DESIGN OF SHAFT

SHAFT:-

It is a rotating number, usually circular in cross section used to transmit power.

TRANSMISSION SHAFT:-

It transmit power from one place to another place.

Ex: Shaft coupled with turbine and generator.

MACHINE SHAFT:-

Integral part of the machine
Example: Crank shaft.

SPINDLE:-

Short shaft is called spindle
example: Drilling m/c spindle
Lathe spindle.

AXLE:-

It is appearence similar to shaft. It's Load carrying member. It may be rotating (or) non-rotating.

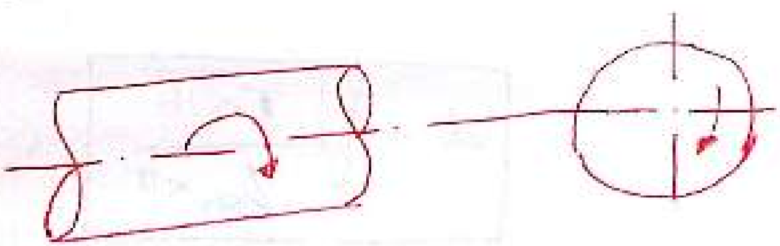
SHAFT MATERIAL:-

→ mild steel
→ cast iron
→ steel - chromium alloy
→ steel - vanadium alloy
→ copper alloy.

Stress induced in shaft material:

→ Shear stress due to transmission of torque.
→ Bending stress due to weight of pulley (or) gear in the shaft.
→ combined shear stress and bending stress
→ combined shear stress, Bending stress and axial stress.

SHAFT SUBJECTED TO TWISTING MOMENT



$$\frac{T}{J} = \frac{\tau_{sh}}{R} = \frac{C\theta}{l}$$

T = Torque (or) Twisting movement - N.mm

J = polar moment of inertia - mm4

τ_{sh} - shear stress - N/mm²

R - Radius of the shaft - mm

C - modulus of Rigidity - N/mm²

θ : angle of twist - radians.

l : length of shaft - mm.

DESIGN OF SHAFT BASED ON STRENGTH

$$\frac{T}{J} = \frac{\tau_{sh}}{R} \quad -(1)$$

$$J = \frac{\pi d^4}{32} \quad -(2)$$

$$R = d/2 \quad -(3)$$

sub eqn (2) & (3) in (1)

$$\frac{T \times 32}{\pi \times d^4} = \frac{\tau_{sh} \times 2}{d}$$

$$T \times \cancel{32}^{16} \times d = \tau_{sh} \times \cancel{2} \times \pi \times \cancel{d^4}^{3}$$

$$T \times 16 = \tau_{sh} \times \pi \times d^3$$

$$\boxed{d = \sqrt[3]{\frac{T \times 16}{\tau_{sh} \times \pi}}}$$

d - diameter of shaft (mm)

DESIGN BASED ON RIGIDITY

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$J = \frac{\pi}{32} d^4$$

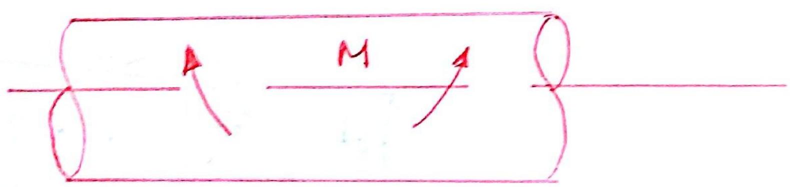
$$\frac{T \times 32}{\pi \times d^4} = \frac{C\theta}{l}$$

$$T \times 32 \times l = C \times \theta \times \pi \times d^4$$

$$d^4 = \frac{T \times 32 \times l}{C \times \theta \times \pi}$$

$$d = \left[\frac{T \times 32 \times l}{C \times \theta \times \pi} \right]^{1/4} \text{mm}$$

SHAFT SUBJECTED TO BENDING MOVEMENT



M - Bending moment - N-mm

I - Moment of Inertia - mm^4

σ_b - Bending stress - N/mm^2

Y - Distance from Neutral axis - mm

E - Young modulus - N/mm^2

R - Radius of Curvature - mm.

