GE 6152 – ENGINEERING GRAPHICS
UNIT – II

PROJECTION OF POINTS, LINES AND PLANE SURFACES.
Orthographic projections – Principles – Principle planes – First angle projections – Projection of Points – Projection of Straight lines (only first angle projections) inclined to both the principle planes – Determination of true lengths and true inclinations by rotating line method and traces – Projection of planes (polygonal and circular surfaces) inclined to both the principle planes by rotating line method.

Introduction – Point

A point is an object that which has simply position but no magnitude. It is generally represented by a very small circle or a dot.

Projection

Any kind of representation of an object on a paper, screen or similar surface by drawing is called the projection of the object.

Type of Projection

1. Pictorial projection
2. Orthographic projection

Pictorial Projection

It is the projection that gives three dimensional view of the object. Pictorial projection gives an overall idea about the shape of the solid, but not above the size.

Orthographic Projection

The projection or view obtained on a plane of projection when the projectors are parallel to each other, but perpendicular to the plane of projection, is known as orthographic projection.

Planes of Projection

The plane which is used for the purpose of projection is called plane of projection.

Type of Planes used for Projection

1. Vertical plane – VP
2. Horizontal plane – HP
3. Auxiliary vertical plane – AVP

Vertical Plane – VP

The plane which is vertical is called vertical plane and is denoted by VP. Vertical plane is also known as frontal plane since front view is projected on this plane.

Horizontal Plane – HP

The plane which is horizontal but at right angle to the VP is called horizontal plane.

Auxiliary Vertical Plane – AVP

A plane perpendicular to both VP and HP is known as auxiliary vertical plane. It is denoted by AVP.

Four Quadrants

If the horizontal and vertical planes of projections are assumed to extend beyond the line of interaction, the four dihedral are formed which are designated as first, second, third and fourth angles or four quadrants.

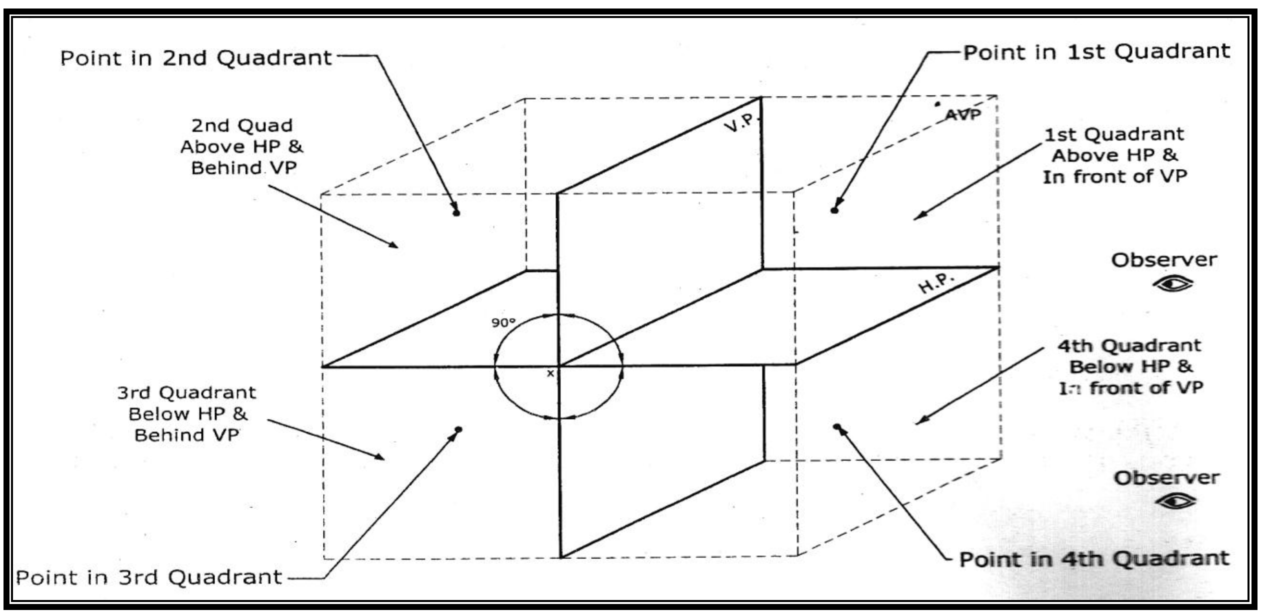


Figure – 1: Four Quadrants.

The position of the object placed in any one of the quadrant is described as below:

1. First quadrant – above **HP** and in-front of **VP**
2. Second quadrant – above **HP** and behind of **VP**
3. Third quadrant – below **HP** and behind of **VP**
4. Fourth quadrant – below **HP** and in-front of **VP**

Rotation of Planes

When the projections of an object have been made on the various planes, they are bought together on a single sheet of paper by rotating the planes.

The standard practice of rotation of planes is to keep the **VP** fixed and to rotate **HP** clockwise away from the object so that they may come in line with **VP**.

Projection of Points

The projection of a point is the graphical representation of elevation and plan of the given point which is positioned at different quadrants.

Getting Projections of a Point

After keeping the point in space projectors are drawn from it perpendicular to those two principle planes (**HP** & **VP**). The meeting point of the projectors with these principle planes is called projections of the points and still is in pictorial view. To make it in single paper space, always the HP is tilted through 90° in clockwise direction, so that the two principle planes are set in line.

Sign Conventions

1. Point in space is denoted by capital letter.
2. The projection obtained on the **HP** is called top view or plan and is denoted by lower case letter.
3. The projection obtained on the **VP** is called front view or elevation and is denoted by lower case letter with a dash.
4. Irrespective of the position of the point in any one of the four quadrants, the observer should be stationed at the right side of the quadrant for the front view.
5. For top view, the observer should be stationed at the tops always.

Hints

1. *Lower case letters with a prime should be used to represent the points of elevation.*
   (viz a’, b’, c’, …., 1’, 2’, 3’, …..)
2. *Lower case letters should be used to represent the points of plan.*
   (viz a, b, c, …., 1, 2, 3, …..)
3. *Lower case letters with double prime should be used to represent the points of side views.*
   (viz a’’, b’’, c’’, …., 1’’, 2’’, 3’’, …..)
4. *Both the elevation and plan of a point must lie in a line, called projector.*

Projections of a Point in the First Quadrant

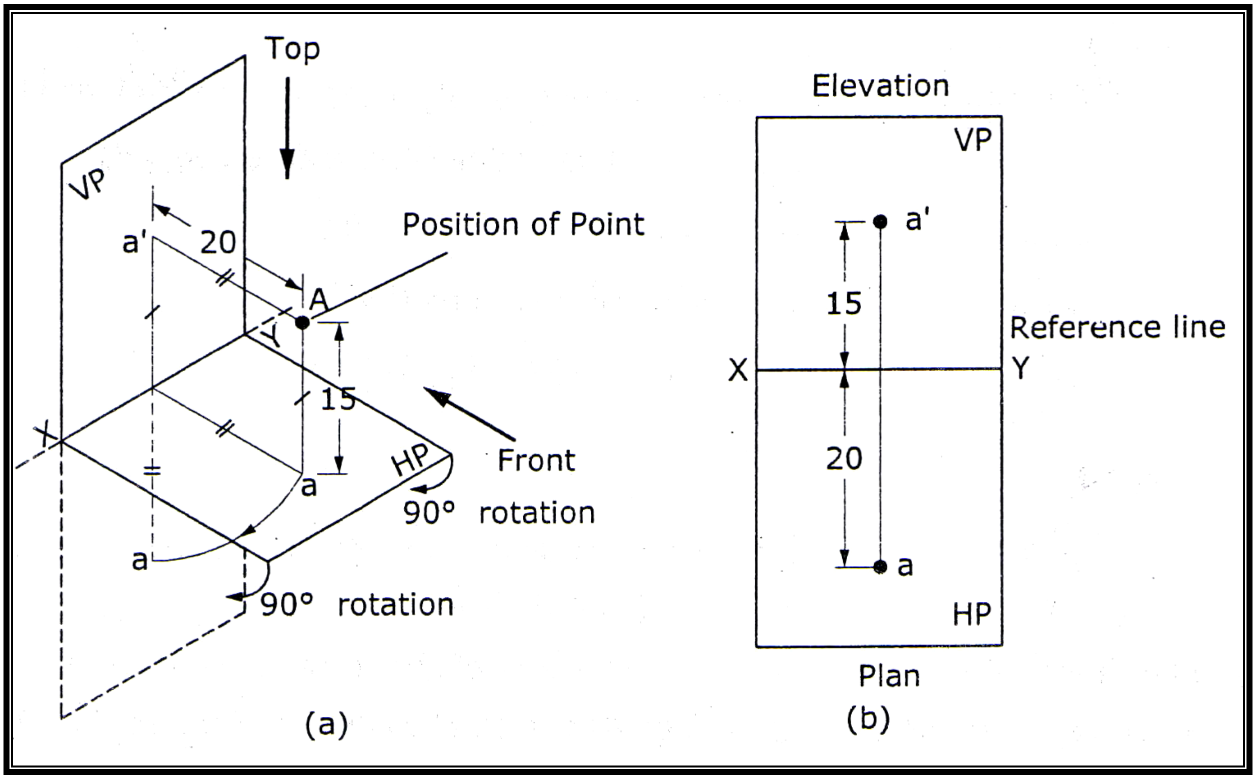


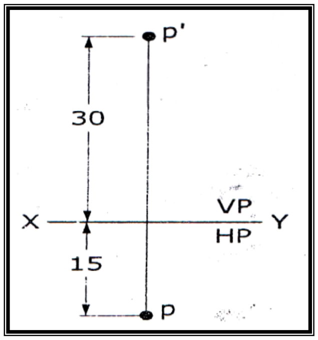
Figure – 2: Projection of points in first quadrant.

The above **figure – 2(a)** represent the point **A** in space in the first quadrant. The height of the point **A** with reference to **HP** is clearly seen from the front. At this position the image obtained on the **VP** is called front view or elevation and is denoted as **a'**. Similarly the distance of **A** with reference to **VP** is clearly seen from the top. At this position the image obtained on the **HP** is called top view or plan and is denoted as **a**.

After getting the projections on the **HP** & **VP** the horizontal plane is tilted through 90° in clockwise direction so that both these planes are brought inline and is as shown in **figure – 2(b)**. Here the line of intersection of the reference planes **HP** and **VP** is denoted as **XY** and is called reference plane.

Now the plane above **XY** line is the vertical plane and the plane below the **XY** line is horizontal plane. The elevation of the point (**a'**) is located at a height 15 mm above **XY** line. The plan of the given point **a** is at a distance of 20 mm below **XY** line.

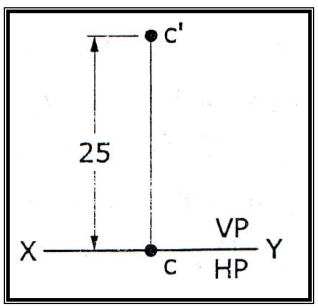
Solved Problem – 1: ***Draw the projection of a point P, which is 30 mm above HP and 15 mm in-front of VP.***



Procedure:

1. Draw the line **XY**.
2. Draw a projector (line perpendicular to **XY**) somewhere at the middle of the **XY** line.
3. Mark point **p'** on it 30 mm above **XY**, which is the front view of the point **P**.
4. On the same projector make **p** 15 mm below **XY** line, which is the top view of the point **P**.

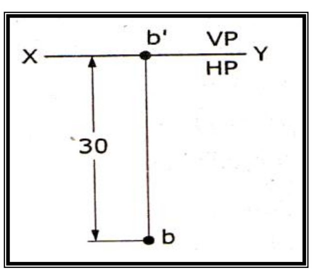
Solved Problem – 2: ***Point C is 25 mm above HP and in VP. Draw the projection of the point.***



Procedure:

1. Draw the line **XY**.
2. Draw a projector. Mark point **c'** on the projector 25 mm above **XY**, which is the front view of the point **C**.
3. Since the point is on **VP** the top view of the point **C** will lie on the **XY** line itself. So mark **c** at the point of intersection of the projector and the **XY** line.

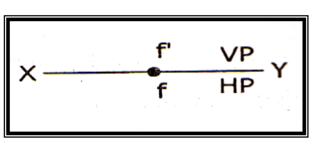
Solved Problem – 3: ***Draw the projection of a point B lying on HP and 30 mm in-front of VP.***



Procedure:

1. Draw the line **XY**.
2. Since the point is lying on **HP** it won't have any height when it is seen from the front. So mark **b'** on **XY** line.
3. Mark **b** 30 mm below **XY** line on the projector from **b'**, which is the plan of **b**.

Solved Problem – 4: ***Draw the projection of F lying on both HP and VP.***



Procedure:

1. Draw the line **XY**.

2. Since the point is lying on **HP** it won't have any height when it is seen from the front. So mark **b'** on **XY** line.
3. Mark **b** 30 mm below **XY** line on the projector from **b'**, which is the plan of **B**.

Projections of a Point in the Second Quadrant

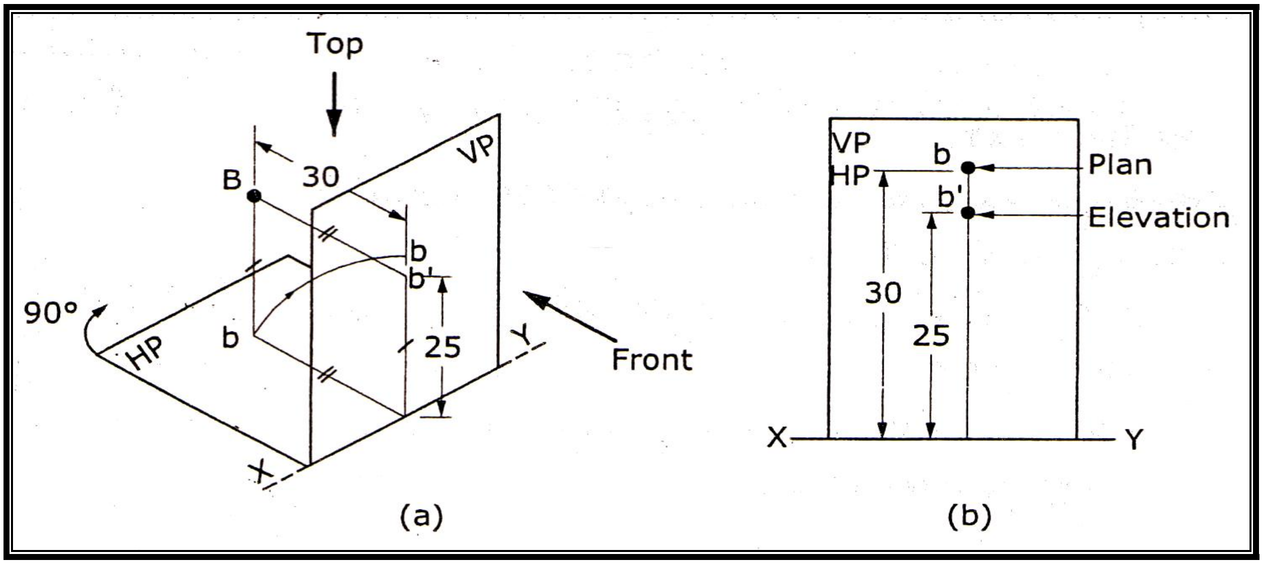
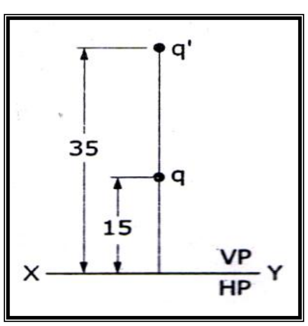


Figure – 3: Projection of points in second quadrant.

The above **figure – 3(a)** represent the point **B** in space in the second quadrant. The projections can be obtained both on **HP** and **VP** by following the steps as for the first quadrant. After getting the projections on the **HP** and **VP** the horizontal plane is tilted through 90° in clockwise direction, so that both these planes are brought inline and is as shown in **figure – 3(b)**. Now the planes **HP** and **VP** are lying above **XY** line. So both the elevation and plan will always be above **XY** line.

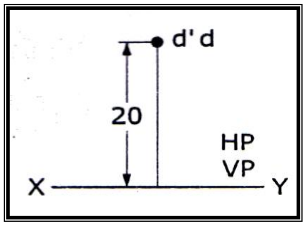
Solved Problem – 1: ***Draw the projections of a point Q, which is 35 mm above HP and 15 mm behind VP.***



Procedure:

1. Draw the line **XY**.
2. Draw a projector somewhere in the middle of the **XY** line.
3. Mark point **q'** on it 35 mm above **XY**, which is the front view of the point **Q**.
4. On the same projector mark **q** 15 mm above **XY** line, which is the top view of the point **Q**.

Solved Problem – 2: Draw the projection of a point D 20 mm away from both the reference planes and is in the second quadrant.



Procedure:

1. Draw the line **XY**.
2. Draw a projector somewhere in the middle of the **XY** line.
3. Since the point **D** is in the second quadrant mark points **d** and **d'** 20 mm above **XY**, which is the projection of **D**.

Projections of a Point in the Third Quadrant

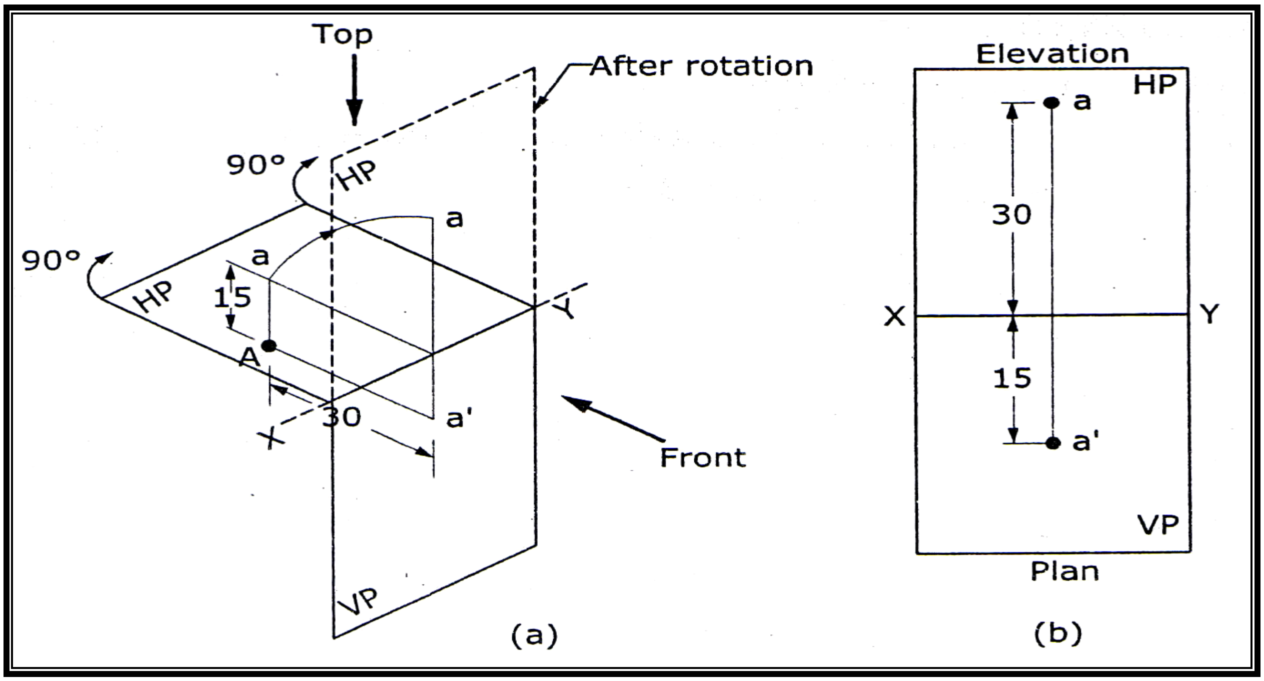
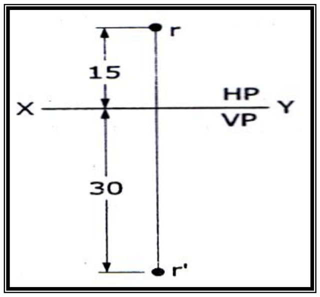


Figure – 4: Projection of points in third quadrant.

The above **figure – 4(a)** represent the point **A** in space in the third quadrant. The projections can be obtained both on **HP** and **VP** by following the steps as for the first quadrant. After getting the projections on the **HP** and **VP** the horizontal plane is tilted through 90° in clockwise direction so that both these planes are brought inline and is as shown in **figure – 4(b)**. Now the plane above **XY** line is the horizontal plane and the plane below the **XY** line is vertical plane. The elevation of the point (**a'**) is located below **XY** line. The plan (**a**) is above **XY** line.

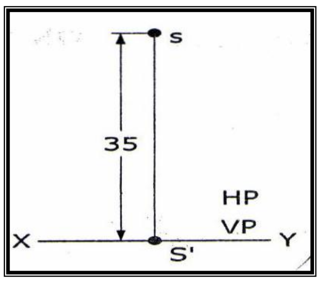
Solved Problem – 1: ***Draw the projection of a point R, which is 30 mm below HP and 15 mm behind VP.***



Procedure:

1. Draw the line **XY**.
2. Draw a projector somewhere in the middle of the **XY** line.
3. Mark point **r'** on it 30 mm above **XY**, which is the front view of the point **R**.
4. On the same projector mark **r** 15 mm above **XY** line, which is the top view of the point **R**.

Solved Problem – 2: ***Draw the projection of a point S, which is in HP and 35 mm behind VP.***



Procedure:

1. Draw the line **XY**.
2. Mark **s'** on **XY**, since it is in **HP**.
3. Draw a projector through **s'** and mark **s** 35 mm above **XY**, which is the top view of the point **S**.

Projections of a Point in the Fourth Quadrant

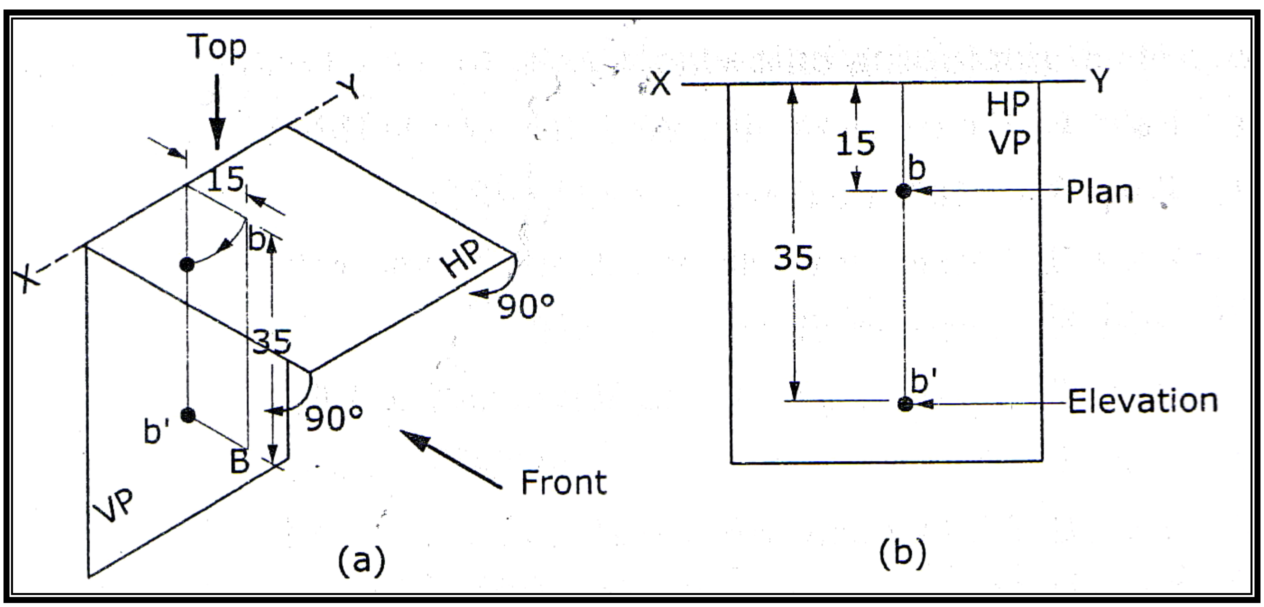
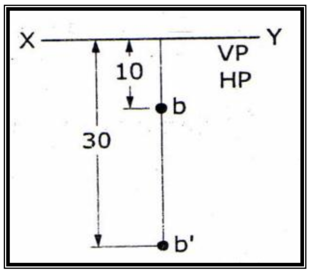


Figure – 5: Projection of points in fourth quadrant.

