7) calculation of b, d_1 and v:-

(i) $b = 10 m_n = 10 \times 5 = 50\,mm \Rightarrow \boxed{b = 50\,mm}$

(ii) Pcd : $d_1 = \frac{m_n}{\cos\beta} \times z_1 = \frac{5}{\cos 15} \times 20$ [From 8.22]

$$\boxed{d_1 = 103.53\,mm}$$

(iii) PLV : $v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 103.53 \times 10^{-3} \times 1400}{60}$

$$\boxed{v = 7.59\ m/s}$$

8) Recalculation of the beam strength (F_s)

$$F_s = 646.35 \times m_n^2$$

$$= 646.35 \times 5^2$$

$$\boxed{F_s = 16{,}158.75\ N}$$

9) calculation of accurate dynamic load (F_d):

From PSGTDB 8.51

$$F_d = F_t + \left[\frac{0.164\, V_m\, (c b \cos^2\beta + F_t) \cos\beta}{0.164\, V_m + 1.485 \sqrt{c b \cos^2\beta + F_t}} \right]$$

(i) $F_t = \frac{P}{v} = \frac{15 \times 10^3}{7.59} = 1976.28\,N$

$$\boxed{F_t = 1976.28\ N}$$

(ii) $V_m = v \times 60 = 7.59 \times 60 = 455.4$ m/min

(iii) $c = 11860\,e$ [For steel pinion steel gear, 20° FD]
From PSGTDB 8.53, T-41

(iv) $e = 0.025$ [carefully cut gears]

$c = 11860 \times 0.025 = 296.5$ mm $= 296.5 \times 10^{-3}$ m

$$\boxed{c = 296.5 \times 10^{-3}\ m}$$

(v) $b = 50$ mm $\Rightarrow 50 \times 10^{-3}$ m

$$\boxed{b = 50 \times 10^{-3} m}$$

$$F_d = 1976.28 + \frac{\left[0.164 \times 455.4 \times \cos 15 \left[296.5\times10^{-3} \times 50\times10^{-3} \times \cos 15 \times \cos 15 + 1976.28\right]\right]}{\left[0.164 \times 455.4 + 1.485\sqrt{296.5\times10^{-3}\times50\times10^{-3} \times \cos 15 \times \cos 15 + 1976.28}\right]}$$

$$\boxed{F_d = 2989.56\,N}$$

10) check for beam strength (or tooth breakage):

compare F_s and F_d

$F_s = 16158.75\,N$, $F_d = 2989.56\,N$

$F_s > F_d$

$\therefore$ The design is safe and satisfactory.

11) calculation of the maximum wear load (F_w):

From 8.51, Helical gears

$$F_w = \frac{b d_1 Q K}{\cos^2\beta}$$

(i) $b = 50\,mm$

(ii) $d_1 = 103.53\,mm$

(iii) $Q = \frac{2i}{i+1}$ [same as spur gear, 8.51]

$$Q = \frac{2 \times 3}{3+1} = 1.5$$

(iv) $$K = [\sigma_c^2] \frac{\sin \alpha_n}{1.4} \left[\frac{1}{E_1} + \frac{1}{E_2} \right]$$ [From 8.51, Helical gear]

$$K = [800^2] \times \frac{\sin 20}{1.4} \left[\frac{1}{2 \times 10^5} + \frac{1}{2 \times 10^5} \right]$$

$$\boxed{K = 1.56}$$

sub. all the value in Fw

$$F_W = \frac{b d_1 Q K}{\cos^2 \beta}$$

$$= \frac{50 \times 103.53 \times 1.5 \times 1.56}{\cos 15 \times \cos 15}$$

13) Calculation of basic dimensions of pinion and gear:

From 8.22, T-26

(i) Normal module $m_n = 5mm$

(ii) Number of teeth: $z_1 = 20$ & $z_2 = 60$

(iii) pitch circle diameter: $d_1 = 103.53mm$

$$d_2 = \frac{m_n}{\cos\beta} \times z_2$$

$$= \frac{5}{\cos 15^\circ} \times 60$$

$$\boxed{d_2 = 310.58mm}$$

iv) centre distance: $a = \frac{m_n}{\cos\beta} \times \left(\frac{z_1 + z_2}{2}\right)$

$$= \frac{5}{\cos\beta} \times \left(\frac{20+60}{2}\right)$$

$$\boxed{a = 207.05 \text{ mm}}$$

v) Height factor: $f_0 = 1$

vi) Bottom clearance $c = 0.25m_n = 0.25 \times 5 = 1.25mm$

PART – B (ADDITIONAL PROBLEMS)

1. In a spur gear drive for a stone crusher, the gears are made of C40 steel. The pinion is transmitting 30 kW at 1200 rpm. The gear ratio is 3. Gear is to work 8 hours per day, six days a week and for 3 years. Design the drive. **(A/M 2011)**
2. A cast steel 24 teeth spur pinion operating at 1150 rpm transmit 3 kW to a cast steel spur wheel. The gear ratio is 2.25. The tooth profile is 20º full depth involute. Design the gears and stresses.
3. Design a spur gear drive to transmit 22.5 kW at 900 rpm. Speed reduction is 2.5. Materials for pinion and wheel are C15 steel and cast iron grade 30 respectively. Take pressure angle of 20º and working life of the gears as 10000 hrs.
4. Design a spur gear drive required to transmit 45 kW at a pinion speed of 800 rpm. The velocity ratio is 3.5: 1. The teeth are 20º full depth involute with 18 teeth on the pinion. Both the pinion and gear are made of steel with a maximum safe static stress of 180 N/mm^2. Assume medium shock conditions.
5. Design a helical gear to transmit 15 kW at 1400 rpm to the following specifications: Speed reduction is 3; Pressure angle is 20º; Helix angle is 15º; The material of both the gears is C45 steel. Allowable static stress 180 N/mm^2; Surface endurance limit is 800 N/mm^2; Young's modulus of material = 2×105 N/mm^2.
6. For intermittent duty of an elevator, two cylindrical gears have to transmit 12.5 kW at a pinion speed of 1200 rpm. Design the gear pair for the following specifications: Gear ratio 3.5, pressure angle 20º, involute full depth, helix angle 15º. Gears are expected to work 6 hours a day for 10 years.
7. Design a straight spur gear drive. Transmitted power 8 kW. Pinion speed 764 rpm. Speed ratio is 2. The gears are to be made of C45 steel. Life is to be 10000 hours. **(M/J 2013)**
8. Design a spur gear drive for a heavy machine tool with moderate shocks. The pinion is transmitting 18 kW at 1200 rpm with a gear ratio of 3.5. Design the drive and check for elastic stresses and plastic deformation. Make a sketch and label important dimensions arrived. **(M/J 2012)**
9. Design a pair of helical gears to transmit 10 kW at 1000 rpm of the pinion. Reduction ratio of 5 is required. Give details of the drive in a tabular form. **(M/J 2013)**
10. A general purpose enclosed gear train is based on parallel helical gears, specified life is 36000 hours. Torque at driven shaft is 411 N-m. Driving shaft speed is 475 rpm. Velocity ratio is 4. It is desired to have standard centre distance. Design the gear drive. **(N/D 2011)**