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad -(1)$$

$$I = \frac{\pi d^4}{64} \quad -(2)$$

$$y = d/2 \quad -(3)$$

Sub. eqn (2) & (3) in (1)

$$\frac{M \times 64}{\pi \times d^4} = \frac{\sigma_b \times 2}{d}$$

$$M \times 64 \times \cancel{d} = \sigma_b \times 2 \times \pi \times \cancel{d^4}$$

$$M \times 32 = \sigma_b \times \pi \times d^3$$

$$d^3 = \frac{M \times 32}{\sigma_b \times \pi}$$

$$\boxed{d = \sqrt[3]{\frac{M \times 32}{\sigma_b \times \pi}} \quad \text{— N-mm}}$$

~~SUBJECT~~ SHAFT SUBJECTED TO COMBINED BENDING MOVEMENT AND TWISTING MOVEMENT.

$$T_e = \sqrt{M^2 + T^2}$$

T_e = Equivalent Twisting moment – N-mm

$$M_e = \frac{1}{2}\left[\left(M + \sqrt{M^2 + T^2} \right)\right] \quad \text{– N-mm}$$

$$Me = \frac{1}{2}[M + Te]$$

Me = Equivalent Bending moment - N-mm

SHAFT SUBJECTED TO FLUCTUVATING LOADS

$$Te = \sqrt{(k_b \times m)^2 + (k_t \times T)^2} \text{ N-mm}$$

$$Me = \frac{1}{2}\left[k_b \times M + \sqrt{(k_b \times M)^2 + (k_t \times T)^2}\right] \text{ - N-mm}$$

k_b - combined shock and Fatigue Factor for bending

k_t - combined shock and Fatigue Factor for Twisting.

POWER TRANSMITTED SHAFT (P)

$$P = \frac{2\pi N T}{60} \text{ W (or) } \frac{N\text{-}m}{s} \text{ (or) J/s}$$

N - speed - rpm

T - Torque (or) Twisting moment (or) Torsion (or) Turning moment.

1) A Line shaft rotating at 300rpm is to transmit 20kW. Determine the diameter of the shaft, if the permissible shear stress of the shaft is 42MPa. Neglect bending moment on the shaft.

Given:-

$P = 20kW = 20 \times 10^3 W$ | $N = 300rpm$

$\tau_{sh} = 42 MPa = 42 N/mm^2$

To find:-

$d = ?$

solution:- $P = \frac{2\pi NT}{60} \Rightarrow T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 300}$

$T = 636.62 Nm$

$T = 636.62 \times 10^3 \; N\text{-}mm$

$T = \frac{\pi}{16} \times d^3 \times \tau_{sh}$

$T \times 16 = \pi \times d^3 \times \tau_{sh}$

$d = \sqrt[3]{\frac{T \times 16}{\pi \times \tau_{sh}}} = \sqrt[3]{\frac{636.62 \times 10^3 \times 16}{\pi \times 42}}$

$d = 42.58mm$

From PSGDB: 7.20

$\boxed{d \approx 45mm}$

(2) A solid circular shaft is subjected to a bending moment of 3000N-m and a torque of 10000 N-m. The shaft is made of 45C8 steel having ultimate tensile stress of 700MPa and a ultimate shear stress of 500MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

given:-

$M = 3000\ N\text{-}m = 3000 \times 10^3\ N\text{-}mm = 3 \times 10^6\ N\text{-}mm$

$T = 10000\ N\text{-}m = 10{,}000 \times 10^3\ N\text{-}mm = 10 \times 10^6\ N\text{-}mm$

$\sigma_{t(u)} = 700\ mpa = 700\ N/mm^2$

$\tau_u = 500 mpa = 500\ N/mm^2$

$F.S = 6$

To find:-

The diameter of the shaft

Solution:-

Allowable tensile stress

$$\sigma_t \text{ (or) } \sigma_b = \frac{\sigma_{tu}}{F.S} = \frac{700}{6} = 116.7 N/mm^2$$