The above **figure – 5(a)** represent the point **B** in space in the fourth quadrant. The projections can be obtained both on **HP** and **VP** by following the steps as for the first quadrant. After getting the projections on the **HP** and **VP** the horizontal plane is tilted through 90° in clockwise direction so that these planes are brought inline and is as shown in **figure – 5(b)**. Now the planes **HP** and **VP** are lying below **XY** line. So both the elevation and plan will always be below the **XY** line.

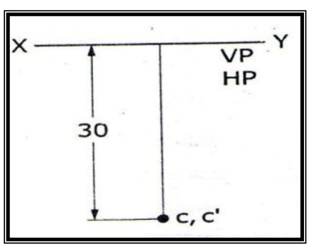
Solved Problem – 1: ***Draw the projections of a point B, which is 30 mm below HP and 10 mm in-front of VP.***



Procedure:

1. Draw the line **XY**.
2. Draw a projector somewhere in the middle of the **XY** line.
3. Mark point **b'** on it 30 mm above **XY**, which is the front view of the point **B**.
4. On the same projector mark **b** 10 mm above **XY** line, which is the top view of the point **B**.

Solved Problem – 2: ***Draw the projection of a point C 30 mm away from both the reference planes and is in the fourth quadrant.***



Procedure:

1. Draw the line **XY**.
2. Draw a projector somewhere in the middle of the **XY** line.
3. On it mark point's **c** and **c'** 30 mm below **XY**, which is the projection of **C**.

Exercise – 1: ***Draw the projections for the following points***

a. Point A, 10 mm in-front of VP 45 mm above HP.
b. Point B, on HP and 50 mm in-front of VP.
c. Point C, 10 mm above HP and 25 mm behind VP.
d. Point D, in VP and 35 mm below HP.
e. Point E, 35 mm below HP and 30 mm behind VP.
f. Point F, 45 mm away from the reference planes and is in the third quadrant.
g. Point G, 50 mm below HP and 25 mm in-front of VP.
h. Point H, 20 mm in below HP and in VP.

Exercise – 2: ***Projections of various points are given in the below mentioned figure – 6. State their positions with respects to the reference planes.***

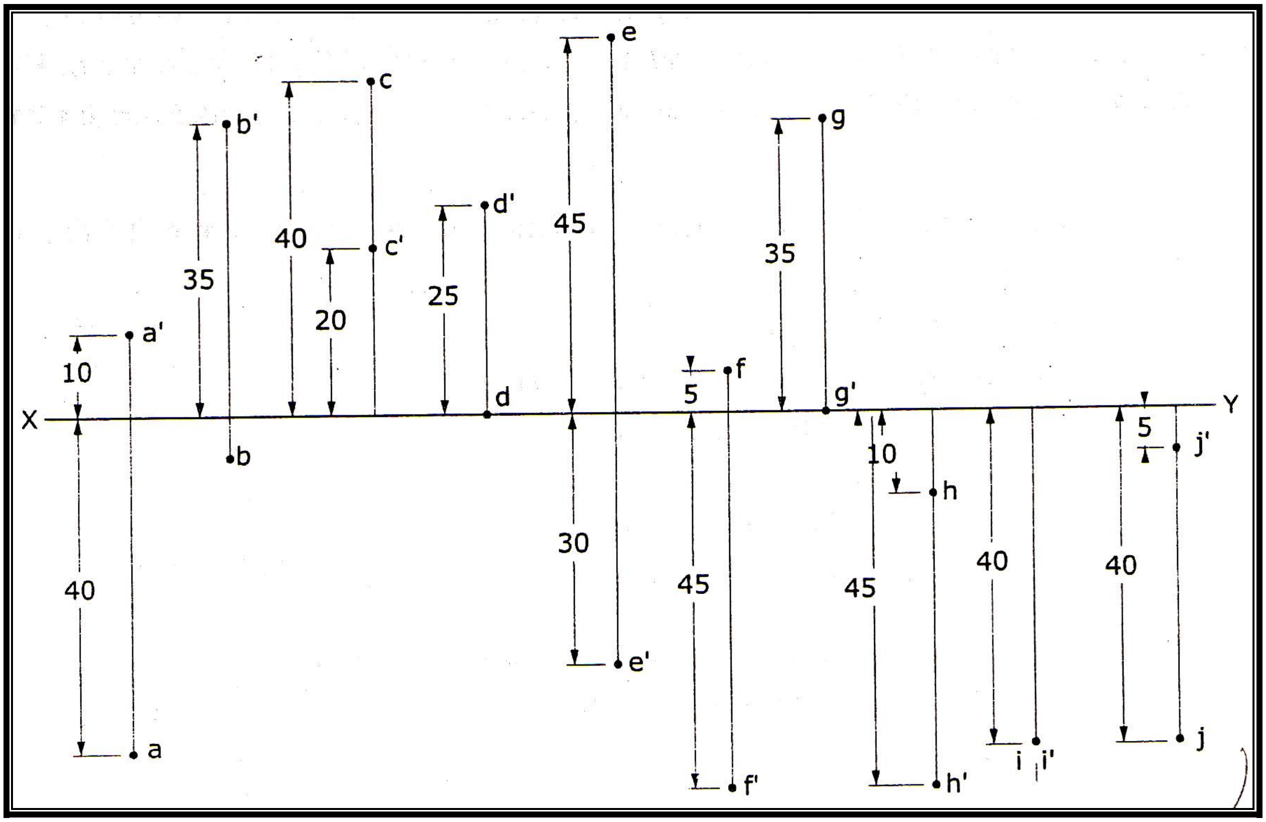
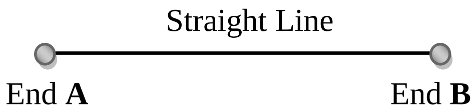


Figure – 6: Projection of various points.

PROJECTION OF STRAIGHT LINES

Introduction – Straight Line

A straight line is an entity which defined as the shortest distance between two points. It has two end points. It has length but negligible thickness.



Projection of Straight Lines

Drawing the front view, top view of a straight line is called projections of a straight line.

Position of Straight Lines

The position of a straight line in space can be described with respect to the two reference planes (**VP** and **HP**) are as follows.

1. Line parallel to both **HP** and **VP**.
2. Line parallel to one plane and perpendicular to other.
   - Line parallel to **HP** and perpendicular to **VP**.
   - Line parallel to **VP** and perpendicular to **HP**.
3. Line parallel to one plane and inclined to other.
   - Line parallel to **HP** and inclined to **VP**.
   - Line parallel to **VP** and inclined to **HP**.
4. Line contained by one or both the planes.
   - Line in **HP**.
   - Line in **VP**.
   - Line both in **HP** and **VP**.
5. Line inclined to both the planes.
   - Line inclined to both the planes with one end on **XY** plane.

1. LINE PARALLEL TO BOTH HP AND VP

Rule – 1:	*A straight line will represent its true length in that plane to which it is parallel.*

When the straight line **AB** is parallel to both, then the distance of the end points are equally away from **HP** and **VP**. From the below mentioned figure – 1, the front view or elevation is denoted as **a'b'**. The top view or plan is denoted as **ab**. The length of the elevation and plan are equal to the true length of line **AB** and parallel to reference line **XY**.

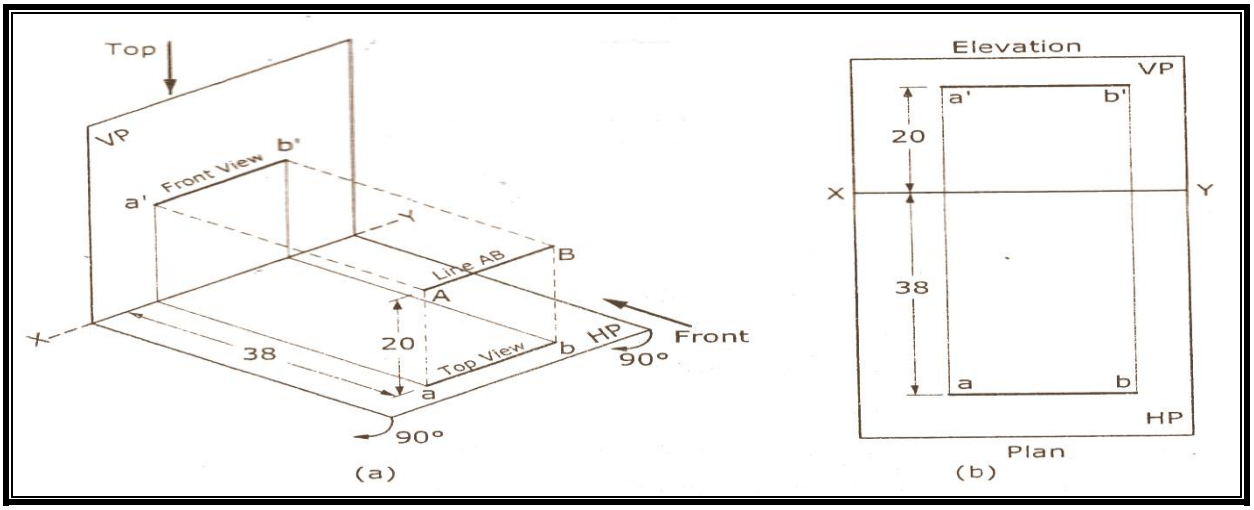


Figure – 1: Straight line parallel to both HP and VP.

2. LINE PARALLEL TO ONE PLANE AND PERPENDICULAR TO OTHER

Rule – 2:	*A straight line will represent at a point in that plane to which plane it is perpendicular.*

Line **AB** 25 mm parallel to **VP** and perpendicular to **HP**. *(Refer figure – 2 (a) (i))*
Line **CD** 21 mm parallel to **HP** and perpendicular to **VP**. *(Refer figure – 2 (a) (ii))*

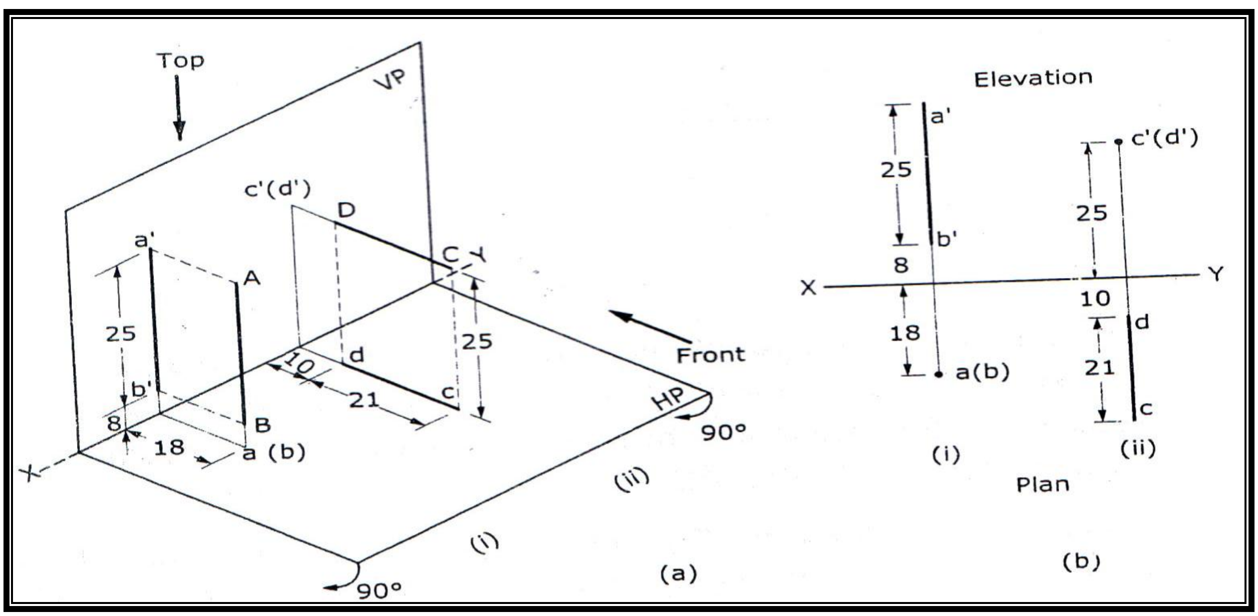


Figure – 2: Straight line parallel to one plane and perpendicular to other.

Figure (a) – Parallel to VP and perpendicular to HP.

Figure (b) – Parallel to HP and perpendicular to VP.

The above figure – 2 shows the line **AB** parallel to **VP** and perpendicular to **HP**. So the elevation **a'b'** is a line perpendicular to **XY** showing true length of **AB**. In the plan, both the ends of the line **AB** are merged together and seen as a point with the visible one point which hides the other point. The rule to find the hidden point is that, the points that are nearer to **XY** line in one view are not visible in other. So the point **b** is hidden here.

The line **CD** is parallel to **HP** and perpendicular to **VP**. So the plan of the line **CD** is the line **cd** perpendicular to **XY** showing the true length and the front view of the line **CD** is a point **c'(d')**.

3. LINE PARALLEL TO ONE PLANE AND INCLINED TO OTHER PLANE

When a straight line is inclined to one plane and parallel to the other, its projection on the plane to which it is inclined will be a straight line, shorter than its true length but parallel to **XY** line. Simultaneously its projection on the plane to which it is parallel will be a straight line equal to its true length and inclined to **XY** line at its true inclination.

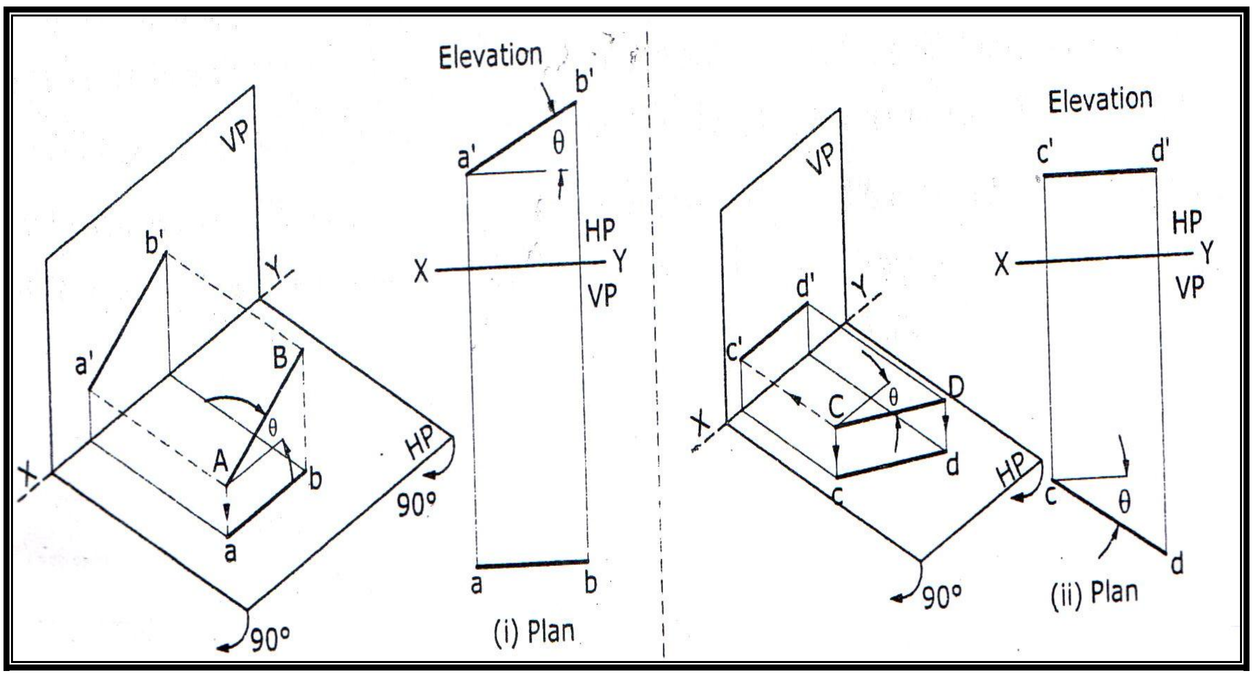


Figure – 3: Straight line parallel to one plane and inclined to other.

Figure (a) – Parallel to VP and inclined to HP. ***Figure (b) – Parallel to HP and inclined to VP.***

a) **Straight line inclined to HP and parallel to VP:** From the above mentioned figure – 3 (a), the straight line **AB** is inclined at an angle **θ** to **HP** and parallel to **VP**. Its front view **a'b'** is equal to the straight line **AB** and its inclination **θ** is in its true form. Its top view **ab** is shorter than the line **AB**.

b) **Straight line inclined to VP and parallel to HP:** Considering figure – 3 (b), the straight line **CD** is inclined at an angle **θ** to **VP** and its inclination is in its true magnitude. Its front view **c'd'** is shorter than the line **CD**. Its top view **cd** is equal to the straight line **CD** and its inclination **θ** is in true form.

4. LINE CONTAINED BY ONE OR BOTH THE PLANES

Rule – 3:	*A straight line will represent its true length in that plane to which plane the straight line is contained.*

Case – 1:

Line AB is in the VP and inclined to HP: Its elevation **a'b'** shows the true length and true inclination and the plan **ab** is shorter than the line **AB** and on the **XY** line. *(Refer Figure (b) – i)*

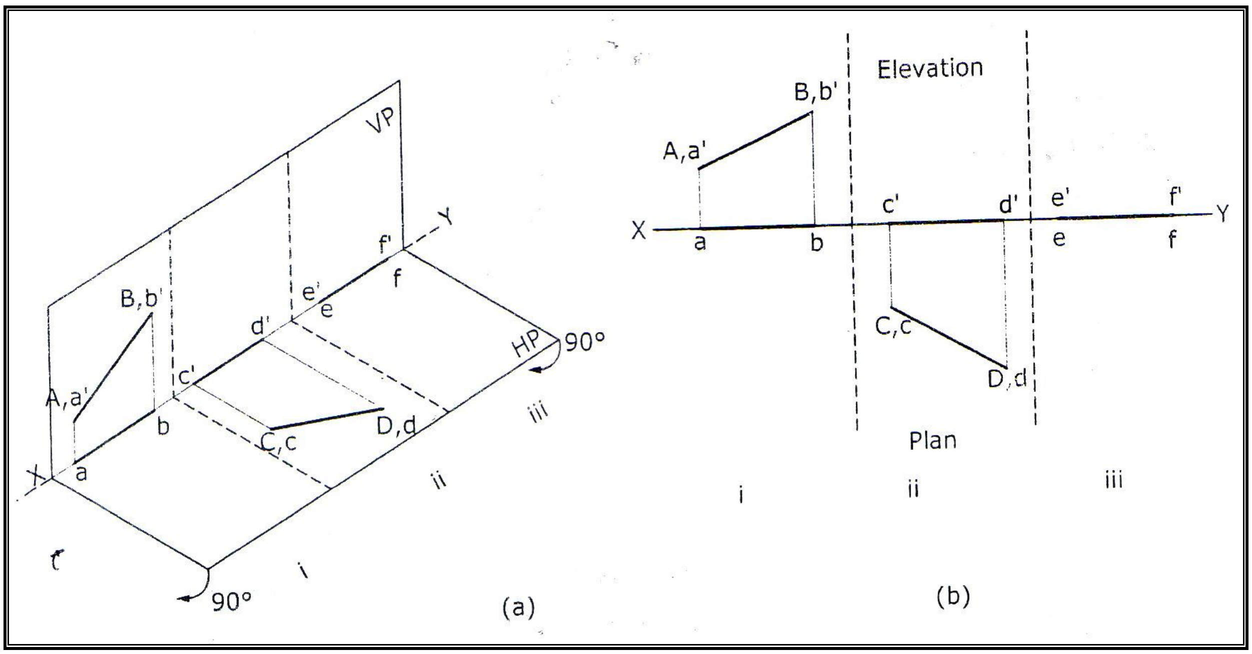


Figure – 4: Straight line contained by one or both the planes.

Figure (a) – Position of lines in three cases. ***Figure (b) – Projection of lines in three cases.***

Case – 2:

Line CD is in the HP and inclined to VP: Its plan **cd** shows the true length and true inclination and the elevation **c'd'** is shorter than the line **CD** and on **XY** line. *(Refer Figure (b) – ii)*

Case – 3:

Line EF is both in HP and VP: Here both the elevation **e'f'** and the plan ef coincides on **XY**. *(Refer Figure (b) – iii)*

5. LINE INCLINED TO BOTH THE REFERENCE PLANES

The figure – 5, shows the straight line **AB** inclined to both **HP** and **VP**. The elevation and plan of the inclined line both **HP** and **VP** are duly projected and shown in the same figure. Here both the elevation / front view **a'b'** and top view / plan **ab** are inclined to **XY** line and are shorter than the true length.

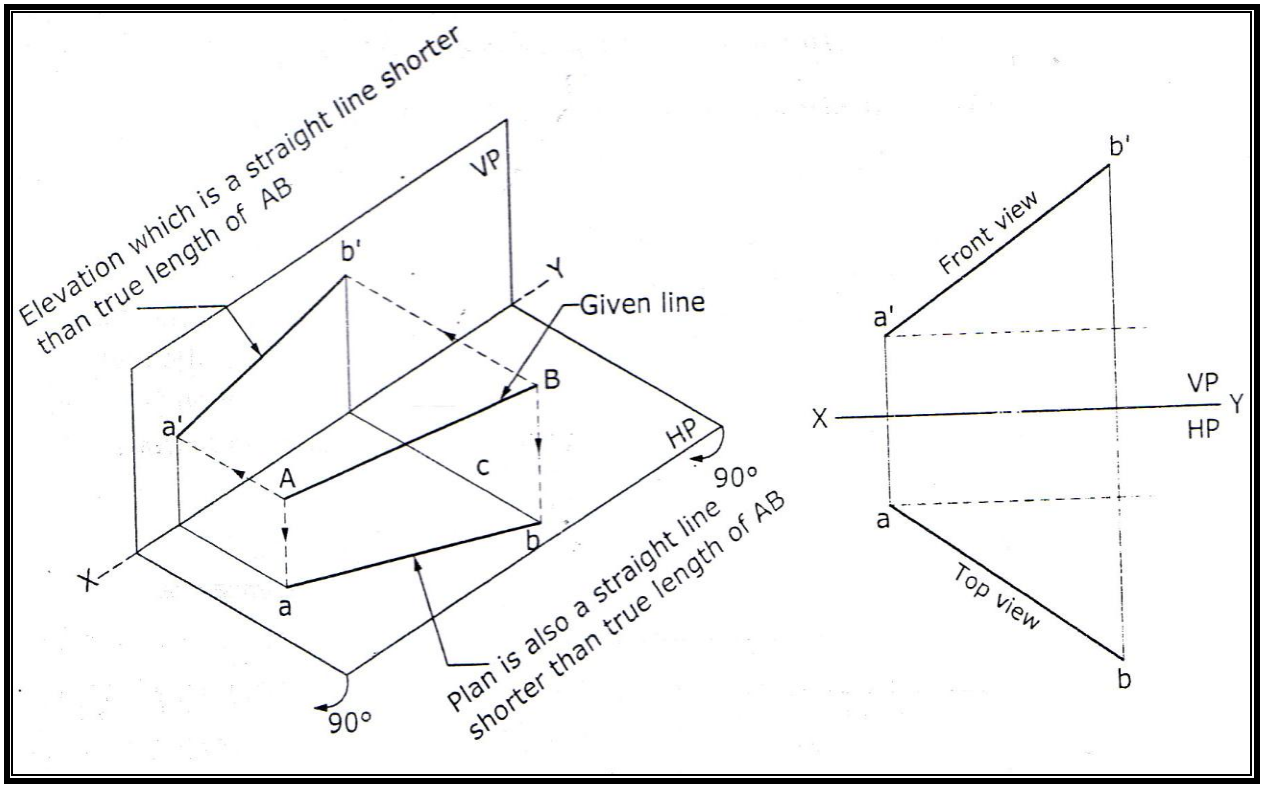


Figure – 5: Straight line inclined to both the reference planes.

Note:

- If the line is parallel to a reference plane then the projection obtained on it gives true length and true inclination.
- When a line is inclined to both the **HP** and **VP**, then the projections are shorter than the true length and the inclination angles with **XY** line are greater than the true inclination of the line. The greater angles with the **XY** line are called apparent angles.
- When a line is inclined to both the **HP** and **VP** it will neither show its true length nor true inclination in top view or front view and is termed as oblique line.