Allowable shear stress

$$\tau = \frac{\tau_u}{F.S} = \frac{500}{6} = 83.13 N/mm^2$$

Equivalent twisting moment

$$T_e = \sqrt{M^2+T^2} = \sqrt{(3\times10^6)^2+(10\times10^6)^2}$$

$$\boxed{T_e = 10.44\times10^6 \text{ N-mm}}$$

W.K.T equivalent twisting moment (T_e)

$$T_e = \pi/16 \times \tau \times d^3$$

$$10.44\times10^6 = \pi/16 \times 83.3 \times d^3$$

$$= 16.36\, d^3$$

$$d^3 = \frac{10.44\times10^6}{16.36}$$

$$\boxed{d = 86\text{mm}}$$

equivalent bending moment

$$M_e = \frac{1}{2}\left(M+\sqrt{M^2+T^2}\right)$$

$$= \frac{1}{2}(M+T_e)$$

$$= \frac{1}{2}(3\times10^6 + 10.44\times10^6)$$

$$\boxed{M_e = 6.72\times10^6 \text{ N-mm}}$$

$$\frac{M}{I} = \frac{\sigma_b}{Y}$$

$$I = \frac{\pi}{64} d^4$$

$$Y = d/2$$

$$Me = \frac{\sigma_b \times I}{Y}$$

$$= \frac{\sigma_b \times \pi \times \cancel{d^4} \times \cancel{2}}{\cancel{64}_{32} \times \cancel{d}}$$

$$Me = \frac{\sigma_b \times \pi \times d^3}{32}$$

$$d^3 = \frac{Me \times 32}{\sigma_b \times \pi} = \sqrt[3]{\frac{Me \times 32}{\sigma_b \times \pi}}$$

$$d = \sqrt[3]{\frac{Me \times 32}{\sigma_b \times \pi}}$$

$$d = \sqrt[3]{\frac{6.72 \times 10^6 \times 32}{116.7 \times \pi}}$$

$$\boxed{d = 83.7\,mm}$$

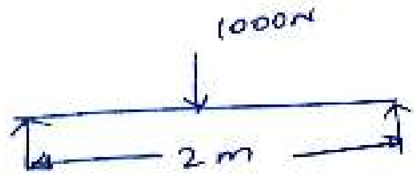
compare to both

$d = 86mm$

From PSGDB : T.20

$d = 90mm$

(5) A mild steel shaft transmits 20kW at 200rpm. It carries a central load of 1000N and is simply supported b/w the bearing 2.0m apart. Determine the size of the shaft, if the allowable shear stress is 42Mpa and the maximum tensile (or) compressive stress is not to exceed 56Mpa. what size of the shaft is required, if it is subjected to gradually applied loads?



given:-

$P = 20kW = 20\times10^3 W$

$N = 200rpm$

$W = 1000N$

$L = 2m = 2\times10^3 = 2000mm$

$\tau_{sh} = 42 Mpa = 42 N/mm^2$

$\sigma_b = 56 mpa = 56 N/mm^2$

To find:-

When gradual load is applied, $d = ?$

solution:-

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2 \times \pi \times 200}$$

$$T = 954.92\ N\text{-}m$$

$$\boxed{T = 954.92 \times 10^3\ N\text{-}mm}$$

SSB with point load at mid span

$$M = \frac{W \times l}{4} = \frac{1000 \times 2000}{4}$$

$$\boxed{M = 500 \times 10^3\ N.mm}$$

$T = 954.92 \times 10^3\ N\text{-}mm$ $M = 500 \times 10^3\ N\text{-}mm$

equivalent Twisting moment -

$$Te = \sqrt{M^2 + T^2}$$

$$= \sqrt{(500 \times 10^3)^2 + (954.92 \times 10^3)^2}$$

$$Te = 1.0779 \times 10^6\ N\text{-}mm$$

$$Te = \pi/16 \times d^3 \times \tau_{sh}$$

$$d = \sqrt[3]{\frac{T_e \times 16}{\pi \times \tau_{sh}}}$$

$$d = \sqrt[3]{\frac{1.0779 \times 10^6 \times 16}{\pi \times 42}}$$

$$d = 50.75 \text{ mm}$$

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

$$M_e = \frac{1}{2}\left[M + T_e\right]$$

$$= \frac{1}{2}\left[500 \times 10^3 + 1.0779 \times 10^6\right]$$

$$M_e = 788950 \text{ N-mm}$$

$$\frac{M_e}{I} = \frac{\sigma_b}{y}$$

$$d = \sqrt[3]{\frac{M_e \times 32}{\pi \times \sigma_b}}$$

$$= \sqrt[3]{\frac{788950 \times 32}{\pi \times 56}}$$

$d = 52.35\ mm$

Gradually applied Load

From PSGDB : T.21

$k_b = 1.5$, $k_t = 1$

$$T_e = \sqrt{(k_b \times m)^2 + (k_t \times T)^2}$$

$$= \sqrt{(1.5 \times 500 \times 10^3)^2 + (1 \times 954.92 \times 10^3)^2}$$

$$T_e = 1214237.29\ N\text{-}mm$$

$$T_e = \frac{\pi}{16} d^3 \tau_{sh}$$

$$d = \sqrt[3]{\frac{T_e \times 16}{\pi \times \tau_{sh}}}$$

$$d = \sqrt[3]{\frac{1214237.29 \times 16}{\pi \times 42}}$$

$$\boxed{d = 52.80\ mm}$$

$$M_e = \frac{1}{2}\left[k_b \times M + \sqrt{(k_b \times M)^2 + (k_t \times T)^2}\right]$$

$$M_e = \frac{1}{2}\left[(1.5 \times 500 \times 10^3) + \sqrt{(1.5 \times 500 \times 10^3)^2 + (1 \times 954.92 \times 10^3)^2}\right]$$

$$\boxed{M_e = 982118.65 \text{ N-mm}}$$

$$d = \sqrt[3]{\frac{(M_e \times 32)}{\pi \times \sigma_b}}$$

$$d = \sqrt[3]{\frac{982118.65 \times 32}{\pi \times 56}}$$

$d = 56.31$ mm

Compare

$d = 56.31$ mm selected

From PSGDB: T.20

$$\boxed{d = 60 \text{mm}}$$

Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200N and is located at 300mm from the centre of the bearing. The diameter of the pulley is 200mm and the maximum power transmitted is 1kw at 120rpm. The angle of lap of the belt is 180° and coefficient of friction b/w the belt and the pulley is 0.3. The shock and fatigue factors for bending and twisting are 1.5 and 2.0 respectively. The allowable shear stress in the shaft may be taken as 35 Mpa.

Given:-

$W_p = 200N$

$L = 300mm$

$D = 200mm$, $R = 100mm$

$P = 1KW = 1\times10^3 W = 1000W$

$N = 120rpm$

$\theta = 180° = 180 \times \pi/180 = \pi$ radian $= 3.14$ radian.

$\mu = 0.3$

$K_b = 1.5$

$K_t = 2$

$\tau_{sh} = 35\ mpa = 35\ N/mm^2$

To find:-

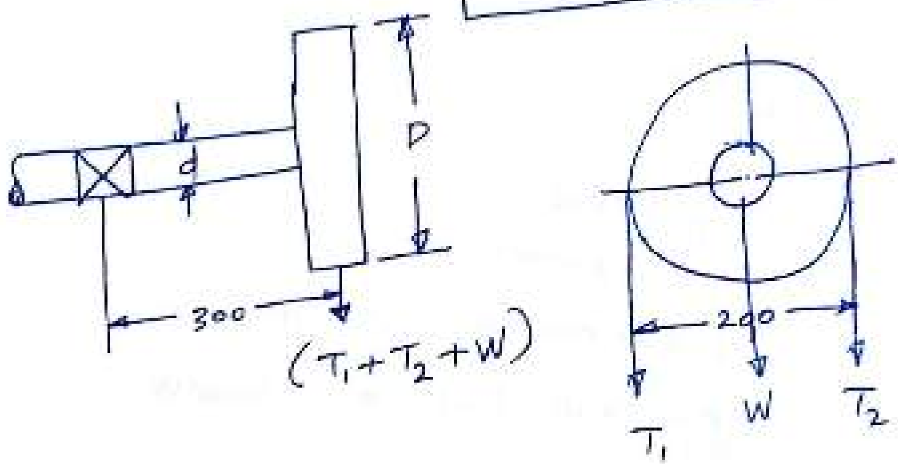
Design a shaft (i.e) diameter of the shaft)