THE NOMENCLATURE OF THE VARIOUS PARAMETERS WHEN THE LINE IS INCLINED TO BOTH HP AND VP

[Rotating Line Method]

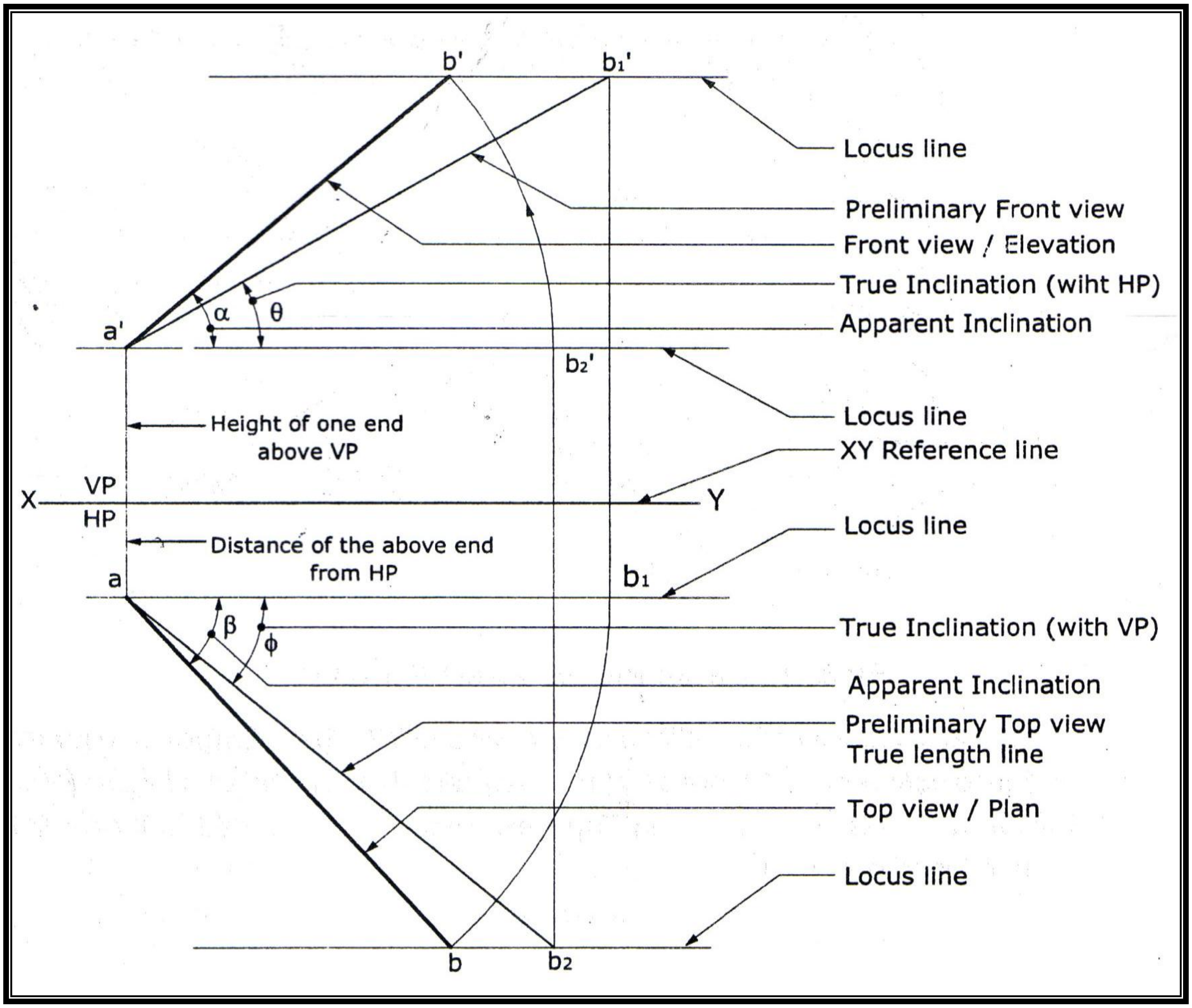


Figure – 6: Straight line inclined to both HP and VP.

Note:

- The apparent inclination of the elevation / front view (denoted as α) is always greater than the true inclination of the line with **HP (θ)**.
- The apparent inclination of the plan / top view (denoted as **β**) is always greater than the true inclination of the line with **VP (ϕ)**.

SUMMARY FOR PROJECTION OF LINES

SL.NO	POSITION OF LINE	FRONT VIEW OR ELEVATION	TOP VIEW OR PLAN
1.	Line parallel to **HP** and **VP**	True length and parallel to **XY**	True length and parallel to **XY**
2.	Line perpendicular to **HP** and parallel to **VP**	True length and perpendicular to **XY**	Point
3.	Line perpendicular to **VP** and parallel to **HP**	Point	True length and perpendicular to **XY**
4.	Line in **HP** and **VP**	True length and coincide on **XY**	True length and coincide at **XY**
5.	Line in **HP** and inclined at **VP**	Shorter than true length and lies in **XY**	True length and inclined at **θ°** to **XY**
6.	Line in **VP** and inclined at **θ°** to **HP**	True length and inclined at **θ°** to **XY**	Shorter than true length and lies in **XY**
7.	Line parallel to **HP** and inclined at **θ°** to **VP**	True length and inclined at **θ°** to at **XY**	Shorter than true length and parallel to **XY**
8.	Line inclined to **HP** at **ϕ°** and parallel to **VP**	Shorter than true length and parallel to **XY**	True length and inclined at **ϕ°** to **XY**
9.	Line inclined **θ°** to **VP** and inclined at **ϕ°** to **HP**	Neither true length not true inclination	Neither true length not true inclination
10.	Line contained by plane perpendicular to **HP** and **VP**	Shorter than true length and perpendicular to **XY**	Shorter than true length and perpendicular **XY**

Table – 1: Summary for projection of lines.

STRAIGHT LINE INCLINED TO BOTH THE PLANES WITH ONE END ON XY LINE

Rule – 4:	*When the end positions of a straight line are not given in problem, then for solving, the problem, one end of the line should be taken on **XY** line.*

The line **AB** 60 mm long has its end a in both the **HP** and **VP**. It is inclined at 45° to **HP** 30° to **VP**. Draw the projections of the line **AB** and determine its traces.

If the end positions of a straight line are not mentioned in the problem, then one end point of the line may be assumed to be either in **HP** or in **VP** or both. In this case, one end of the line has been taken in both the planes i.e., on **XY** line. Here neither its top view nor its front view will show the true and inclination of the line.

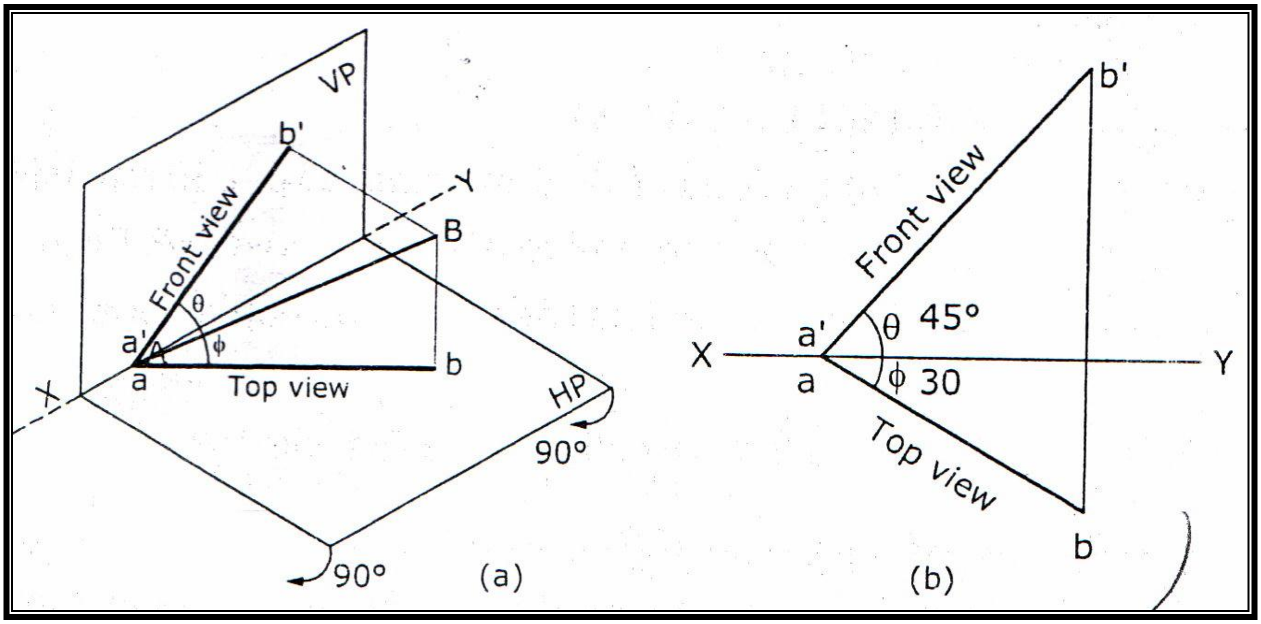


Figure – 7: Straight line inclined to both the planes with one end on XY plane.

Solved Problems – 1: ***One end P of a line PQ, 55 mm long is 35 mm in-front of VP and 25 mm above the HP. The line is inclined at 40º to HP and 30º to VP. Draw the projections of PQ.***

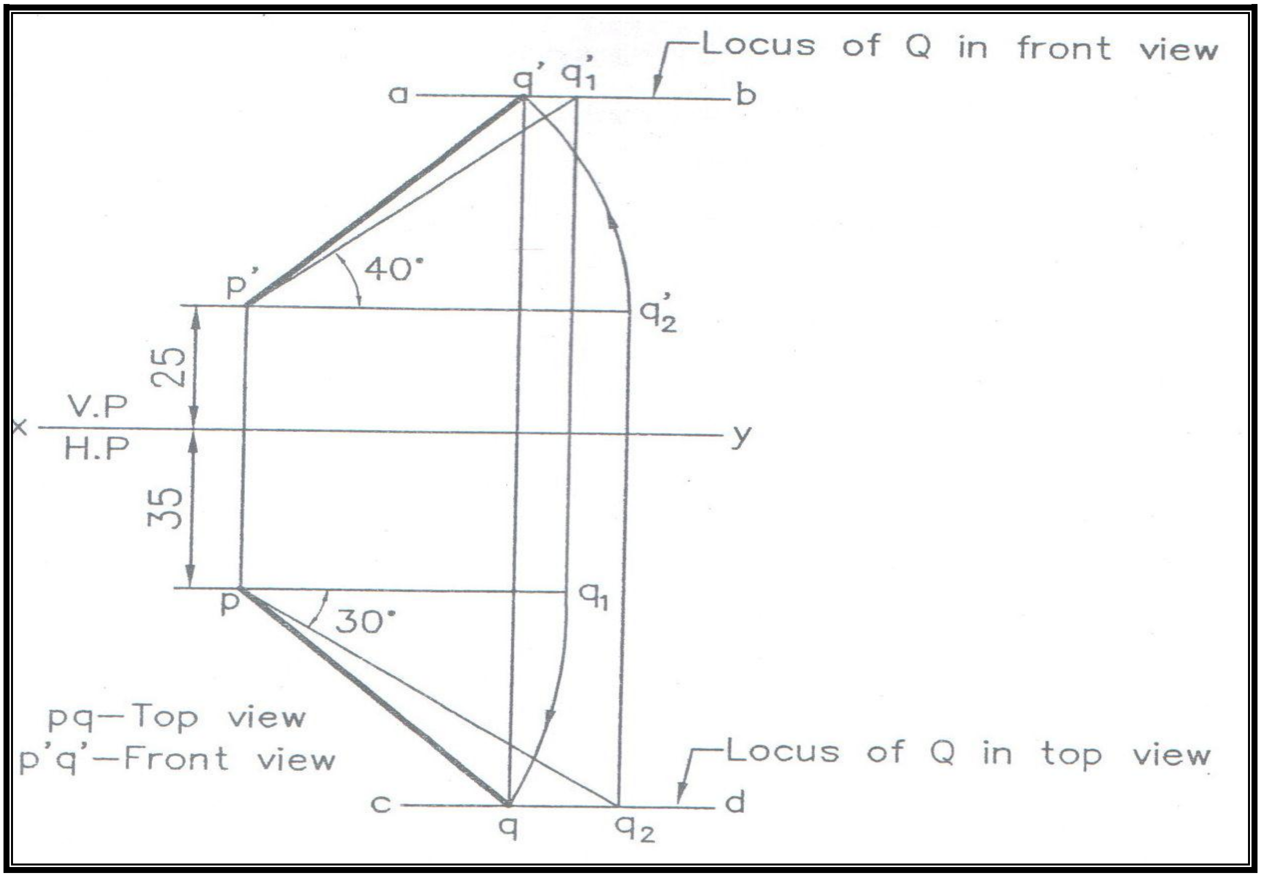


Figure – 8: Straight line inclined to both the planes.

1. Locate the top view **p** of **P**, 35 mm below **XY** and front view **p'**, 25 mm above **XY**.
2. It is next required to find the length of the top view and front view of the line and also the paths of the end **Q** in the top view and front view.
3. The line is first assumed to be parallel to the **VP** and inclined at 40º to the **HP**.
4. Then, the front view will have the true length and will be inclined at 40º to **XY**. Accordingly, draw **pq$_1$**' inclined at 40º to **XY** and make **p'q$_1$'** equal to 55 mm.
5. Project **q$_1$'** to **q$_1$** on the line drawn parallel to **XY** through **p**. Then, **pq$_1$** will give the length of top view of the line.
6. Next, the line assumed to be parallel to **HP** and inclined at 30º to **VP**. Then, the top view will have the true length and will be inclined at 30º to **XY**. Draw **pq$_2$** inclined at 30° to **XY** and make **pq$_2$** equal to 55 mm.
7. Project **q$_2$** to **q$_2$'** on the line drawn parallel to **XY** through **p'**. **p'q$_2$'** will represent the length of the front view of the line.

8. Draw a line **ab** through **q_1'** parallel to **XY**. This line is the path of the end **Q** in the front view. Similarly, draw **cd** through **q_2** parallel to **XY** to represent the path of **Q** in the top view.
9. With **p'** as centre and **p'q_2'** as radius, draw an arc to cut the path **ab** at **q'**. Draw a line joining **p'** and **q'**.
10. Similarly, with p as centre and **pq_1** as radius, draw an arc meeting the path **cd** at **q**. Draw a line joining **p** and **q**.
11. Then, **pq** is the required top view and **p'q'**, the front view of the line **PQ**.

Solved Problems – 2: ***One end S of a line SR, 70 mm long is in both the HP and the VP. The line is inclined at 40° to the HP and at 35° to the VP. Draw its projections.***

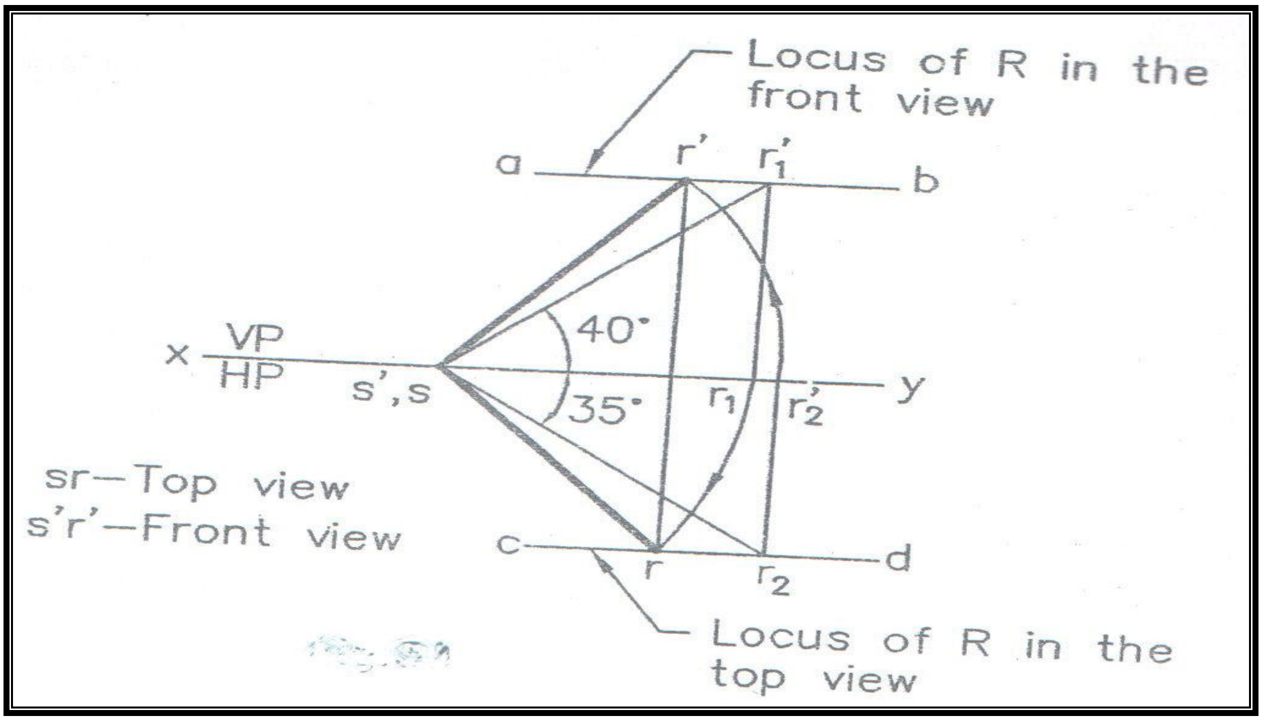


Figure – 9: Straight line inclined to both the planes with one end on XY plane.

1. Both the top view and front view of **S** coincide and lie in **XY**. To find the length of the top view, the line is assumed to be parallel to the **VP** and inclined at 40° to the **HP**. The line will lie in the **VP** and hence the top view will lie in **XY**. Accordingly, draw **s'r_1'** inclined at 40° to **XY** and to have a length of 70 mm (true length). Project **r_1'** to **r_1** on **XY**. Then, **sr_1** is the length of the top view.
2. Similarly, draw **sr_2** inclined at 35° to **XY** to have a length of 70 mm. Project **r_2** to **r_2'** on **XY**. **s'r_2'** is the length of the front view.

3. Draw **ab** through **r_1'**, parallel to **XY** and **cd** through **r_2**, parallel to **XY**. Then, **ab** and **cd** are the paths of the end **R** in the front view and the top view respectively.
4. With **s'** as center and radius **s'r_2'**, draw an arc to cut **ab** at **r'**. With the same centre and radius **sr_1**, draw an arc to meet **cd** at **r**. Draw a line joining **s'** and **r'**. Draw **sr**. **sr** and **s'r'** are the required projections of the line **SR**.

Solved Problems – 3: ***A line NS, 80 mm long has its end N, 10 mm above the HP and 15 mm in-front of the VP. The other end S is 65 mm above the HP and 50 mm in-front of the VP. Draw the projections of the line and find its true inclinations with the HP and VP.***

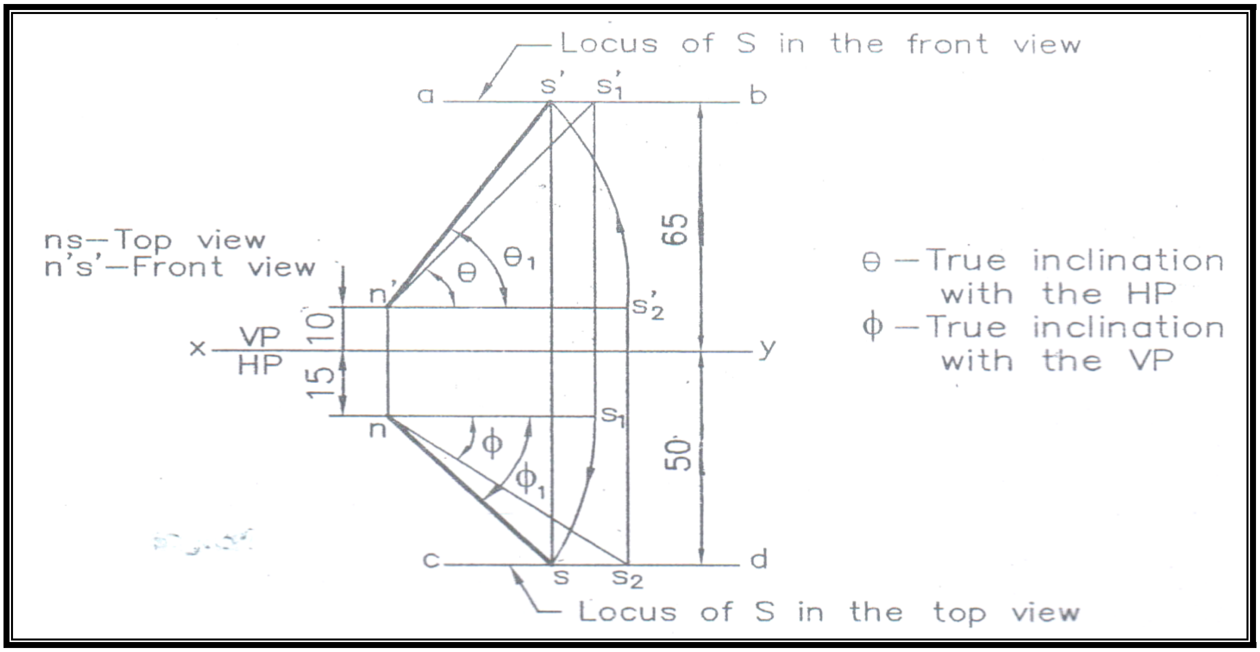
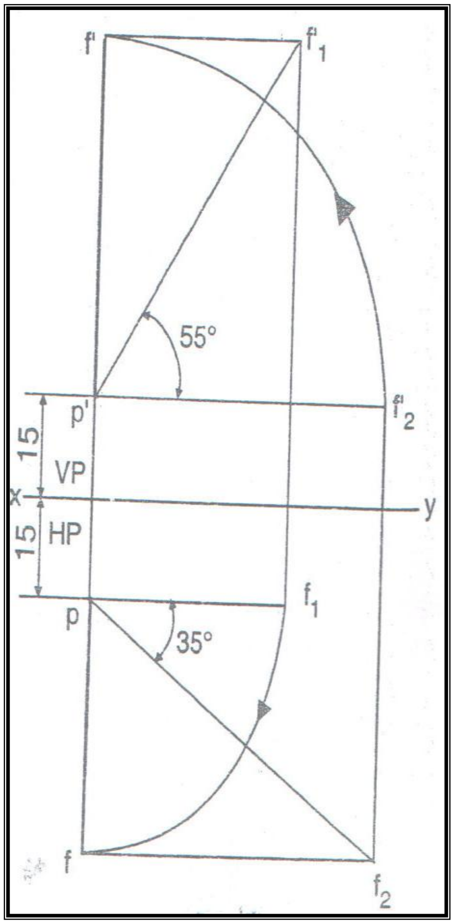


Figure – 10: Straight line inclined to both the planes.

[Here θ = 42°; ϕ = 26°]

1. Make the projections of one of the points, say **N**. Draw **ab** and **cd** parallel to **XY**, 65 mm above **XY** and 50 mm below **XY** respectively. **ab** and **cd** are the loci of the end **S**.
2. With **n'** as centre and 80 mm as radius, draw an arc meeting **ab** at **s_1'**. Project **s_1'** to **s_1** on the line drawn through **n** parallel to **XY**.
3. With **n** as centre and radius **ns_1**, draw an arc to cut **cd** at **s**. Project **s** to **s'** on **ab**. Join **n's'** and **ns**. Then, **ns** and **n's'** are the top view and front view respectively of the line **NS**. The angle of inclination **θ** of **n's_1'** with **XY** is the true inclination of the line with the **HP**.
4. With **n** as centre and 80 mm as radius, draw an arc to cut **cd** at **s_2**. Then, the angle **ϕ** that **ns_2** makes with the horizontal is the true inclination of the line with the **VP**.

Solved Problems – 4: ***A line PF, 65 mm long has its end P, 15 mm above the HP and 15 mm in-front of the VP. It is inclined at 55° to the HP and 35° to the VP. Draw its projections.***



1. Since the sum of inclinations (θ+ϕ) of the line with the principle planes is 90°, the projections of the line lie in the same projector. The projections of **P** are marked. The lengths of the top view and front view are determined in the usual manner.
2. The line is first assumed to be parallel to the **VP** and inclined at 55° to the **HP**. Draw **p'f$_1$'** of length 65 mm inclined at 55° to **XY**. Project **f$_1$'** to **f$_1$** on a line drawn through **p** parallel to **XY**. **pf$_1$** is the length of the top view.
3. Next, find the length of the front view assuming the line to be parallel to the **HP** and inclined at 35° to the **VP**. For this, draw **pf$_2$** of length 65 mm inclined at 35° to **XY**. Project **f$_2$** to **f$_2$'** on the line drawn through **p'** parallel to **XY**.
4. With **p** as centre and **pf$_1$** as radius, draw an arc to cut the path through **f$_2$** at **f**. With **p'** as centre and **p'f$_2$'** as radius, draw an arc to cut the path through **f$_1$'** at **f'**.
5. Draw lines **pf** and **p'f'** to get the required projections. It is found that **pf** and **p'f'** lie in the same projector perpendicular to **XY**.

Figure – 11: Straight line inclined to both the planes.