Solution:-

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{P \times 60}{2\pi N} = \frac{1000 \times 60}{2 \times \pi \times 120} = 79.6 \ N\text{-}m$$

$$\boxed{T = 79.6 \times 10^3 \ N\text{-}mm}$$



T_1 & T_2 - tight & slack side

Torque transmitted (T)

$$T = (T_1 - T_2) R$$

$$76.6 \times 10^3 = (T_1 - T_2) \times 100$$

$$T_1 - T_2 = \frac{79.6 \times 10^3}{100} = 796 N$$

$$\boxed{T_1 - T_2 = 796 N} \quad \text{— (1)}$$

W.K.T $\frac{T_1}{T_2} = e^{\mu\theta}$

$$\frac{T_1}{T_2} = e^{(0.3\times\pi)}$$

$$\Rightarrow \frac{T_1}{T_2} = 2.57$$

$$T_1 = 2.57\,T_2 \quad -(2)$$

sub (2) in eqn (1)

$$2.57\,T_2 - T_2 = 796$$

$$(2.57-1)\,T_2 = 796$$

$$T_2 = \frac{796}{1.57} = 507$$

$$\boxed{T_2 = 507\,N} \quad -(3)$$

sub (3) in eqn (1)

$$T_1 - 507 = 796 \Rightarrow T_1 = 796 + 507$$

$$\boxed{T_1 = 1303\,N}$$

Total Force acting on the pulley (F)

$$= T_1 + T_2 + W_p$$

$$F = 1303 + 507 + 200$$

$$\boxed{F = 2010\,N}$$

Bending moment acting on the shaft

$$M = \overset{W}{F} \times L$$

$$= 2010 \times 300$$

$$\boxed{M = 603\times10^3\ N\text{-}mm}$$

$$T_e = \sqrt{(K_b \times M)^2 + (K_t \times T)^2}$$

From

$$T_e = \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2}$$

$$\boxed{T_e = 918 \times 10^3 \, N\text{-}mm}$$

W.K.T equivalent twisting moment (T_e)

$$T = \frac{\pi}{16} d^3 \times \tau_{sh}$$

$$T_e = \frac{\pi}{16} d^3 \times \tau_{sh}$$

$$d = \sqrt[3]{\frac{T_e \times 16}{\pi \times \tau_{sh}}}$$

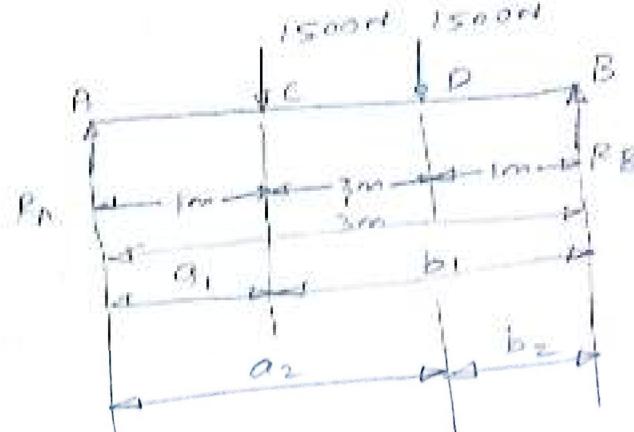
$$= \sqrt[3]{\frac{918 \times 10^3 \times 16}{\pi \times 35}}$$

$$d = 51.1 \, mm$$

From PSGDB : 7.20

$$\boxed{d = 53 mm}$$

A mild steel shaft transmit 100kW at 300rpm. The supported length of the shaft is 3m. It carries two pulleys each weighing 1500N mounted at a distance of 1m from the ends respectively. Determine the size of the shaft if the allowable shear stress for the shaft material is 60N/mm².



$a_1 = 1m, b_1 = 2m$ | $R_A = \frac{W_1 b_1}{l} + \frac{W_2 b_2}{l}$

$a_2 = 2m$ $b_2 = 1m$ | $R_B = \frac{W_1 a_1}{l} + \frac{W_2 a_2}{l}$

given:-

$P = 100kW = 100 \times 10^3 W$

$N = 300rpm$

$l = 3m$

$W_C = 1500N$

$W_D = 1500N$

$\tau_{sh} = 60 N/mm^2$.

To find:-

size of the shaft (i.e $d=?$)

Solution:-

$$R_A = \frac{w_1 b_1}{l} + \frac{w_2 b_2}{l}$$

$$= \frac{w_C b_1}{l} + \frac{w_D b_2}{l}$$

$$R_A = \frac{1500 \times 2}{3} + \frac{1500 \times 1}{3}$$

$$R_A = 1000 + 500 = 1500N$$

$$\boxed{R_A = 1500N}$$

$$R_B = \frac{w_1 a_1}{l} + \frac{w_2 a_2}{l}$$

$$= \frac{w_C a_1}{l} + \frac{w_D a_2}{l}$$

$$= \frac{1500 \times 1}{3} + \frac{1500 \times 2}{3}$$

$$R_B = 500 + 1000 = 1500N$$

$$\boxed{R_B = 1500N}$$

moment

$M_A = 0$

$M_B = 0$

$M_C = R_A \times 1 = 1500 \times 1 = 1500\ N\text{-}m$

$M_D = R_B \times 1 = 1500 \times 1 = 1500\ N\text{-}m.$

$M_{max} = M_C = M_D = M = 1500\ N\text{-}m$

$= 1500 \times 10^3\ N\text{-}mm.$

$$P = \frac{2\pi N T}{60} \Rightarrow T = \frac{P \times 60}{2\pi N}$$

$$= \frac{100 \times 10^3 \times 60}{2 \times \pi \times 300}$$

$T = 3183.09\ N\text{-}m$

$$\boxed{T = 3183.09 \times 10^3\ N\text{-}mm}$$

$$T_e = \sqrt{M^2 + T^2}$$

$$= \sqrt{(1500 \times 10^3)^2 + (3183.09 \times 10^3)^2}$$

$$\boxed{T_e = 3518815.42\ N\text{-}mm}$$

$$d = \sqrt[3]{\frac{Te \times 16}{\pi \times \tau_{sh}}}$$

$$d = \sqrt[3]{\frac{3518815.42 \times 16}{\pi \times 60}}$$

$d = 66.85\,mm \simeq 70\,mm$

From PSGDB: 7.20

$$\boxed{d = 71\,mm}$$

~~$Me = \frac{1}{2}\left[M + Te\right]$~~

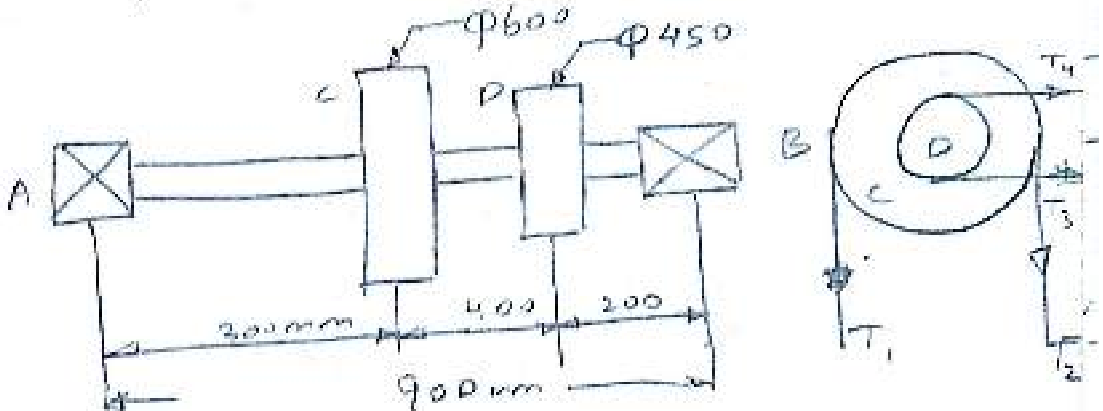
~~$= \frac{1}{2}\left[1500 \times 10^3 + 3518815.42\right]$~~

~~$Me = 2509407.71$~~

~~$d = \sqrt{\frac{Me \times 16}{}}$~~

A mild steel shaft 900mm b/w bearings support, a 600mm pulley 300mm to the right of the left hand bearing and a belt drives the pulley directly below. Another pulley of 450mm diameter is located 200mm to the left of the right hand bearing and the belt is driven from a pulley horizondal to the right.