Solved Problems – 5: ***The end P of a line PQ, 70 mm long is 15 mm above the HP and 20 mm in-front of the VP. Q is 40 mm above the HP. The top view of the line is inclined at 45° to the VP. Draw the projections of the line and its true inclination with the VP and the HP.***

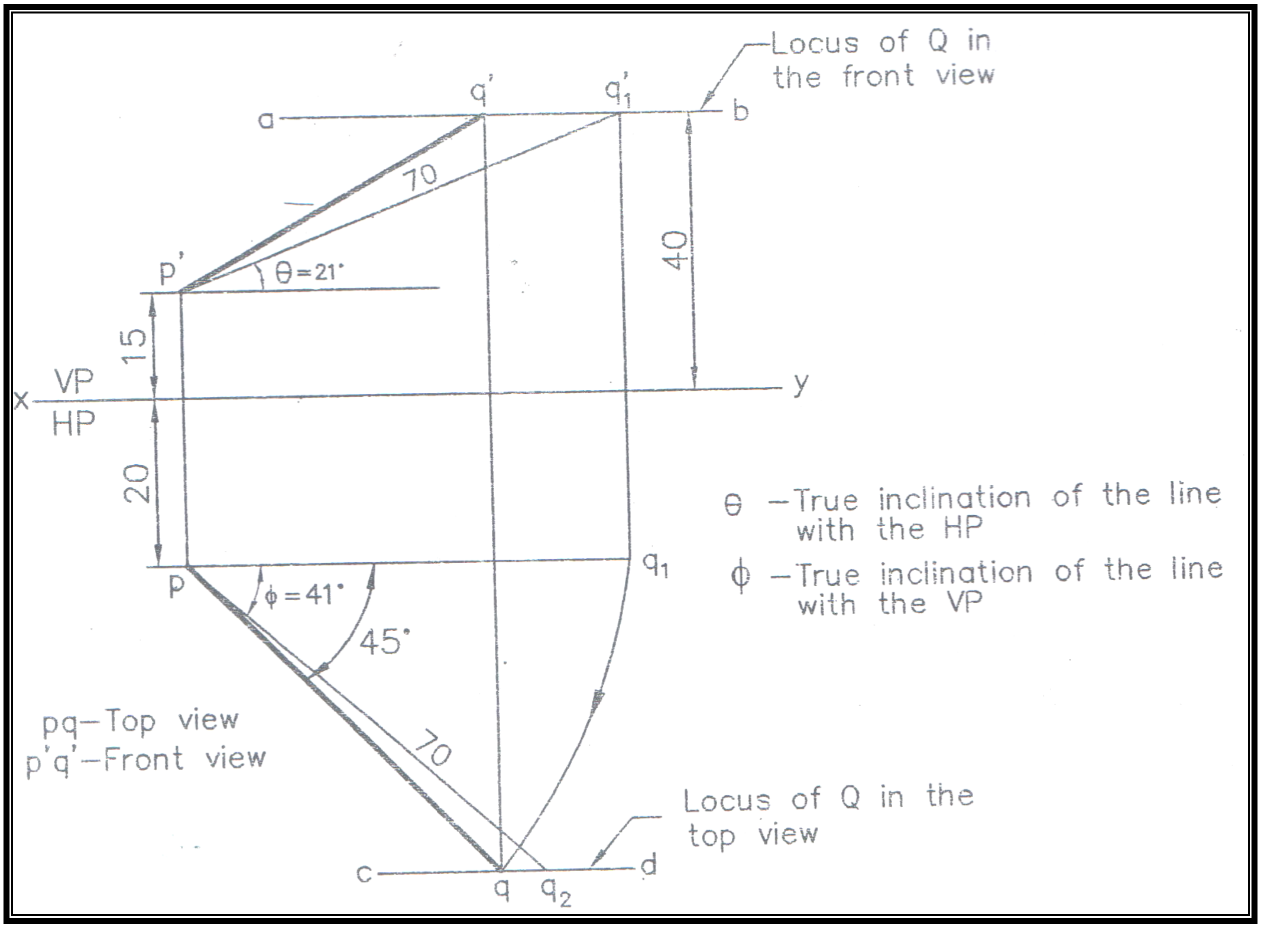


Figure – 12: Straight line inclined to both the planes.

[Here $\theta = 21°$; $\phi = 41°$]

1. Mark the projections of **P**. **Q** is 40 mm above the **HP**. Draw **ab** parallel to **XY** at a distance 40 mm above **XY** to represent the path of **Q** in the front view.
   The line is assumed to be parallel to the **VP** and inclined to the **HP**. The front view will show the true length and true inclination with the **HP**.
2. With **p'** as centre and true length 70 mm as radius, draw arc to meet the path **ab** at **q_1'**. **ϕ**, the inclination of **p'q_1'** with **XY** shows the true inclination of the line with the **HP**.
3. Project **q_1'** to **q_1** on a line drawn through **p**, parallel to **XY**. **pq_1** gives the length of the top view. Tilt **pq_1** by 45° to **pq**.
4. Project **q** to **q'** on **ab**. Draw lines **pq** and **p'q'**. Then, **pq** and **p'q'** give the top view and front view respectively of **PQ**.

5. With **p** as centre and radius 70 mm, draw an arc to meet **cd** at $\mathbf{q_2}$. Then, the inclination **ϕ** of $\mathbf{pq_2}$ with the horizontal shows the true inclination of the line with the **VP**.

Solved Problems – 6: ***A line EF, 85 mm long has its end E, 25 mm above the HP and 20 mm in-front of the VP. The top and front views of the line have lengths of 55 mm and 70 mm respectively. Draw the projections of the line and find its true inclinations with the VP and HP.***

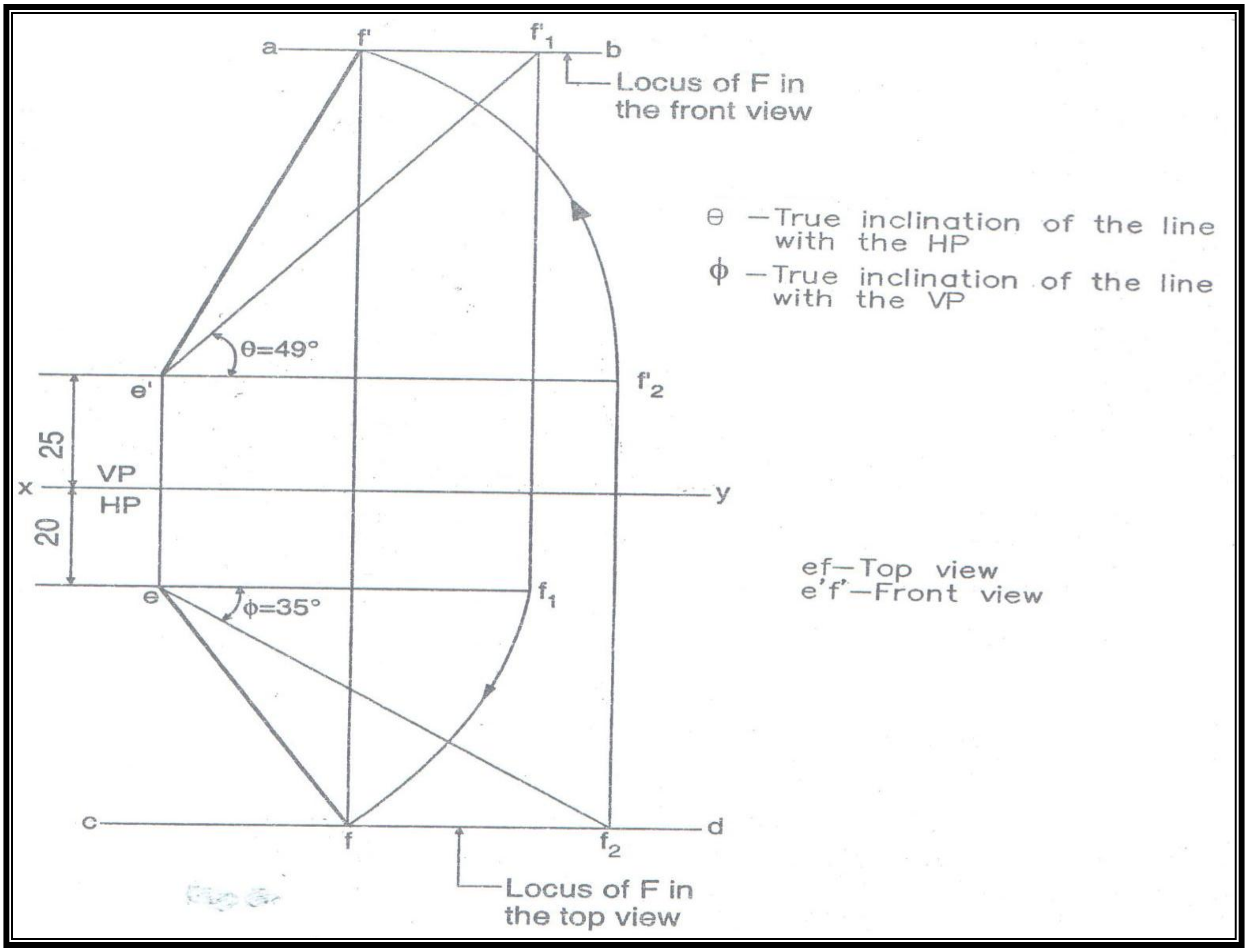


Figure – 13: Straight line inclined to both the planes.

1. Locate **e**, the top view of **E**, 20 mm below **XY** and **e'**, its front view, 25 mm above **XY**.
2. Assume the line to be parallel to the **VP** first. Its top view will be parallel to **XY** and its front view will have true length. Hence, draw $\mathbf{ef_1}$ of length 55 mm (length of the top view) parallel to **XY**. Draw a projector through $\mathbf{f_1}$. With **e'** as centre and 85 mm (true length) as radius, draw an arc to cut the projector through $\mathbf{f_1}$ at $\mathbf{f_1'}$. Then, the inclination **θ** of the line $\mathbf{e'f_1'}$ represents the true inclination of the line with the **HP**.
3. Draw **ab**, the path of **F** in the front view, parallel to **XY** through $\mathbf{f_1'}$. Repeat the construction with the front view. Draw $\mathbf{e'f_2'}$ parallel to **XY** and of length 70 mm (given). Draw a

projector down through **f_2'**. With e as centre and radius 85 mm, draw an arc to intersect the projector through **f_2'** at **f_2**. The inclination **ϕ** of **ef_2** with **XY** shows the true inclination of the line with the **VP**.

4. Draw **cd**, the path of **F** in the top view.
5. With **e** as centre and **ef_1** as radius, draw an arc to cut **cd** at **f**. With **e'** as centre and radius **e'f_2'**, draw arc to meet **ab** at **f'**. Draw lines **ef** and **e'f'** as the required projections.
6. Here, **θ** = 49° and **ϕ** = 35°

Solved Problems – 7: ***A line PQ has its end P, 10 mm above the HP and 20 mm in-front of the VP. The end Q is 35 mm in-front of the VP. The front view of the lines measures 75 mm. The distance between the end projectors is 50 mm. Draw the projections of the line and find its true length and its inclinations with the VP and the HP.***

1. Locate the projections **p** and **p'** of **P**. Draw a projector perpendicular to **XY** at 50 mm from the projector **pp'**. On this projector, mark **q** at a distance of 35 mm below **XY**.
2. With **p'** as centre and 75 mm as radius, draw an arc to cut the projector through **q** at **q'**. **pq** and **p'q'** are the required projections of **PQ**.
3. Assume the line to be parallel to **VP**. The top view will be parallel to **XY**. The front view will show the true length and the true inclination of the line with the **HP**.
   With **p** as centre and **pq** as radius, draw an arc to cut the line drawn through p parallel to **XY** at **q1**. Project **q1** to **q1'** on **ab**, the path of **Q** in the front view. Then, **pq1'** shows the true length of the line. The inclination **θ** of **p'q_1'** with **XY** shows the true inclination of the line with the **HP**.
4. Next, the line is assumed to be parallel to the **HP** and inclined to **VP**. With centre **p'** and radius **p'q'**, draw an arc to meet the line drawn through **p'** parallel to **XY** at **q2'**. Project **q2'** to **q2** on **cd**, the path of **Q** in the top view. Then, the inclination **θ** of **pq2** with **XY** shows the true inclination of the line with the **VP**. It can be checked that **p'q_1' = pq_2**.
   Here, **θ** = 45° and **ϕ** = 11°.

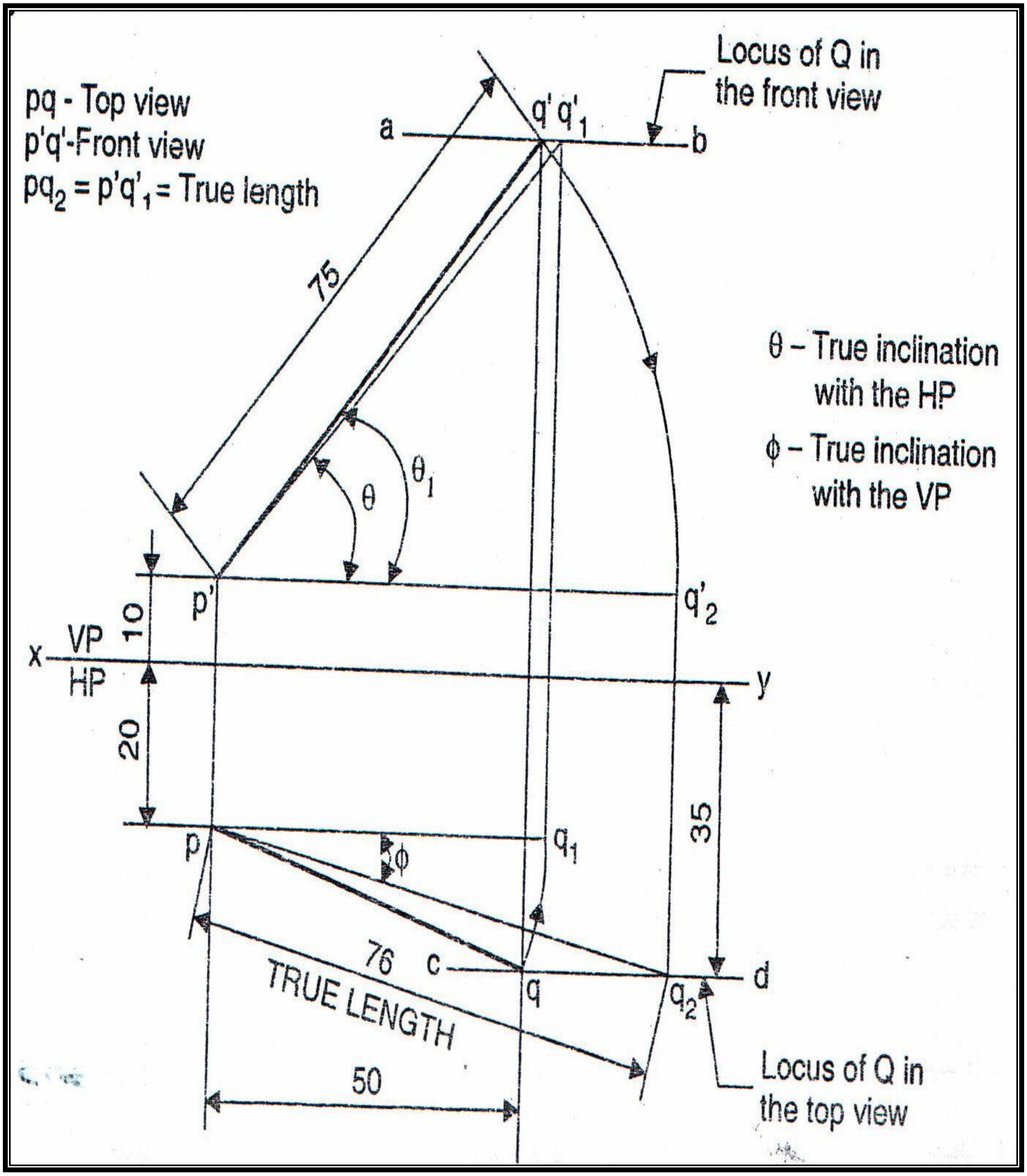


Figure – 14: Straight line inclined to both the planes.

Solved Problems – 8: ***A straight line ST has its end S, 10 mm in-front of the VP and nearer to it. The mid-point m of the line is 50 mm in-front of the VP and 40 mm above the HP. The front and top views measures 100 mm and 120 mm respectively. Draw the projections of the line. Also find its true length and true inclinations with the HP and the VP.***

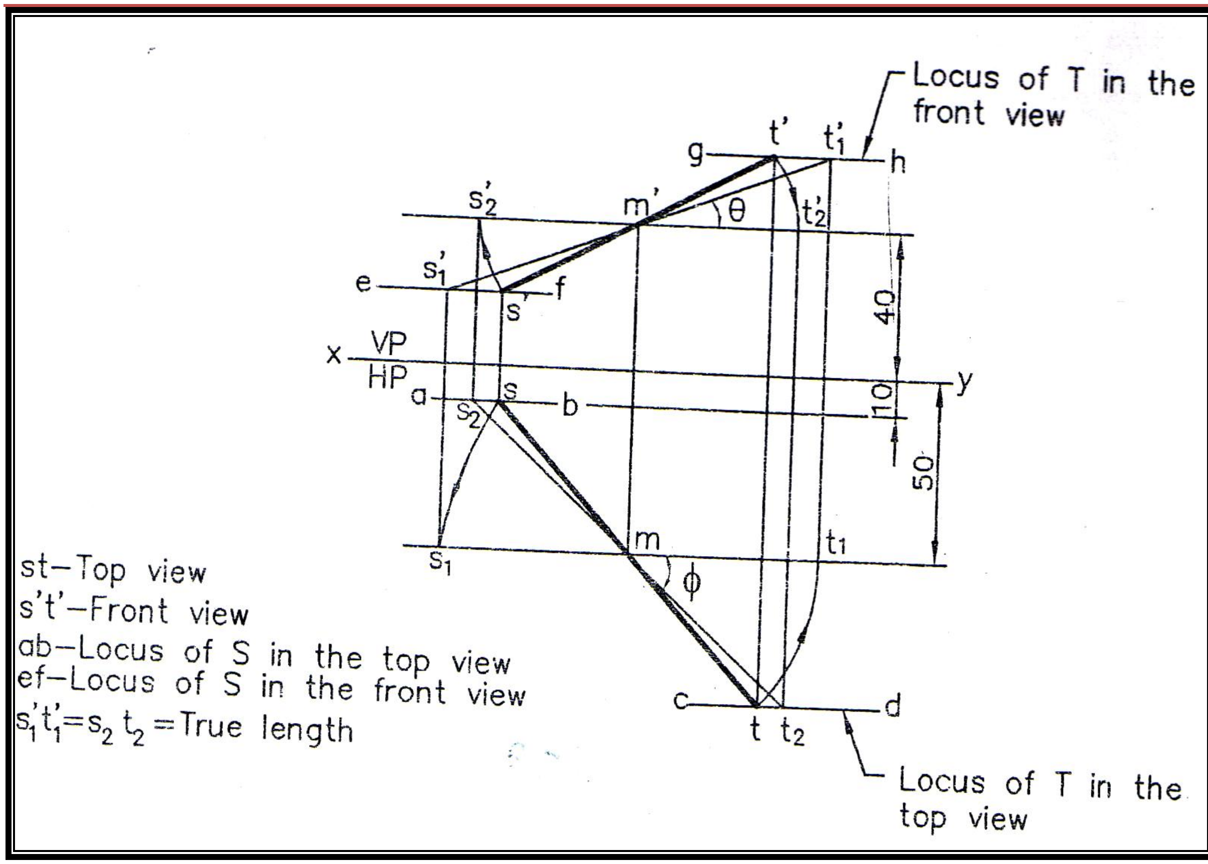


Figure – 15: Straight line inclined to both the planes.

1. Locate the projections of the mid-point **M**. Draw the locus **ab** of **S** in the top view, 10 mm below **XY**.
2. With **m** as centre and half the length of top view (60 mm) as radius, draw an arc to cut the locus **ab** at s. Join **sm** and produce it to t such that **sm** = **mt**. Through **t**, draw **cd**, parallel to **XY** to denote the locus of **T** in the top view.
3. Draw the projectors upwards through **s** and **t**.
4. With **m'** as centre and half the length of the front view (50 mm) as radius, draw arcs to cut the projector through **s** at **s'** and that through **t** at **t'**.
5. Then, **st** and **s't'** are the top and front view of the line respectively.

To find the true length and true inclinations

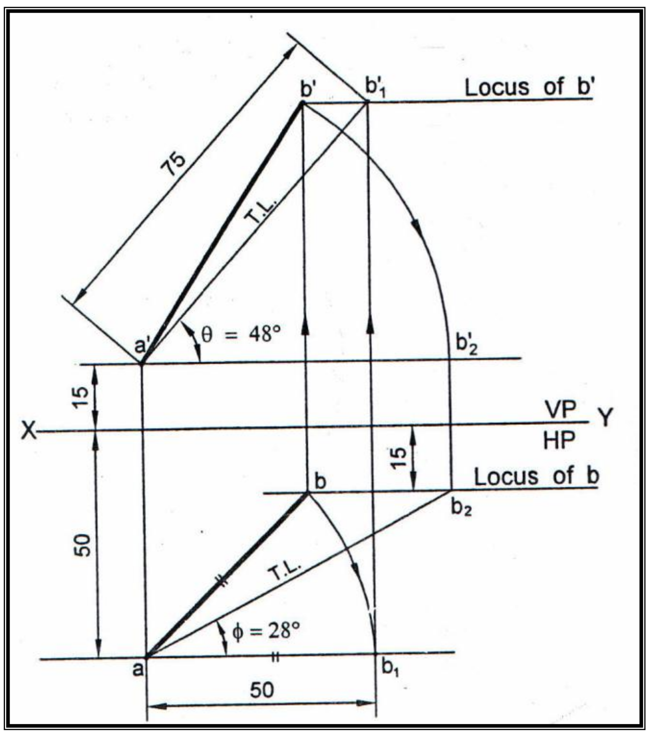
6. Make the top view parallel to **XY** such that the line is parallel to the **VP**. Then, the corresponding front view will give the true length and true inclination with the **HP**.

Accordingly, with **m** as centre and ms as radius draw an arc to meet the horizontal line through **m** at s_1. Similarly, with **m** as centre and **mt** as radius, draw arc to cut the line through **m** at **t1**.

7. Project s_1 to s_1' on **ef**, the path of **S** in the front view. Similarly, project t_1 to t_1' on **gh**, the path of **T** in the front view. Join s_1' and t_1'. Then, $s_1't_1'$ gives the true length of the line and its inclination **θ** with **XY** gives the true inclination of the line with the **HP**.
8. Make the front view **s't'** horizontal. Project s_2' to s_2 on **ab**, the path of **S** in the top view. Project t_2' to t_2 on **cd**, the path of T in the top view. Then, s_2t_2 also shows the true length and its inclination **ɸ** with the horizontal gives the true inclination of the line with the **VP**.
9. Here, the true length ($s_1't_1' = s_2t_2$) is measured as 128 mm. **θ** = 21° and **ɸ** = 38°.

Solved Problems – 9: ***A line AB is 75 mm long. A is 50 mm in-front of VP and 15 mm above HP. B is 15 mm in-front of VP and is above HP. Top view of AB is 50 mm long. Find the front view length and the true inclinations.***

Figure – 16: Straight line inclined to both the planes.



1. Mark **a'** 15 mm above **XY** and **a** 50 mm below **XY**.
2. **b** is 15 mm in-front of **VP**. So, draw a horizontal **15** mm below **XY** to represent the locus of **b**. Top view length is 50 mm. Hence a as centre and 50 mm as radius, draw an arc to cut the locus of **b** at **b**. **ab** is the top view.
3. Now, the top view is made parallel to **XY**. i.e., the line **AB** is made parallel to **VP**. A is fixed, **θ** is fixed. So, the front view corresponding to this top view ab_1 gives true length and true inclination **θ** with **HP**. **a** as centre and **ab** as radius draw an arc to meet the horizontal drawn through **a** at b_1. True length is given as 75 mm. Hence, **a'** as center and 75 mm as radius, draw an arc to intersect the projector drawn from b_1 at b_1'. Join $a'b_1'$. Measure **θ** = 48°.
4. Draw a horizontal line through b_1' to represent the locus of **b'**. From **b** draw a projector to cut the locus of **b'** at **b'**. Join **a'b'** = 66 mm.
5. **a** as centre and 75 mm as radius cut a point b_2 on the locus of **b**. Join ab_2. Measure **ɸ** = 28°.
6. Mark b_2'. b_2' should lie on the projector drawn through b_2 as shown in the above figure.

Solved Problems – 10: ***A line AB 100 mm long has its front view inclined at an angle of 45° to the reference line separating the views. The end A is in VP and 25 mm above HP. The length of the front view is 60 mm. Draw the top view of the line and find the true inclinations of the line with HP and VP.***

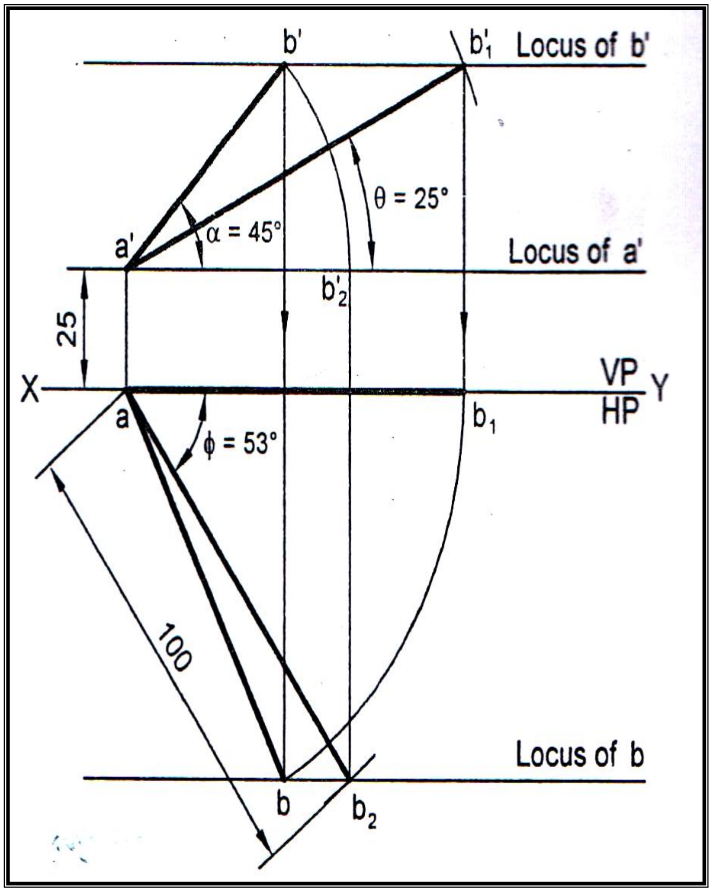
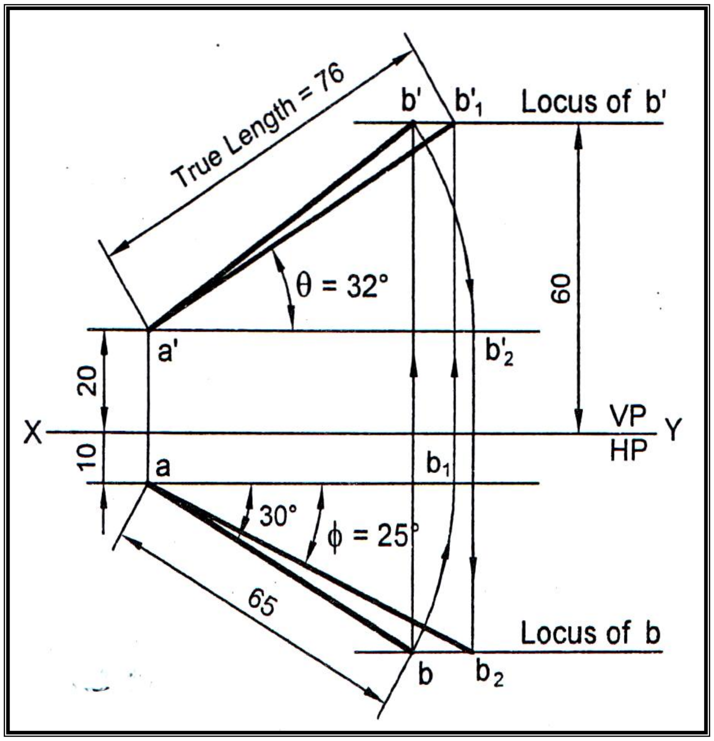


Figure – 17: Straight line inclined to both the planes.

1. Draw **XY**. Mark **a'** 25 mm above **XY** and **a** on **XY**.
2. **α** and **a'b'** are given. Therefore draw **a'b'** = 60 mm at **α** = 45° to **XY**.
3. Draw the locus of **b'**. **a'** as centre and true length 100 mm as radius drawn an arc to cut the locus of **b'** at **b_1'**. Join **a'b_1'**. Measure **θ** = 25°.
4. From **b_1'** draw a projector to cut **XY** at **b_1**. **ab_1** is the top view of **a'b_1'**.
5. **a** as centre and **ab_1** as radius draw an arc to cut the projector from **b'** at **b**.
6. Join **ab** and measure **ab** = 90 mm.
7. Draw the locus of **b**. **a** as centre and true length 100 mm as radius draw an arc to cut the locus of **b** at **b_2**.
8. Join **ab_2**. Measure **ϕ** = 53°.

Solved Problems – 11:	***The top view of a line is 65 mm long and is inclined at 30° to the reference line. One end is 20 mm above HP and 10 mm in-front of VP. The other end is 60 mm above HP and is in-front of VP. Draw the projections and find the true length of the line and its true inclination to HP and VP.***



TL = 76 mm, **θ** = 32° and **ϕ** = 25°

Figure – 18: Straight line inclined to both the planes.

Solved Problems – 12:	***The mid-point of a straight line AB is 60 mm above HP and 50 mm in-front of VP. The line measures 80 mm long and inclined at an angle of 30° to HP and 45° to VP. Draw its projections.***

1. Mark **m’** 60 mm above **XY** and **m** 50 mm below **XY**.

To obtain the loci of a’ and b’

2. To start with, assume the line to be inclined to only one plane, say, **HP** and made parallel to **VP** with reference to mid-point **M**.
3. Thus obtain the loci of **b’** and **a’**. The whole line is now 50 mm in-front of **VP**.
4. Top view corresponding to this will be parallel to **XY** and passes through **m**.
5. Through **m’** draw a line 30° (**θ**) to **XY**.
6. Mark **a_1’b_1’** = 80 mm = True length on this line such that **a_1’m’** = **m’b_1’** = 40 mm.
7. From **a_1’** draw a horizontal to respect the locus of **a’**.

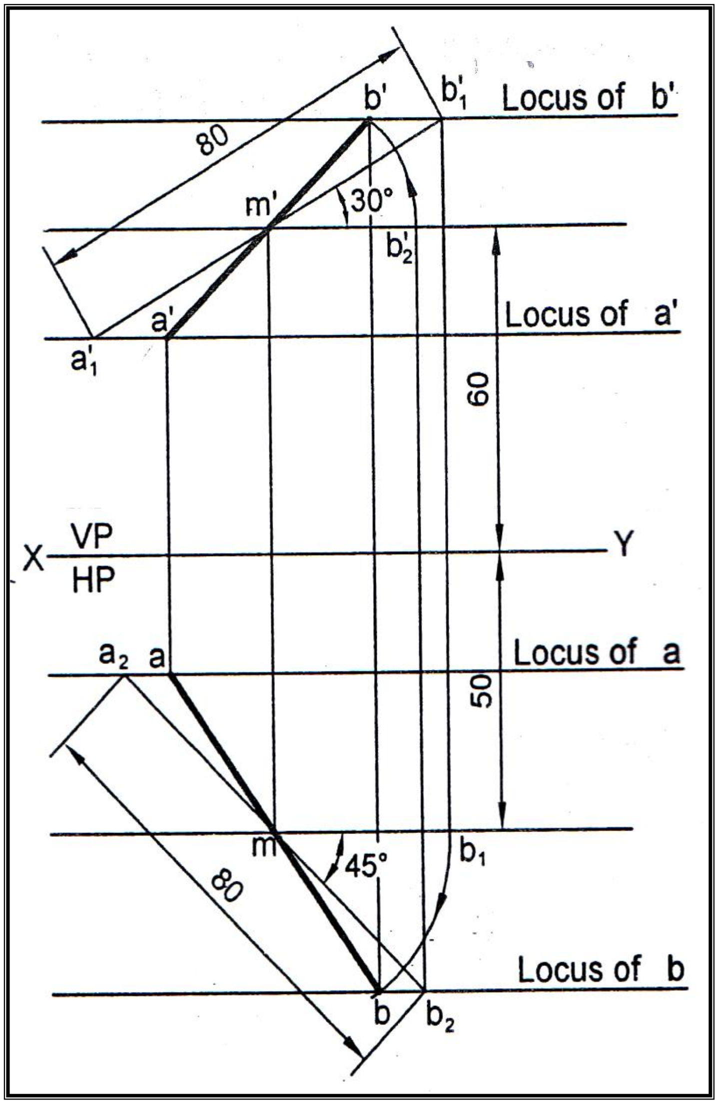


Figure – 19: Straight line inclined to both the planes.

8. From **b$_1$’** draw a horizontal to respect the locus of **b’**.

To obtain the loci of a and b

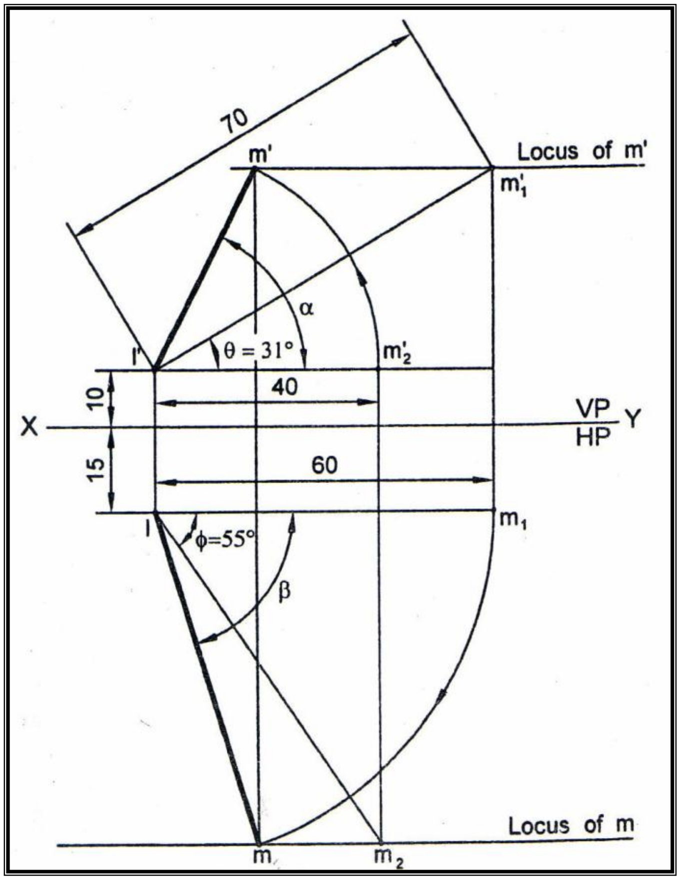
9. Next assume that the line is inclined to **VP** only and made parallel to **HP** with reference to **M**. We can thus obtain the loci of **b** and **a**.
10. Through m draw a line 45° (**ϕ**) to **XY**.
11. Mark **a$_2$b$_2$** = 80 mm = True length on this line such that **a$_2$m** = **mb$_2$** = 40 mm.
12. From **a$_2$** draw a horizontal to respect the locus of **a**.
13. From **b$_2$** draw a horizontal to respect the locus of **b**.

Consider the projection of right half of the line. Projection of remaining half is symmetrical.

14. Now consider the front view **m’b$_1$’** = 40 mm inclined at 30° to **XY**.
15. From **b$_1$’** draw a projector to cut the horizontal drawn through m at **b$_1$**. **mb$_1$** is the top view length for half of the straight line.
16. With **m** as centre and **mb$_1$** as radius drawn an arc to cut the locus of **b** at **b**.
17. From **b** draw a projector to cut the locus of **b’** at **b’**.
18. Join **bm** and extend this till it touches the locus of **a** at **a**. (**amb** is the top view of **AB**)
19. Join **b’m’** and extend this till it touches the locus of **a’** at **a’**. (**a’m’b’** is the front view of **AB**). Check : Now **a’** and **a** will be on the same projector.

Given front and top views, to find true length and true inclination

Solved Problems – 13: *A line LM 70 mm long has its end L 10 mm above HP and 15 mm in-front of VP. Its top and front views measure 60 mm and 40 mm respectively. Draw projections of the line. Find its inclinations with HP and VP.*



Mark **l'** 10 mm above **XY** and l 15 mm below **XY**.

To find θ and l'm'

1. From **l** draw a line parallel to **XY**.
2. Mark **lm**$_1$ = 60 mm = Top view length.
3. From **m**$_1$ draw projector.
4. **l'** as centre and true length 70 mm as radius draw an arc to cut above projector at **m**$_1$**'**.
5. Join **l'm**$_1$**'**. Now measure **θ** = 31°.
6. Draw the locus of **m'**.
7. **l'** as centre and front view 40 mm as radius draw an arc to cut the locus of **m'** at **m'**.
8. Join **l'm'**. This is the front view of **LM**.

To find ф and lm

9. Draw **l'm**$_2$**'** = 40 mm = Front view length and parallel to **XY**.
10. From **m**$_2$**'** draw a projector.

Figure – 20: Straight line inclined to both the planes.

11. **l** as centre and true length 70 mm as radius draw an arc to cut above projector at **m**$_2$.
12. Join **lm**$_2$. Now measure **ф** = 55°.
13. Through **m**$_2$ draw the locus of **m'**.
14. **l** as centre and 60 mm as radius draw an arc to cut the locus of **m** at **m**.
15. Join **lm**. This is the top view of **LM**.

PROJECTION OF PLANES

Introduction – Projection of Planes

A plane is a two dimensional object having length and breadth only. Its thickness is always neglected. Various shapes of plane figures are considered such as square, rectangle, circle, pentagon, hexagon, etc,

Types of Planes

1. Perpendicular planes which have their surface perpendicular to any one of the reference planes parallel or inclined to the other reference plane.
2. Oblique planes which have their surface inclined to both the reference planes.

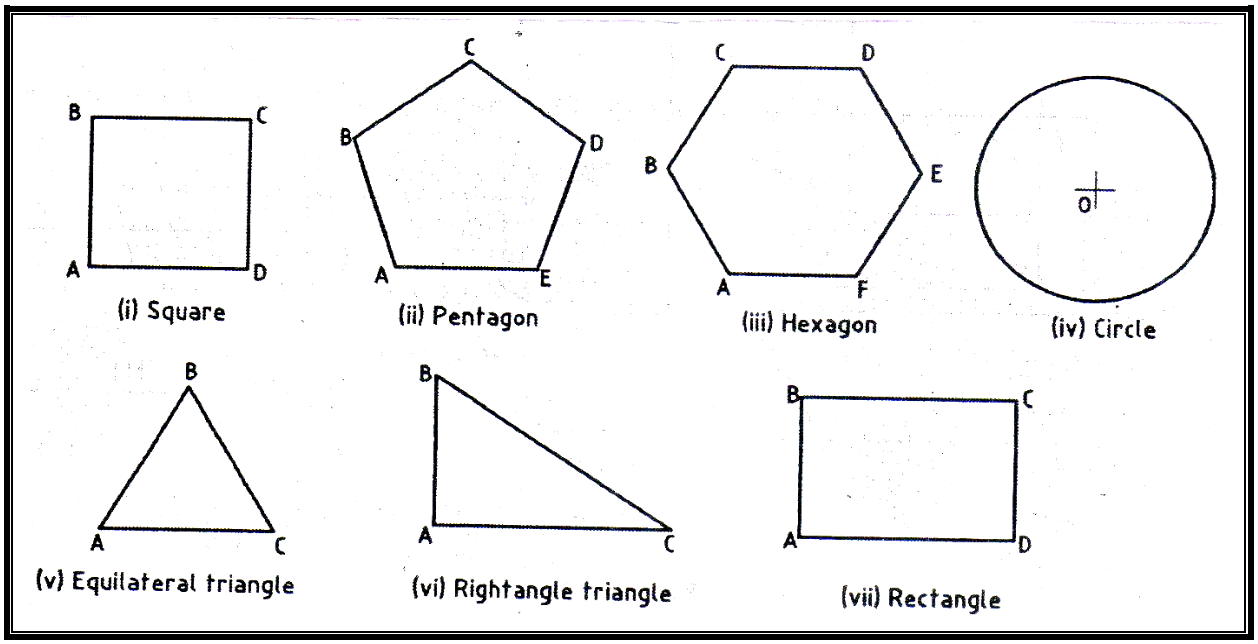


Figure – 1: Various types of planes.

Positions of Planes

A plane figure is positioned with reference to the reference planes by referring its surface in the following possible positions.

1. Surface of the plane kept perpendicular to **HP** and parallel to **VP**.
2. Surface of the plane kept perpendicular to **VP** and parallel to **HP**.
3. Surface of the plane kept perpendicular to both **HP** and **VP**.
4. Surface of the plane kept inclined to **HP** and perpendicular to **VP**.
5. Surface of the plane kept inclined to **VP** and perpendicular to **HP**.
6. Surface of the plane kept inclined to both **HP** and **VP**.

1. PROJECTIONS OF A PLANE SURFACE PERPENDICULAR TO HP AND PARALLEL TO VP

Consider a square plane **ABCD** having its surface perpendicular to **HP** and parallel to **VP** as shown in the below figure – 2 (ii).

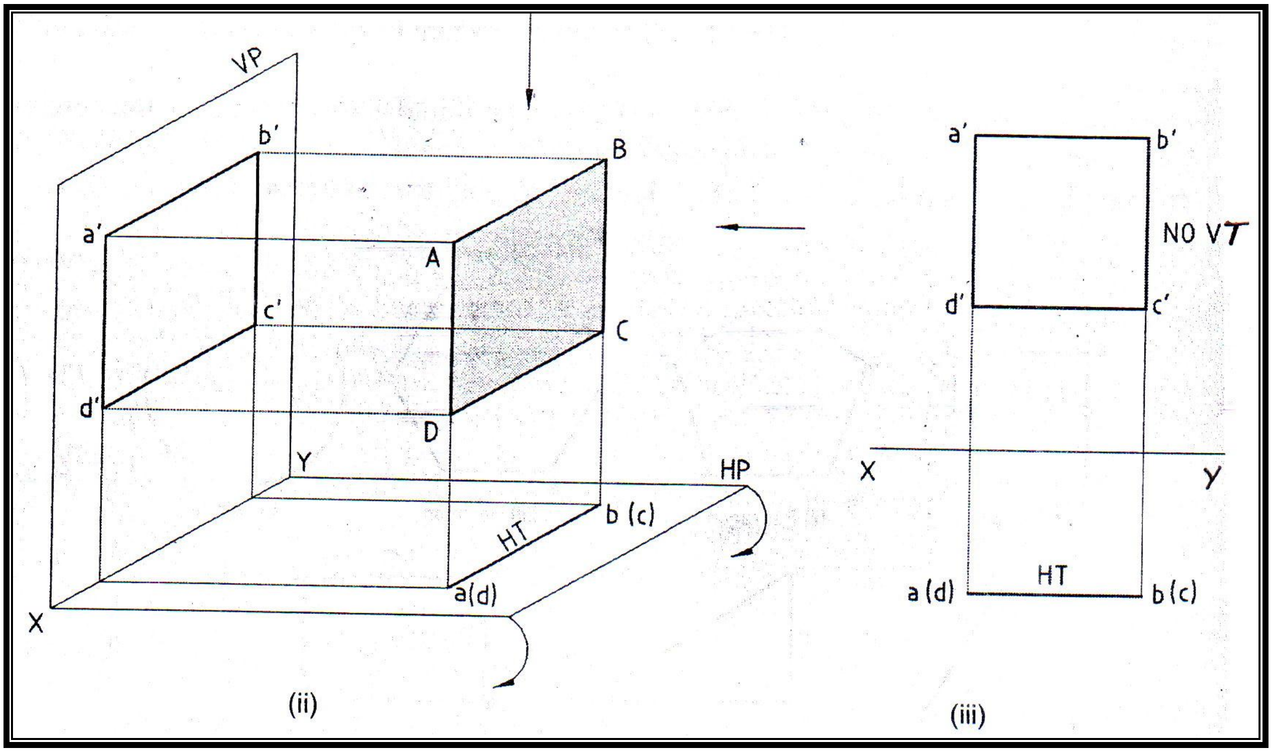


Figure – 2: **Surface of the plane kept perpendicular to HP and parallel to VP.**

The front view is projected onto **VP** which is a square **a' b' c'd'** having the true shape and size. The top view is projected onto **HP** and is a line **ab(c)(d)** parallel to **XY**. The invisible corners are enclosed in ( ).

The plane surface is extended to meet **HP** to get the **HT** which coincides with the top view of the plane. It does not have a **VT** because the plane is parallel to **VP**.

The projections and traces obtained are drawn with reference to the **XY** lines as shown in figure – 2 (iii).

2. PROJECTIONS OF A PLANE SURFACE PERPENDICULAR TO VP AND PARALLEL TO HP

Consider a square plane **ABCD** having its surface perpendicular to **VP** and parallel to **HP** as shown in the below figure – 3 (i).

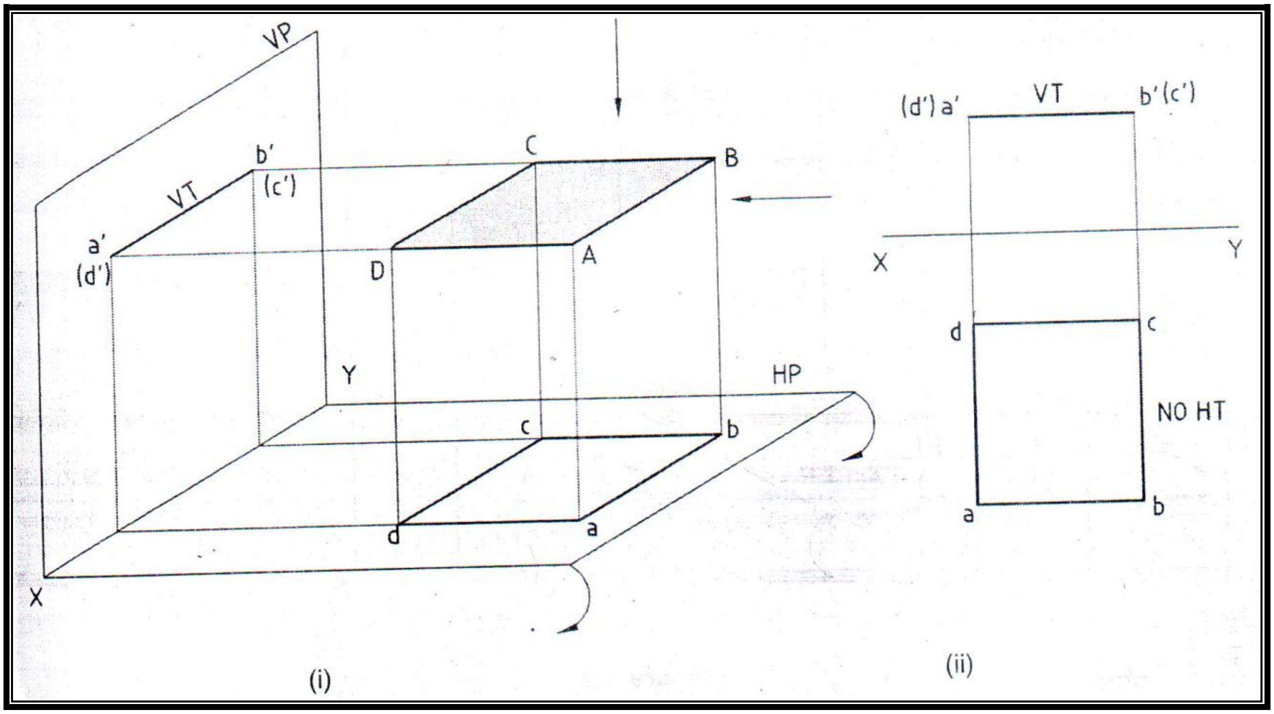


Figure – 3: **Surface of the plane kept perpendicular to VP and parallel to HP.**

The top view is projected onto **HP** which is a square **abcd** having the true shape and size. The front view is projected onto **VP** and is a line **a'b'(c')(d')** parallel to **XY**. The invisible corners are enclosed in ( ).

The plane surface is extended to meet **VP** to get the **VT** which coincides with the top view of the plane. It does not have a **HT** because the plane is parallel to **HP**.

The projections and traces obtained are drawn with reference to the XY lines as shown in figure – 3 (ii).

3. PROJECTIONS OF A PLANE SURFACE PERPENDICULAR TO BOTH HP AND VP

Consider a square plane **ABCD** having its surface perpendicular to both **HP** and **VP** as shown in the below figure – 4 (i).

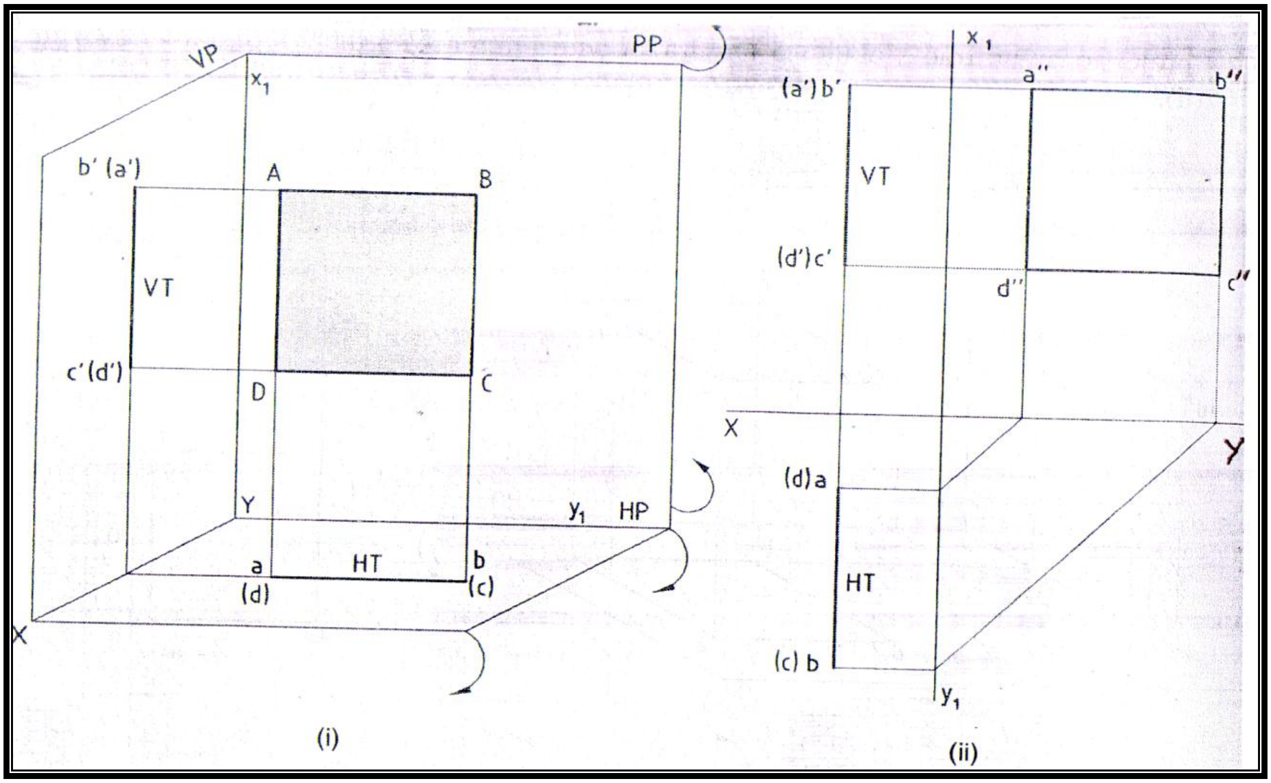


Figure – 4: **Surface of the plane kept perpendicular to both HP and VP.**

The front view **b'c'(d')(a')** and top view **ab(c)(d)** are projected onto **VP** and **HP** respectively. Both the views are lines perpendicular to the **XY** line. The true shape of the plane is obtained in the side view which is projected onto a profile plane **(pp)** which is perpendicular to both **HP** and **VP**. In this case, the left side view **a"b"c"d"** is obtained on the **PP** which is at the right side of the given object (plane).

The plane surface is extended to meet **HP** and **VP** to get **HT** and **VT** which coincides with the top and front views respectively.

The projections and traces obtained are drawn with reference to the **XY** lines as shown in figure – 4 (ii).

4. PROJECTIONS OF A PLANE SURFACE INCLINED TO HP AND PERPENDICULAR TO VP

Consider a square plane **ABCD** having its surface inclined at an angle of **θ** to **HP** and perpendicular **VP** as shown in figure – 5 (i).

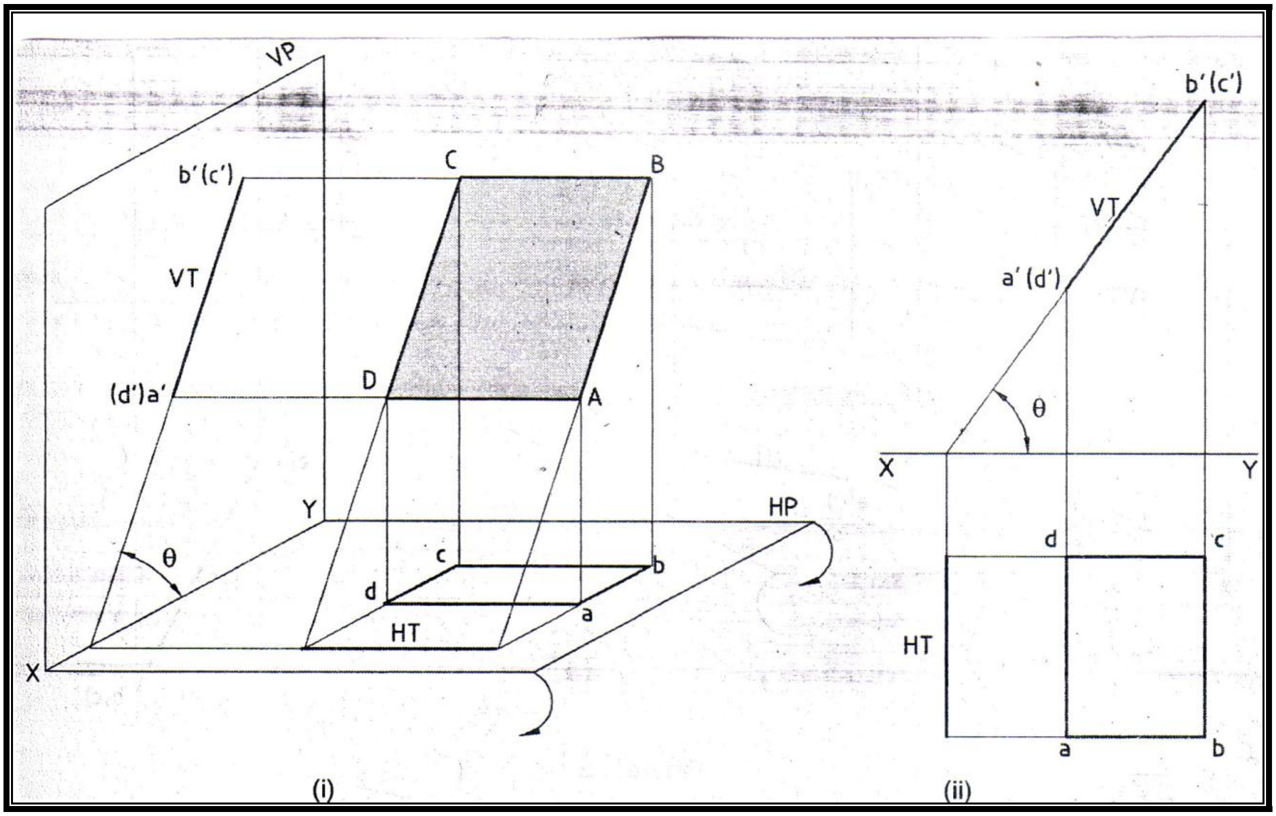


Figure – 5: **Surface of the plane kept inclined to HP and perpendicular to VP.**

The top view **abcd** is projected onto **HP**. It is smaller than the true shape and size. The front view is projected onto **VP** and is a line **a'b'(c')(d')** inclined at an angle **θ** to **XY**. The invisible corners are enclosed in ( ).

The plane surface is extended to meet **HP** to get the **HT** which is a line perpendicular to **XY**. The plane surface is also extended to meet **VP** to get the **VT** which is a line inclined at an angle **θ** to **XY**.

The projections and traces obtained are drawn with reference to the **XY** lines as shown in figure – 5 (ii).

5. PROJECTIONS OF A PLANE SURFACE INCLINED TO VP AND PERPENDICULAR TO HP

Consider a square plane **ABCD** having its surface inclined at an angle of **ϕ** to **VP** and perpendicular **HP** as shown in figure – 6 (i).

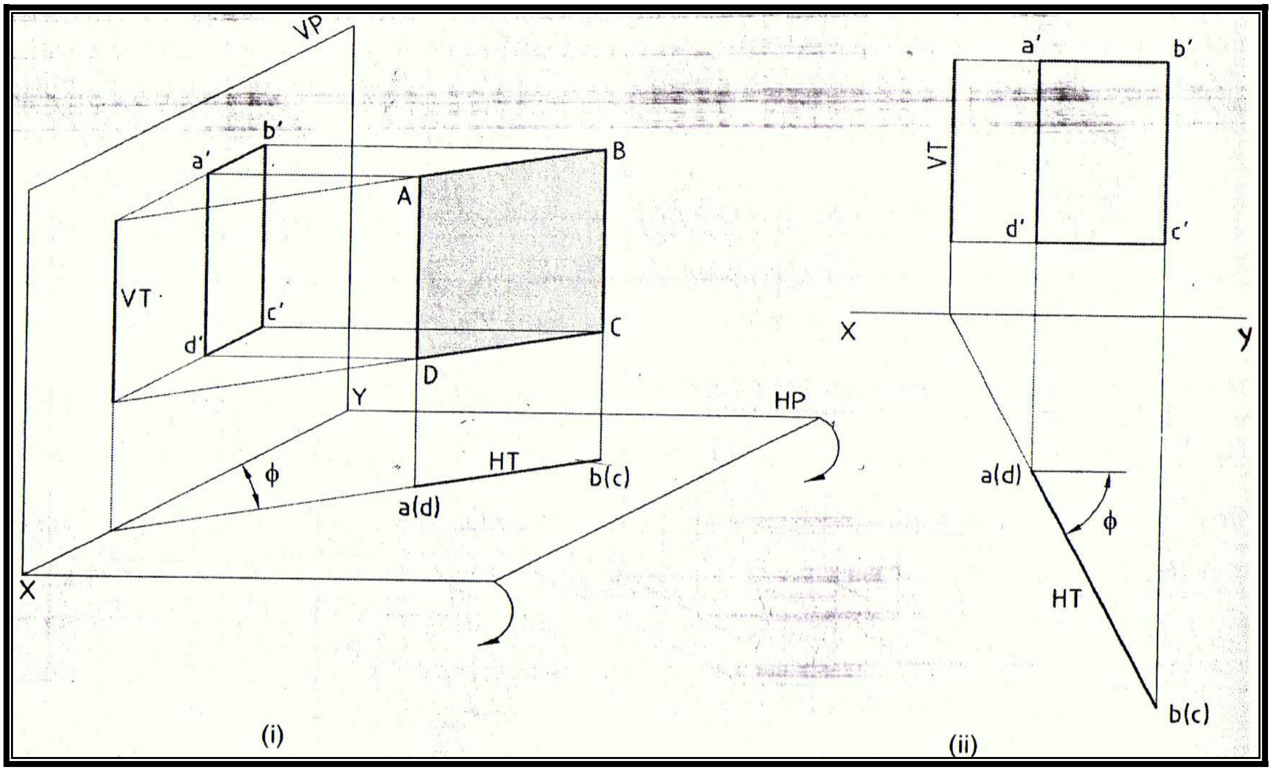


Figure – 6: **Surface of the plane kept inclined to VP and perpendicular to HP.**

The front view **a'b'c'd'** is projected onto **VP**. It is smaller than the true shape and size. The top view is projected onto **HP** and is a line **ab(c)(d)** inclined at an angle **ϕ** to **XY**. The invisible corners are enclosed in ( ).

The plane surface is extended to meet **VP** to get the **VT** which is a line perpendicular to **XY**. The plane surface is also extended to meet **HP** to get the **HT** which is a line inclined at an angle **ϕ** to **XY**.

The projections and traces obtained are drawn with reference to the **XY** lines as shown in figure – 6 (ii).

Solved Problems – 1: ***A circular plate of diameter 50 mm is resting on HP on a point on the circumference with its surface inclined at 45° to HP and perpendicular to VP. Draw its projections.***

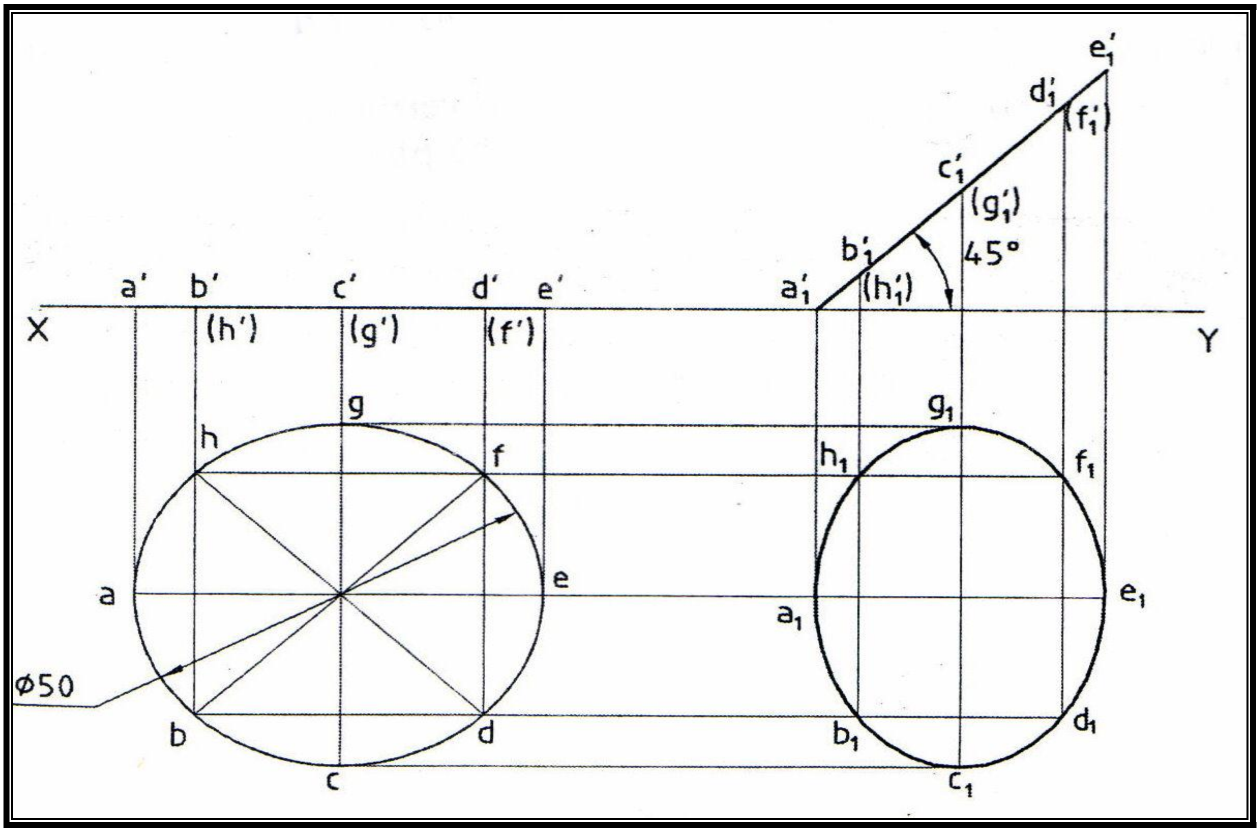


Figure – 7: Projection of planes.

To draw the projections

1. Assume that the plate has its surface parallel to **HP** perpendicular to **VP**. Draw its top view. It is a circle of radius 25 mm.
2. Project and get the front view which is a line on **XY**.
3. As the circle does not have any corners, divide the circle into equal parts, say 8, in such a way that 8 points are marked on its circumference and project them to the front view.
4. Tilt and reproduce the front view to the given angle 45° with **XY** line, in such a way that the end **a'** is on **XY** line.
5. Draw horizontal lines from **a, b, c,** etc., and vertical lines from **a_1', b_1', c_1',** etc., to get the required top view **a_1, b_1, c_1,** etc.,
6. Join **a_1, b_1, c_1,** etc., by drawing a smooth curve to get the top view of the circle as an ellipse.

Solved Problems – 2:	***A rectangular plate of side 50 × 25 mm is resting on its shorter side on HP and inclined at 30° to VP. Its surfaces is inclined at 60° to HP. Draw its projections.***

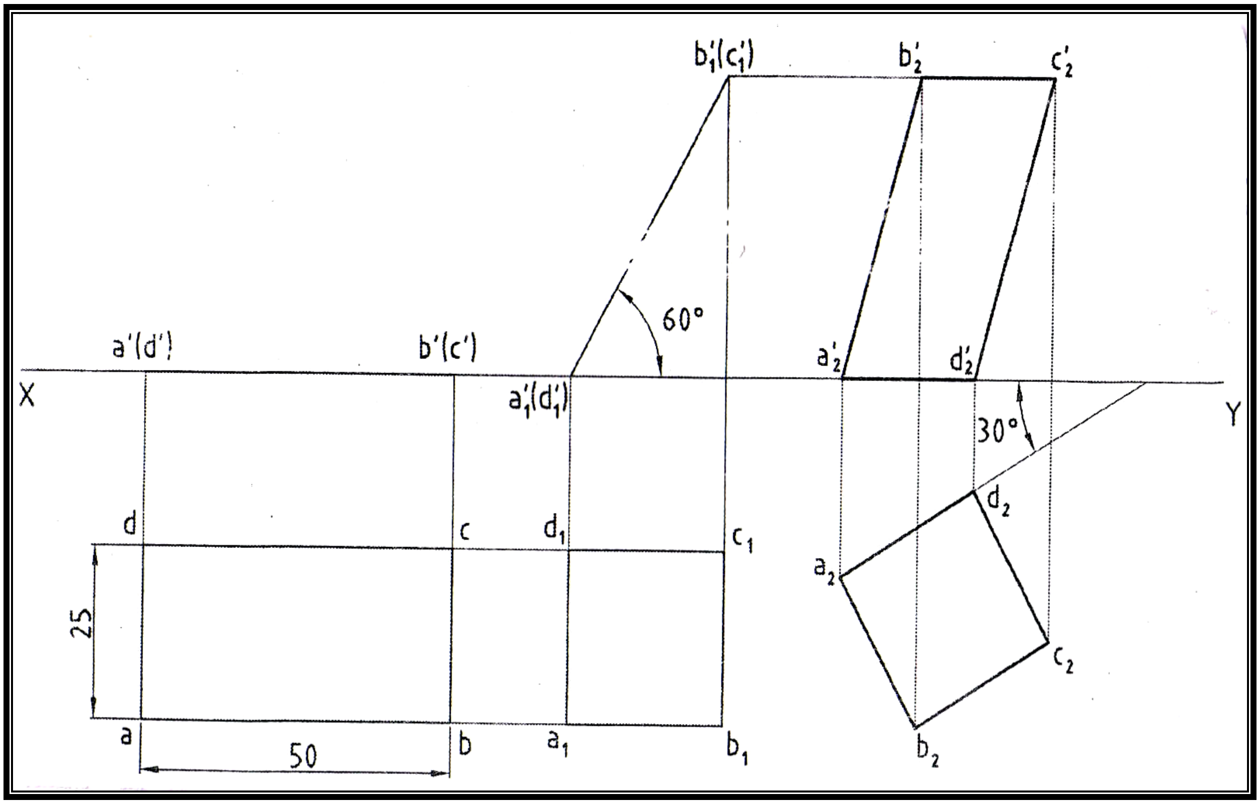


Figure – 8: Projection of planes.

Solution

In this position, the surface of the plate is inclined to both **HP** and **VP**; its projections are obtained in three steps.

Step – 1:

Assume that the plate has its surface parallel to **HP** and perpendicular to **VP**. Draw the top view which will have the true shape and size. Project the front view which will be a line parallel to **XY**.

Step – 2:

Reproduce the front view tilted to the given angle **θ** to **HP** and project the top view of the plate which will be smaller that the true shape and size.

Step – 3:

Reproduce the top view by considering the side of the plate that makes the given angle with **VP**. Project the front view of the plate which is also smaller than the true shape and size.

To draw the projections

1. Draw the top view of the rectangle considering that one of the shorter sides is perpendicular to **XY**. Then only while tilting the surface to the required angle with **HP**, this side of the plate will rest on **HP**.
2. The front view of the plate is projected and obtained on **XY** as a line **a’(d’)b’(c’)**.
3. Tilt and reproduce the front view **a’(d’)b’(c’)** to the given angle 60° with **XY** in such a way that the end **a’(d’)** is on **XY** line.
4. Draw horizontal lines from top view **a, b, c** and **d** and vertical lines from front view **a_1’, b_1’, c_1’** and **d_1’** to get the top view **a_1, b_1, c_1** and **d_1** smaller than the true shape and size.
5. Reproduce the top view **a_1, b_1, c_1** and **d_1** in such a way that the side **a_1d_1** is inclined to the given angle 30° to **VP**.
6. Draw horizontal lines from **a_1’, b_1’, c_1’** and **d_1’** and vertical lines from top view **a_2, b_2, c_2** and **d_2** to get the required front view **a_2’, b_2’ c_2’, d_2’** of the plate smaller than the true shape and size.

Solved Problems – 3: ***A hexagonal plate of side 30 mm is resting on one of its sides on VP and inclined at 40° to HP. Its surface is inclined at 35° to VP. Draw its projections.***

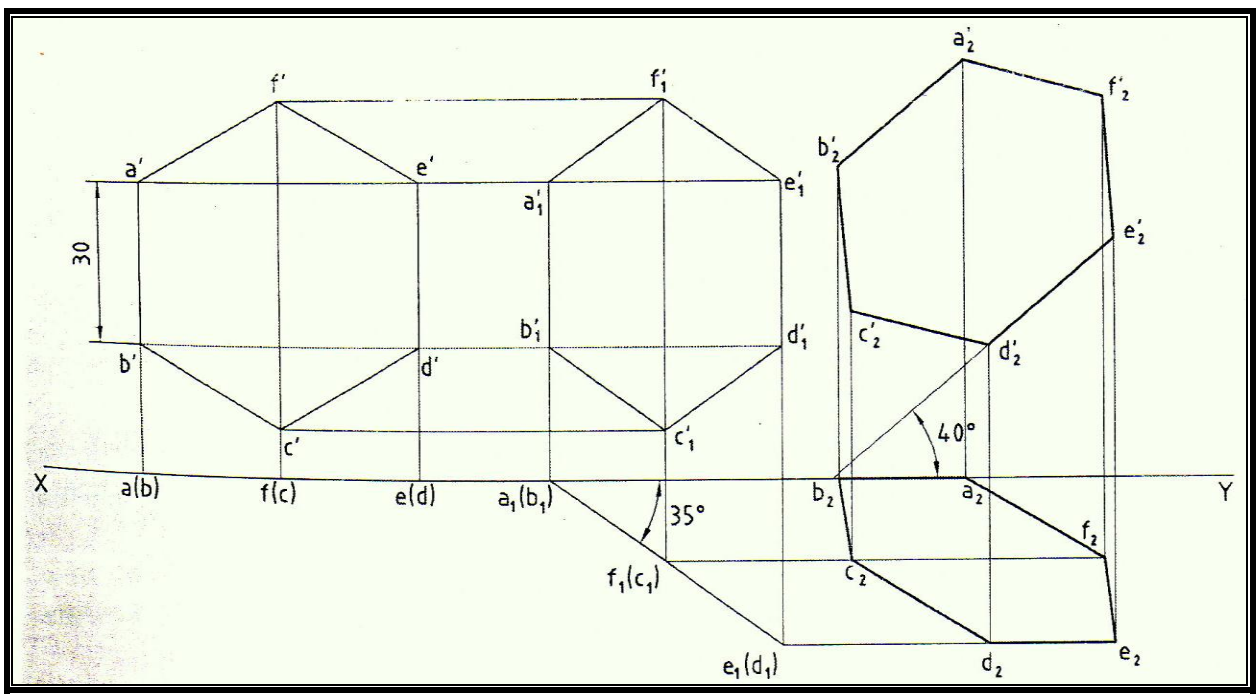


Figure – 9: Projection of planes.

Solution

In this position, the surface of the plate is inclined to both **VP** and **HP**; its projections are obtained in three steps.

Step – 1:

Assume that the plate has its surface parallel to **VP** and perpendicular to **HP**. Draw its front view which will have the true shape and size. Project the top view which will be a line parallel to **XY**.

Step – 2:

Reproduce the top view tilted to the given angle **ϕ** to **VP** and project the front view of the plate which will be smaller that the true shape and size.

Step – 3:

Reproduce the front view by considering the side of the plate that makes the given angle with **HP**. Project the front view of the plate which is also smaller than the true shape and size.

To draw the projections

1. Draw the front view of the hexagon considering one of the sides is perpendicular to **XY**. Then only while tilting the surface to the required angle with **VP**, this side of the plate will rest on **VP**.
2. The top view of the plate is projected and obtained on **XY** as a line.
3. Tilt and reproduce the top view line to the given angle 35° with **XY** in such a way that the end **a(b)** is on **XY** line.
4. Draw horizontal lines from front view **a', b', c',** etc., and vertical lines from top view $\mathbf{a_1}$**,** $\mathbf{b_1}$**,** $\mathbf{c_1}$**,** etc., to get the front view of the plate which is smaller than the true shape and size.
5. Reproduce the front view in such a way that the side $\mathbf{a_1b_1}$ is inclined to the given angle 40° to **HP**.
6. Draw horizontal lines from $\mathbf{a_1}$**,** $\mathbf{b_1}$**,** $\mathbf{c_1}$**,** etc., and vertical lines from $\mathbf{a_2}$**',** $\mathbf{b_2}$**',** $\mathbf{c_2}$**'**, etc., to get the required top view of the hexagonal plate which is smaller than the true shape and size.

Solved Problems – 4:	***A pentagonal plate 30 mm is resting on HP on one its corners with its surface inclined at 45° to HP. The side opposite to the resting corner is parallel to VP and farther away from it. Draw its projections.***

Solution

In this position, the surface of the plate is inclined to both **VP** and **HP**; its projections are obtained in three steps.

Step – 1:

Assume that the plate has its surface parallel to **VP** and perpendicular to **HP**. Draw its front view which will have the true shape ans size. Project the top view which will be a line parallel to **XY**.

Step – 2:

Reproduce the top view tilted to the given angle **ф** to **VP** and project the front view of the plate which will be smaller that the true shape and size.

Step – 3:

Reproduce the front view by considering the side of the plate that makes the given angle with **HP**. Project the front view of the plate which is also smaller than the true shape and size.

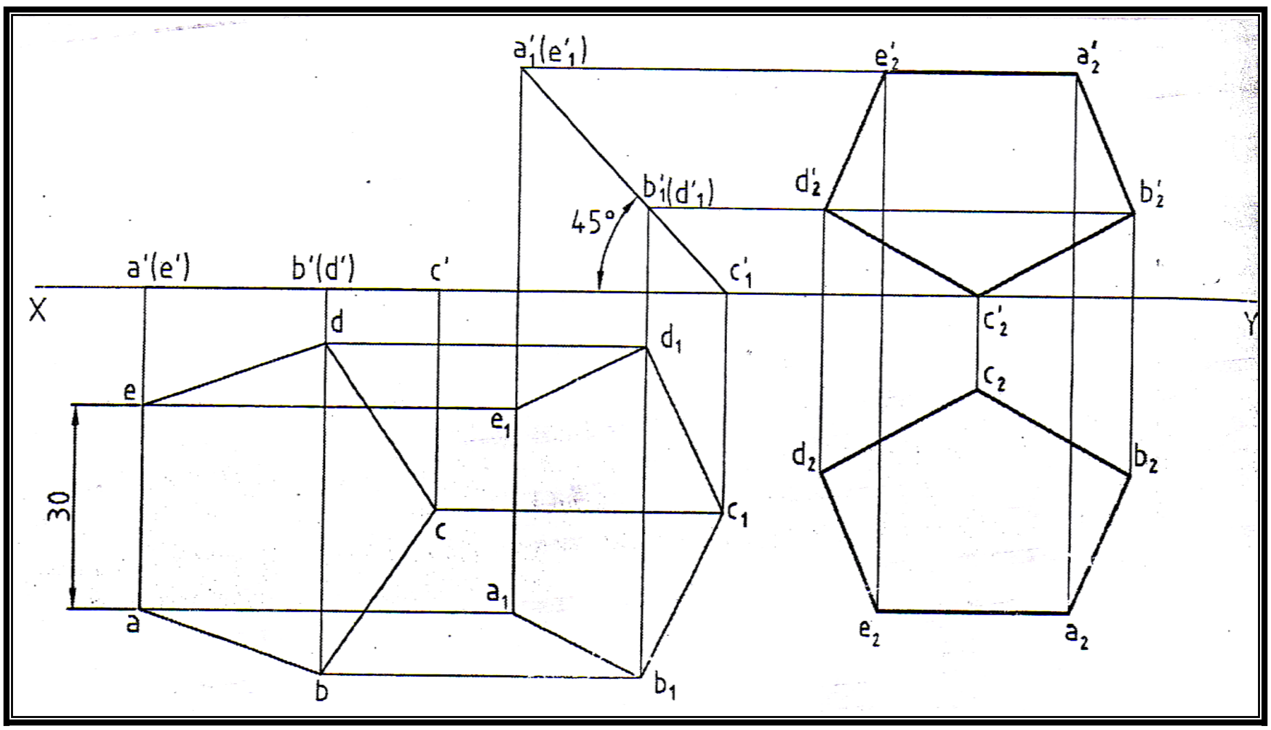


Figure – 10: Projection of planes.

To draw the projections

1. Draw the top view of the pentagonal plate considering one of the sides is perpendicular to **XY**. Then only while tilting the surface to the required angle with **HP**, this side of the plate will rest on **HP**. Name the corners as **a, b, c, d** and **e**.
2. The front view of the plate is projected and obtained on **XY** as a line **a'(e')b'(d')c'**.
3. Tilt and reproduce the front view line **a'(e')b'(d')c'** to the given angle 45° with **XY** in such a way that the corner **c'** is on **XY** line.
4. Draw horizontal lines from top view **a, b, c, d** and **e** and vertical lines from front view a_1', b_1', c_1', d_1' and e_1' to get the top view a_1, b_1, c_1, d_1 and e_1 of the plate which is smaller than the true shape and size.
5. Reproduce the top view a_1, b_1, c_1, d_1 and e_1 in such a way that the side e_1a_1 is inclined to **VP**.
6. Draw horizontal lines from a_1', b_1', c_1', d_1' and e_1' and vertical lines from a_2, b_2, c_2, d_2 and e_2 to get the required front view of the pentagonal plate which is also smaller than the true shape and size.

Solved Problems – 5: ***A square plate ABCD of side 30 mm is resting on HP on one of its corners and the diagonal AC inclined at 30° to HP. The diagonal BD of the plate is inclined at 45° to the VP and parallel to the HP. Draw its projections.***

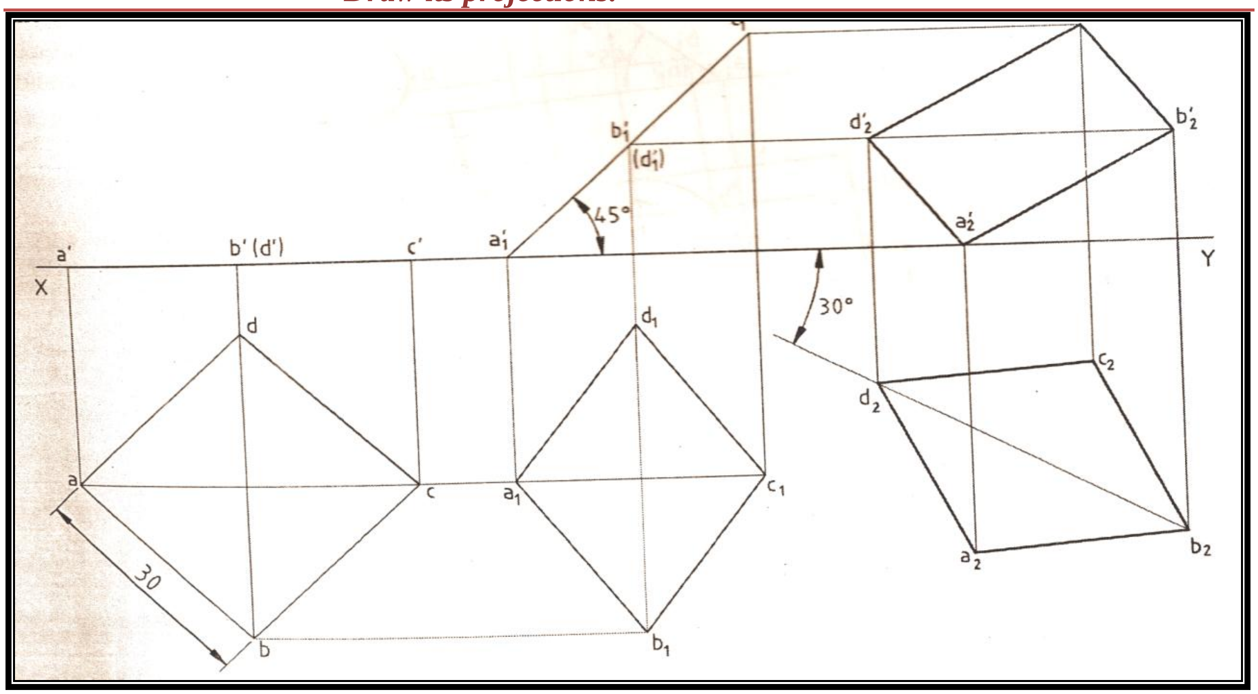


Figure – 11: Projection of planes.

Solution

In this position, the surface of the plate is inclined to both **VP** and **HP**; its projections are obtained in three steps.

Step – 1:

Assume that the plate has its surface parallel to **VP** and perpendicular to **HP**. Draw its front view which will have the true shape and size. Project the top view which will be a line parallel to **XY**.

Step – 2:

Reproduce the top view tilted to the given angle **ɸ** to **VP** and project the front view of the plate which will be smaller that the true shape and size.

Step – 3:

Reproduce the front view by considering the side of the plate that makes the given angle with **HP**. Project the front view of the plate which is also smaller than the true shape and size.

To draw the projections

1. Draw the top view of the square plate considering two of the sides are equally inclined to **XY**. Then only while tilting the surface to the required angle with **HP**, a corner of the plate will rest on **HP**. Name the corners as **a, b, c,** and **d**.
2. The front view of the plate is projected and obtained on **XY** as a line **a' b' (d') c'**.
3. Tilt and reproduce the front view line **a' b' (d') c'** to the given angle 45° with **XY** in such a way that the corner **a'** is on **XY** line.
4. Draw horizontal lines from top view **a, b, c, d** and vertical lines from front view **a_1',b_1', c_1', d_1'** to get the top view **a_1,b_1, c_1, d_1** of the plate which is smaller than the true shape and size.
5. Reproduce the top view **a_1, b_1, c_1, d_1** in such a way that the side **b_1d_1** is inclined at 30° to **VP**.
6. Draw horizontal lines from **a_1', b_1', c_1', d_1'** and vertical lines from **a_2, b_2, c_2, d_2** to get the required front view **a_2', b_2', c_2', d_2'** of the square plate which is also smaller than the true shape and size.

Solved Problems – 6: ***A circular plate of diameter 50 mm is resting on the HP on a point on the circumference. Its surface is kept inclined at 45° to HP. Draw its projections when the line representing its diameter and passing through the resting point is inclined at 30° to the VP.***

Solution

In this position, the surface of the plate is inclined to both **VP** and **HP**; its projections are obtained in three steps.

Step – 1:

Assume that the plate has its surface parallel to **VP** and perpendicular to **HP**. Draw its front view which will have the true shape and size. Project the top view which will be a line parallel to **XY**.

Step – 2:

Reproduce the top view tilted to the given angle **ϕ** to **VP** and project the front view of the plate which will be smaller that the true shape and size.

Step – 3:

Reproduce the front view by considering the side of the plate that makes the given angle with **HP**. Project the front view of the plate which is also smaller than the true shape and size.

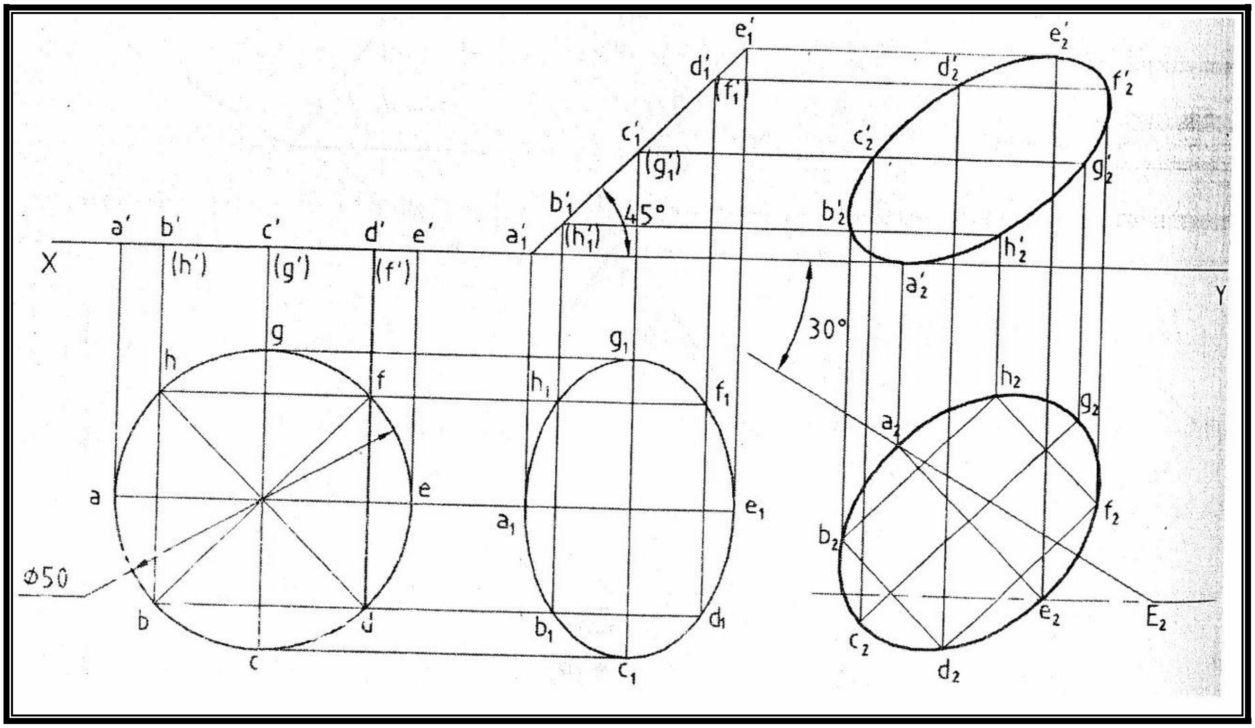


Figure – 12: Projection of planes.

To draw the projections

1. Assume that the circular plate has its surface parallel to **HP** and perpendicular to **VP**. Draw its top view which is a circle of radius 25 mm.
2. The front view of the plate is projected and obtained on **XY** as a line.
3. Divide the circle into 8 equal parts and mark **a, b, c,** etc., and project them to get **a', b', c',** etc., in front view.
4. Tilt and reproduce the front view line **a'e**' to the given angle 45° with **XY** in such a way that the corner **a'** is on **XY** line.

5. Draw horizontal lines from top view **a, b, c,** etc., and vertical lines from front view **a_1', b_1', c_1',** etc., to get the top view **a_1, b_1, c_1,** etc., and join them to get an ellipse.
6. Mark the true length of the diameter **a_2E_2** on the line drawn 30° to **XY**. Draw the locus of the end **E** of the diameter **ae**. Mark the diameter **a_2e_2** on the locus and reproduce the top view **a_2, b_2, c_2,** etc., and draw the ellipse.
7. Draw horizontal lines from **a_1', b_1', c_1',** etc., and vertical lines from **a_2, b_2, c_2,** etc., to get the required front view **a_2', b_2', c_2',** etc., of the plate.

Solved Problems – 7: ***Draw the projections of a circle of 70 mm diameter resting on the HP on a point A of the circumference. The plane is inclined to the HP such that the top view of it is an ellipse of minor axis 40 mm. The top view of the diameter, through the point A is making an angle of 45° with the VP. Determine the inclination of the plane with the HP.***

Answer ; θ = 55°

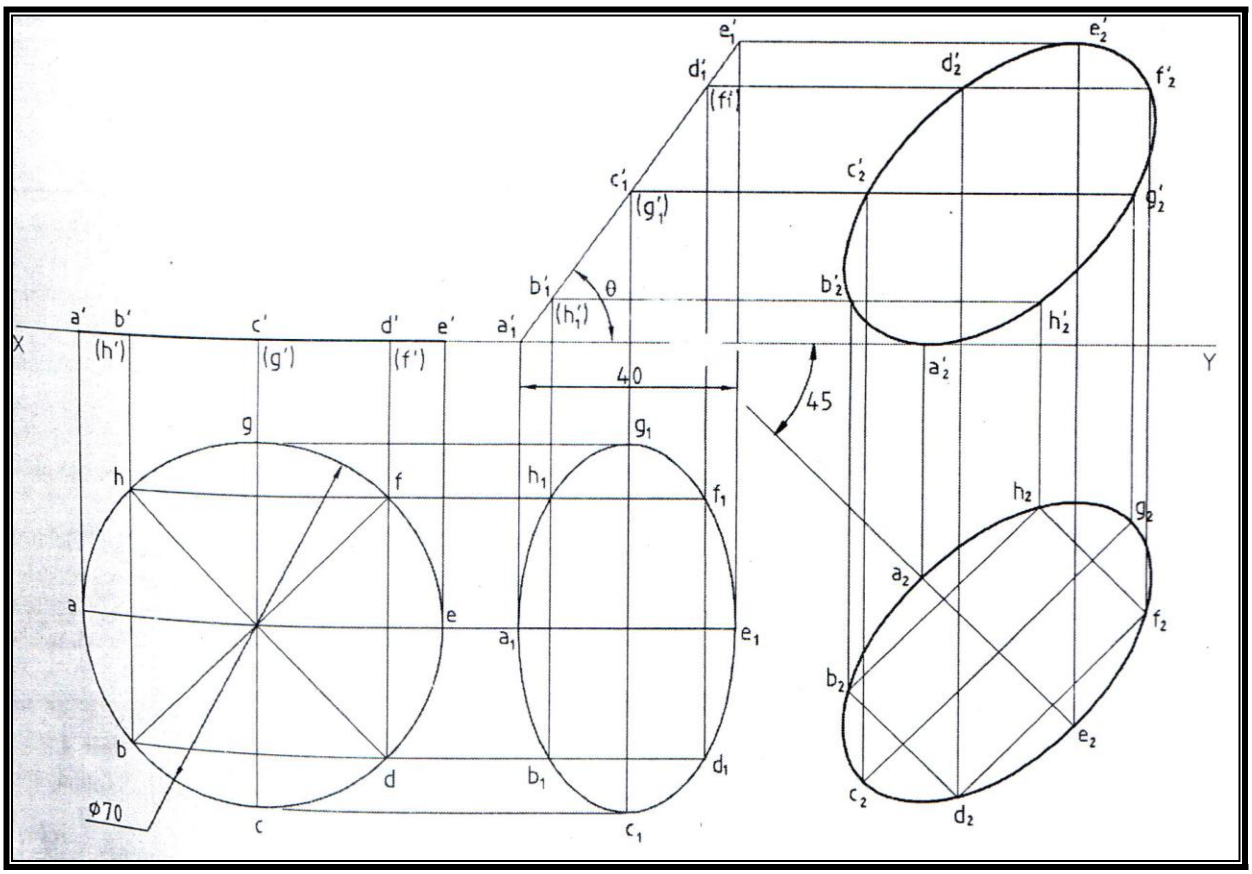


Figure – 13: Projection of planes

Solved Problems – 8: ***A rectangular lamina of size 60 mm × 30 mm is seen as a square in the top view, when it rests on one of its edges on HP and perpendicular to VP. Draw the projections of the lamina. Find true inclination of its surface with HP. Draw the front view of the lamina when the edge about which it is tilted, is inclined at 45° to VP.***

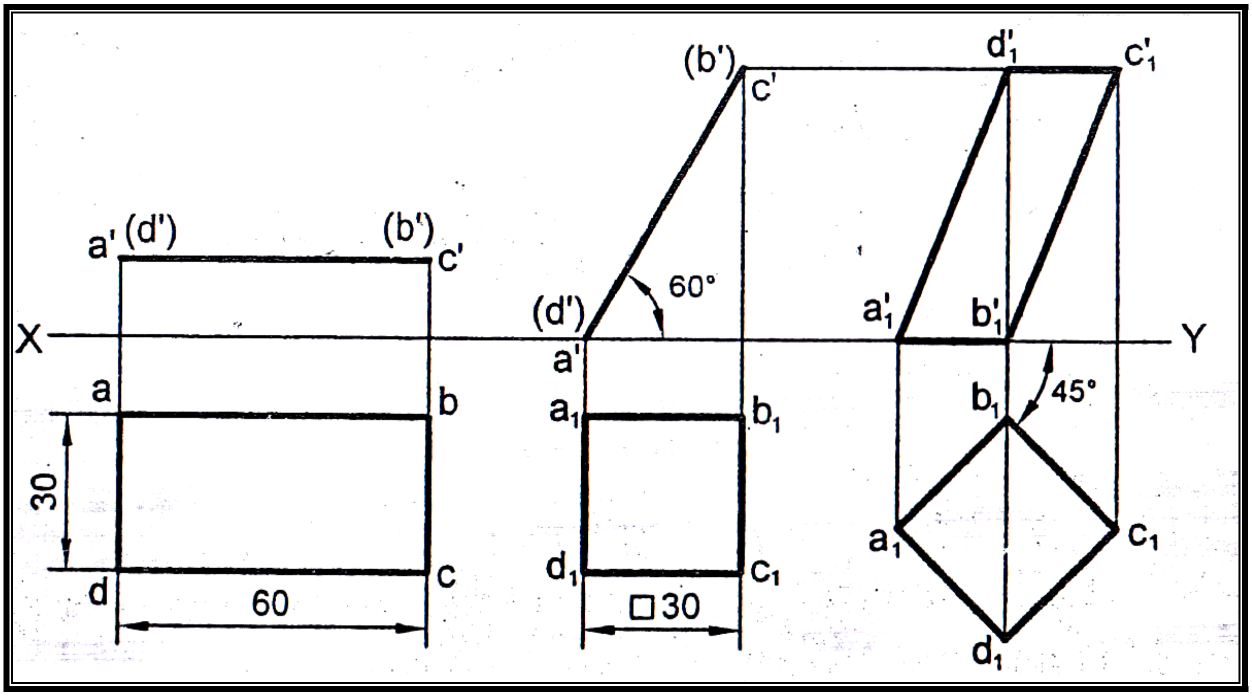


Figure – 14: Projection of planes

Solved Problems – 9:	A thin rectangular plate of sides 60 mm × 30 mm has its shorter side in VP and inclined at 30° to HP. Project its top view, if its front view is square of 30 mm long sides.

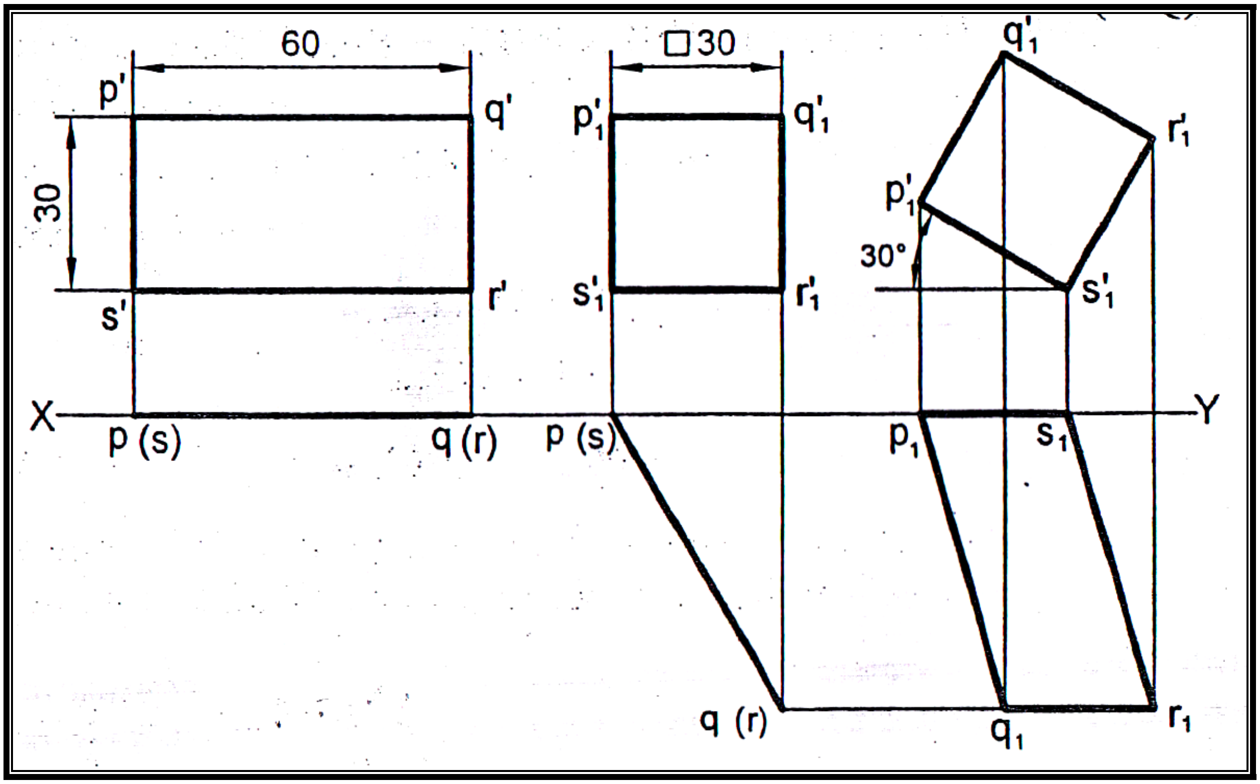


Figure – 15: Projection of planes.

1. Front view of the rectangular plate is a square. Hence, its surface must be inclined to **VP**. Assume the plate to be in **VP** such that its shorter edge is perpendicular to **HP**. Draw front view. Project the corresponding top view.
2. In the top view, the line **ab** should be so inclined to **XY** that the front view becomes a square.

Solved Problems – 10: ***Draw the projections of a pentagonal sheet of 26 mm side having its surface inclined at 30° to VP. Its one side is parallel to VP and inclined at 45° to HP.***

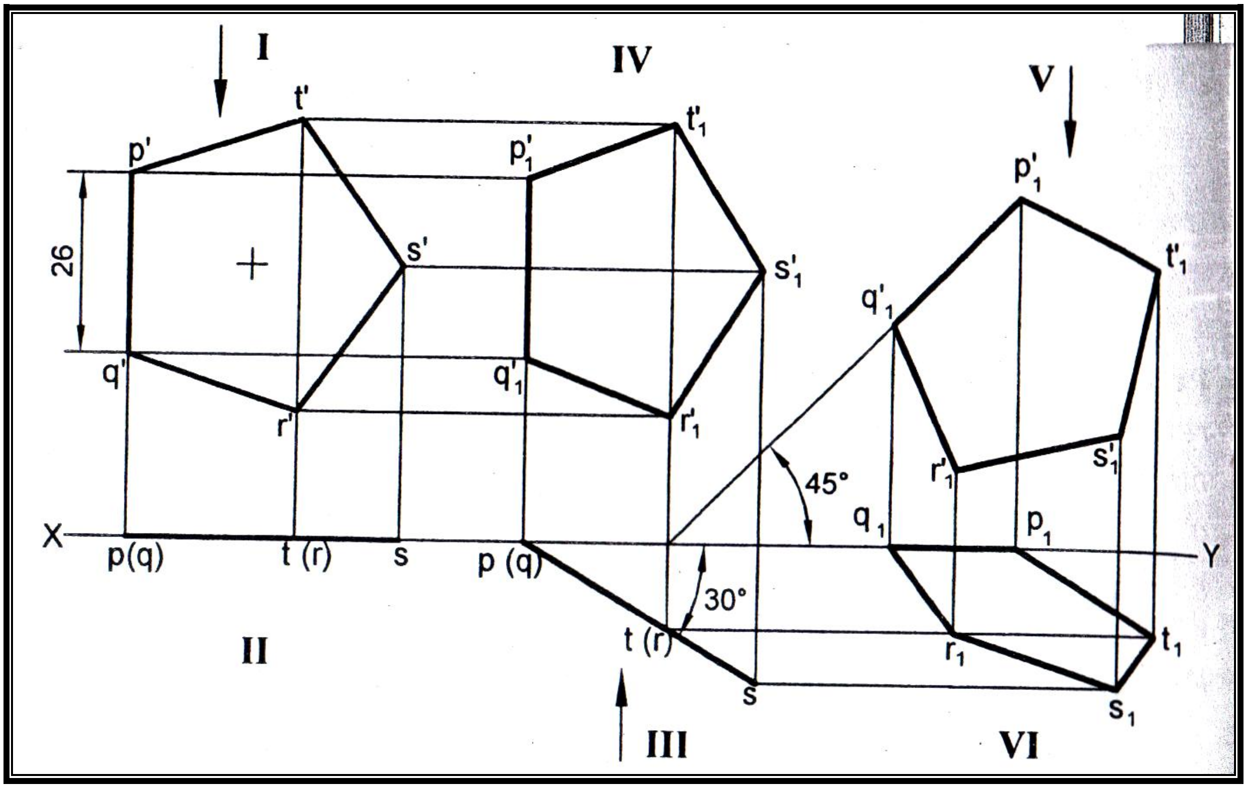


Figure – 16: Projection of planes.

1. Draw the front view as pentagon such that one side (edge) **p'q'** is perpendicular to **XY**.
2. Project first top view from front view.
3. Redraw first top view such that it is inclined at 30° to **XY** and get the second top view.
4. Project second front view from second top view and first front view. Reproduce second front view such that **p_1'q_1'** is inclined at 45° to **XY**. Obtain final front view.
5. Project final top view from final front view and second top view. Note that **p_1q_1** is parallel to **XY**.

Solved Problems – 11: ***A regular pentagonal lamina of 30 mm sides has one edge in HP and inclined at an angle of 30° to VP. Draw its projections whwn its surface is inclined at 45° to HP.***

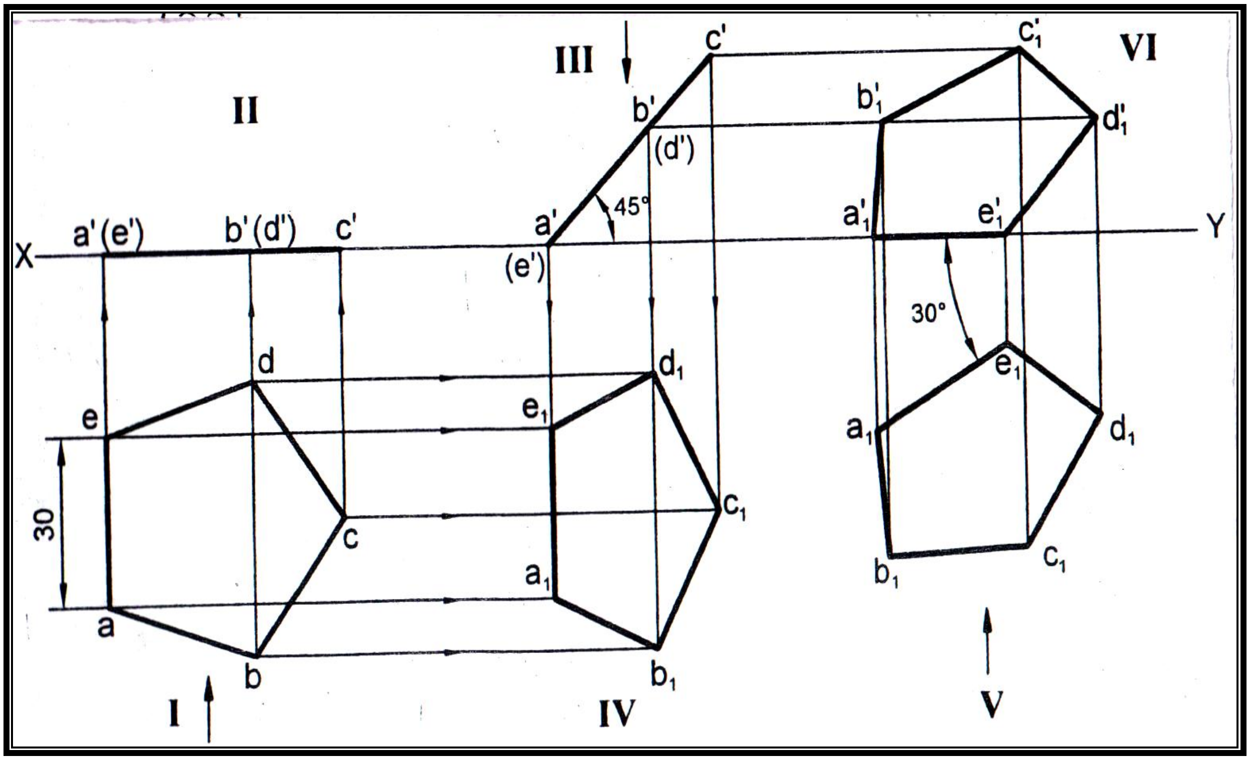


Figure – 17: Projection of planes.

Solved Problems – 12: ***A hexagonal lamina of 20 mm side rest on one of its corners on the HP. The diagonal passing through this corner is inclined at 45° to the HP. The lamina is then rotated through 90° such that the top view of this diagonal is perpendicular to the VP and the surface is still inclined at 45° to the HP. Draw the projections of the lamina.***

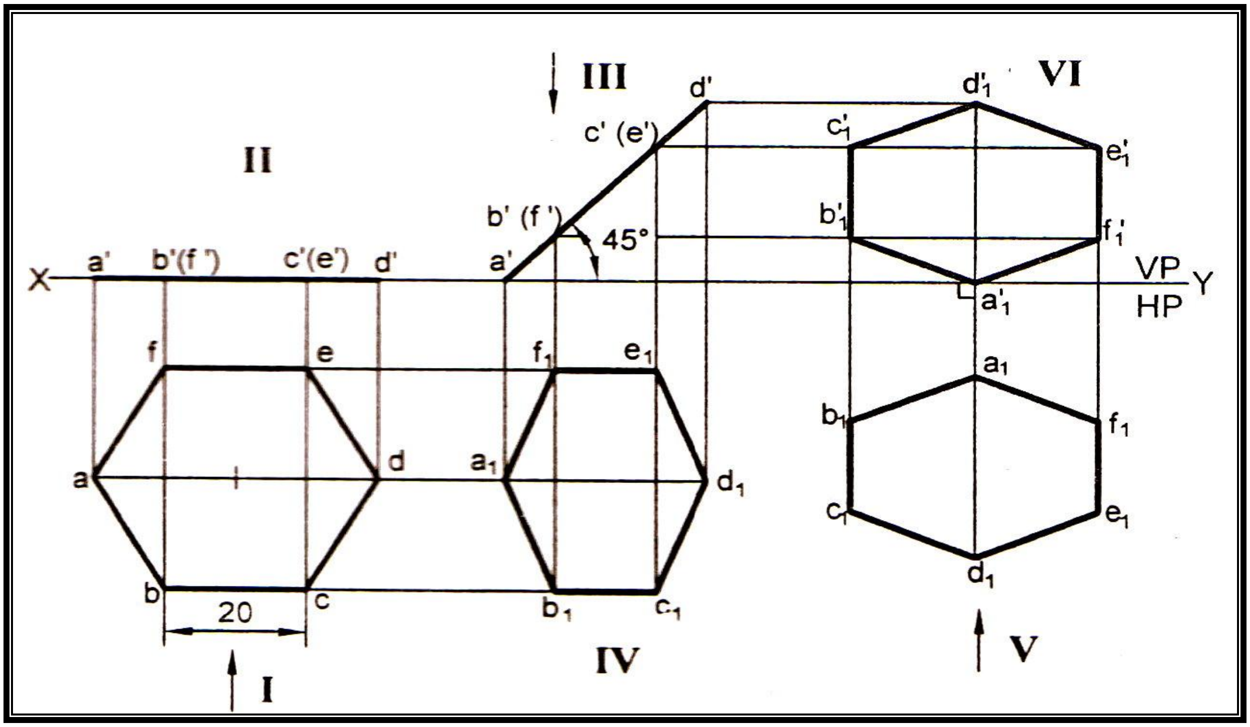


Figure – 18: Projection of planes.

Simple Position:

Assume the lamina to lie on **HP** with one of its diagonals **AD** parallel to **VP**. For this position, draw the first top view **abcdef** below **XY**. Project its corresponding first front view **a'd'** on **XY**. Look at the first top view in the direction of arrow shown. Mark hidden corners in first front view.

Second Position:

Diagonal **AD** of the lamina is inclined at 45° to **HP**. Hence turn first front view **a'd'** about the corner **a'** through 45° to **XY** to obtain the second front view **a'd'**. Project the corresponding second top view $\mathbf{a_1b_1c_1d_1e_1f_1}$.

Final Position:

Then rotate the lamina through 90° such that the top view $\mathbf{a_1d_1}$ of the diagonal **AD** is perpendicular to **VP**. Hence redraw the second top view, turning it through 90° such that $\mathbf{a_1d_1}$ is perpendicular to **XY**. $\mathbf{a_1b_1c_1d_1e_1f_1}$ is the required final top view.

As per the given condition, diagonal **AD** of the lamina still makes 45° to the **HP**. So, from second front view draw horizontal lines and from the final top view draw the vertical projectors. Complete the final front view **$a_1'b_1'c_1'd_1'e_1'f_1'$**.

Solved Problems – 13: ***A hexagonal plate of 25 mm side is resting on HP such that one of its corners touches both HP and VP. It makes 30° with HP and 60° with VP. Draw the projections by change of position method.***

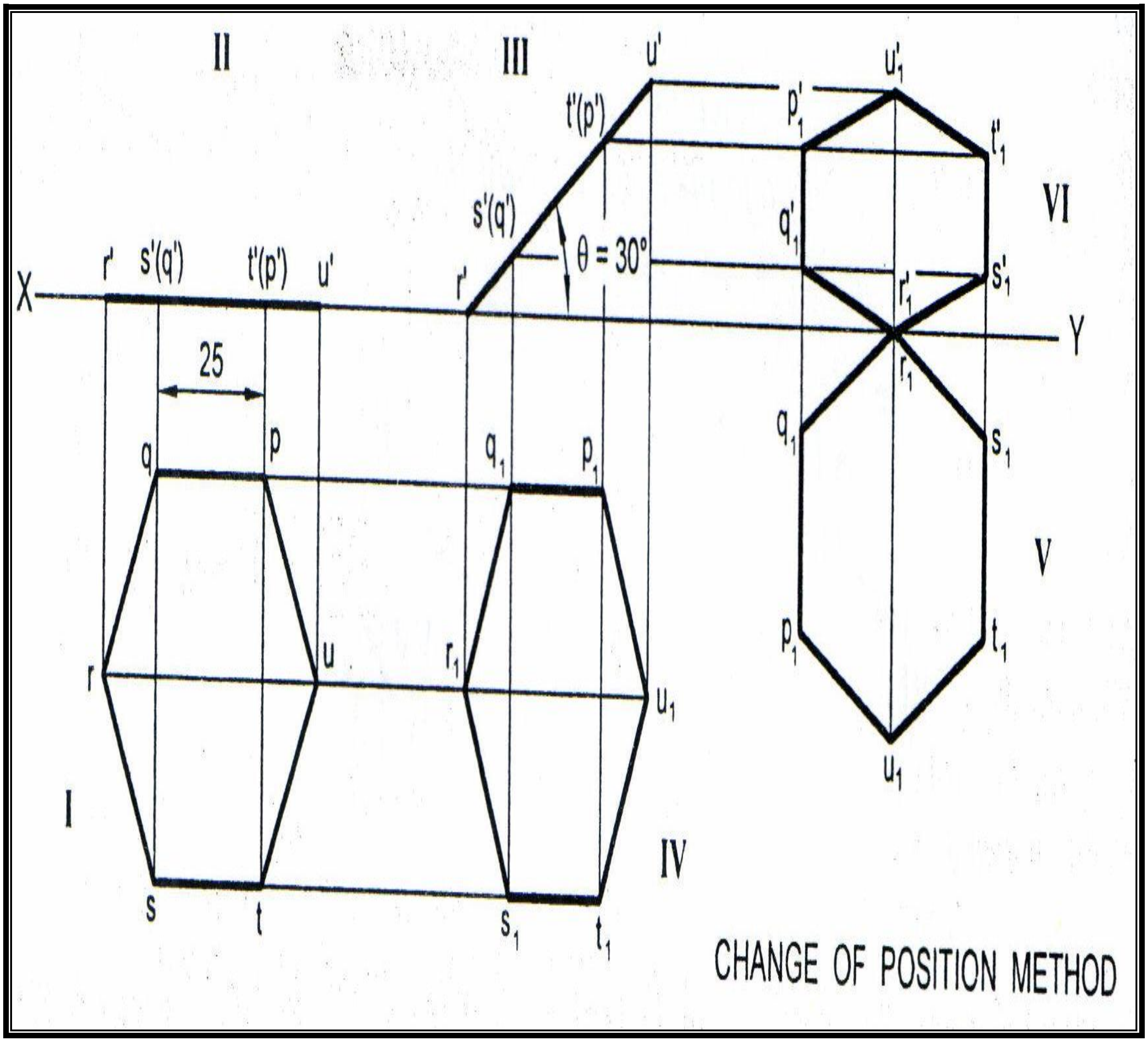


Figure – 19: Projection of planes.

Solved Problems – 14: ***A regular hexagonal lamina of 26 mm side has a central hole of 30 mm diameter. Draw the front and top views when the surface of the lamina is inclined at 45° to HP. A side of lamina is inclined at 35° to VP.***

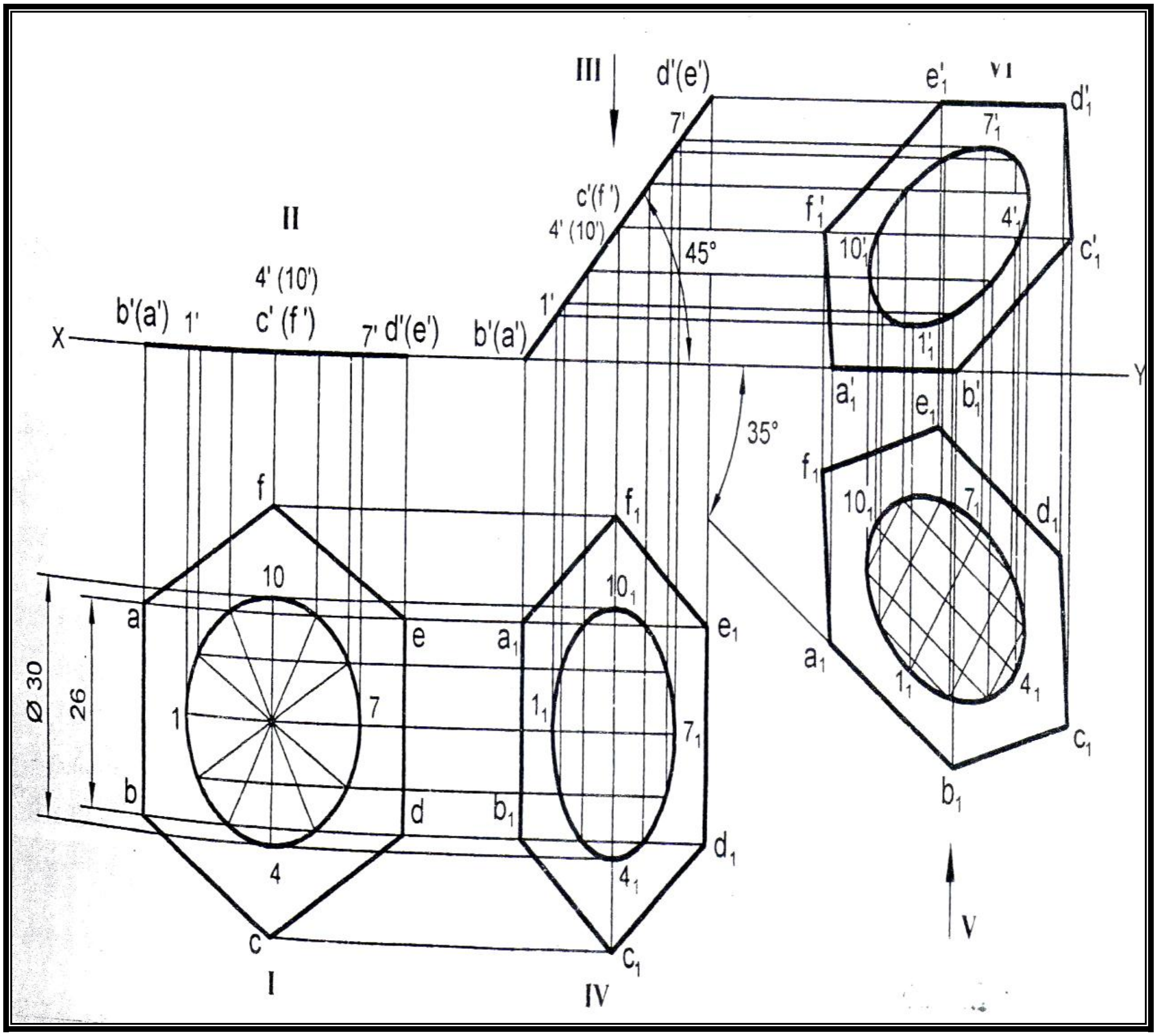


Figure – 20: Projection of plane

Solved Problems – 15: ***A semi-circular lamina of 64 mm diameter has its straight edge in VP and inclined at an angle of 45° to HP. The surface of the lamina makes an angle of 30° with VP. Draw the projections.***

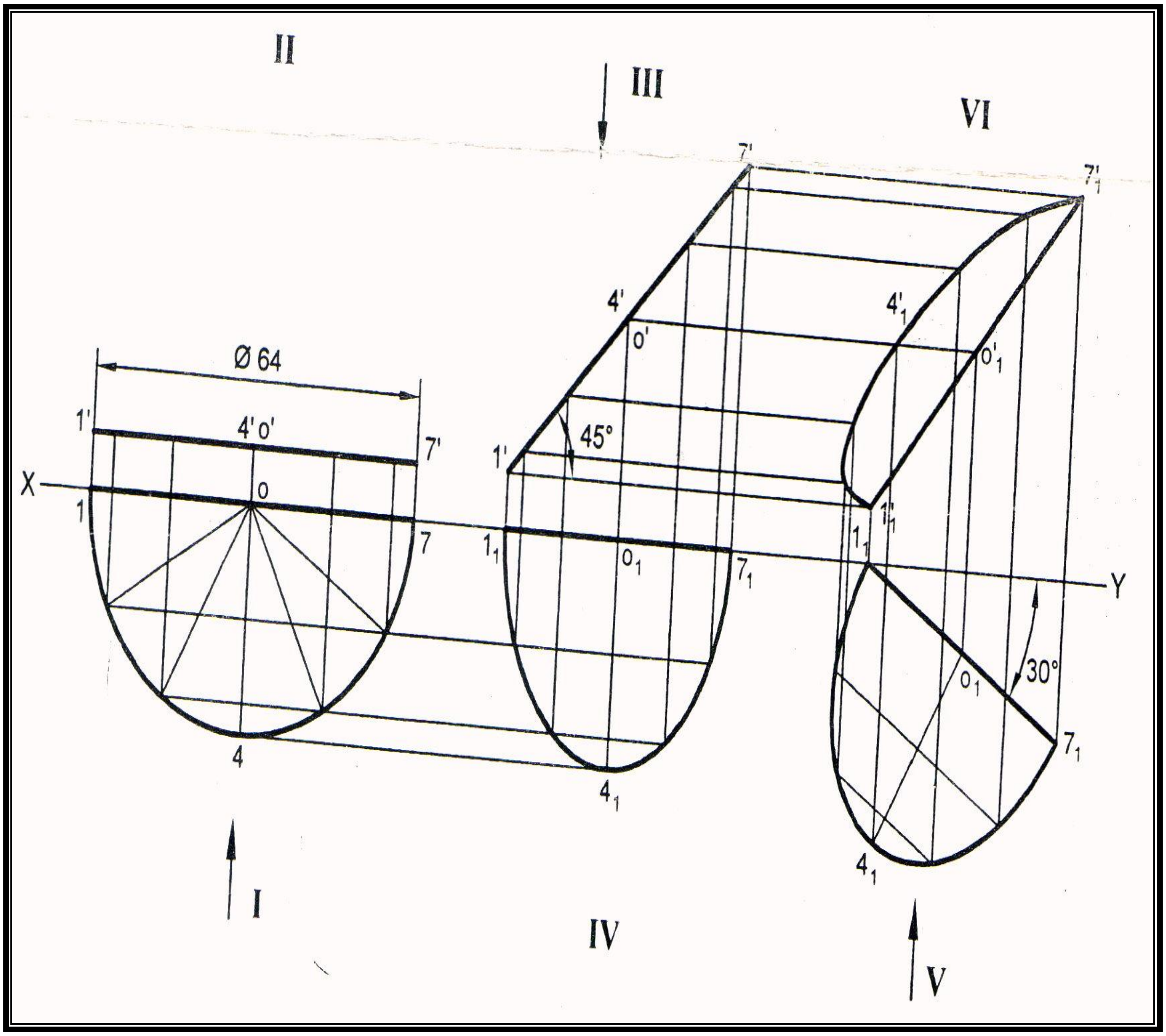


Figure – 21: Projection of planes.