The angle of contact for both the pulleys is 180° and the tension ratio is 2.2. The maximum tension in the belt on 600mm diameter pulley is 2.25 kN. Determine the suitable diameter for the solid shaft, if the permissible tensile stress is 63 N/mm² and the permissible shear stress is 42 N/mm².



given:-

$\theta = 180° = 180 \times \pi/180 = \pi = 3.14$ radian

$\frac{T_1}{T_2} = \frac{T_3}{T_4} = 2.2$

$T_1 = 2.25\,kN = 2.25 \times 10^3\,N$

$\sigma_b = \sigma_t = 63\,N/mm^2$

$\tau_{sh} = 42\,N/mm^2$.

To find:-

Diameter of the shaft (d)

Solution:-

$T_1 = 2.25 \times 10^3 N$

$\frac{T_1}{T_2} = 2.2$

$T_2 = \frac{T_1}{2.2} = \frac{2.25 \times 10^3}{2.2} = 1022.73 N$

$\boxed{T_2 = 1022.73 N}$

Torque on pulley C

$T_C = (T_1 - T_2) R_C$

$T_C = (2.25 \times 10^3 - 1022.73) \times 300$

$\boxed{T_C = 368184 \ N\text{-}mm}$

Torque on both pulley is same

$T_C = T_D$

$T_D = 368184 \ N\text{-}mm$

$T_D = (T_3 - T_4) R_D$

$T_3 - T_4 = \frac{T_D}{R_D} = \frac{368184}{225}$

$T_3 - T_4 = 1636.37$ — (1)

$\frac{T_3}{T_4} = 2.2$

$T_3 = 2.2 T_4$ — (2)

sub eqn (2) in (1)

$2.2T_4 - T_4 = 1636.37$

$(2.2-1) T_4 = 1636.37$

$T_4 = \frac{1636.36}{1.2} = 1363.63N$ — (3)

sub T_4 in eqn (2)

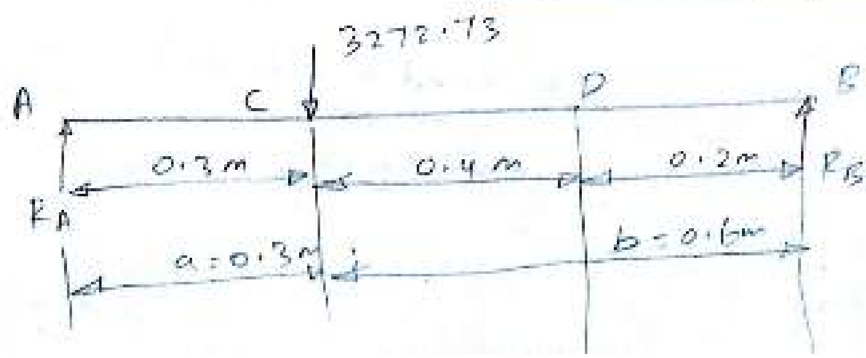
$T_3 = 2.2 \times 1363.63 = 3000N$

$\boxed{T_3 = 3000N}$

vertical force at c = $T_1 + T_2$

$F_v = 2.25 \times 10^3 + 1022.73$

$W_C = \boxed{F_C = 3272.73\ N}$



Take moment about A

$$R_{AV} = \frac{W_c \times b}{l} = \frac{3272.73 \times 0.6}{0.9}$$

$$\boxed{R_{AV} = 2181.82\ N}$$

$$R_{BV} = \frac{W_c \times a}{l} = \frac{3272.73 \times 0.3}{0.9}$$

$$\boxed{R_{BV} = 1090.91\ N}$$

<u>moment</u>

$M_{AV} = 0$

$M_{BV} = 0$

$M_{CV} = R_A \times 0.3$

$= 2181.82 \times 0.3$

$M_{CV} = 654.54\ N\text{-}m$

$M_{DV} = R_B \times 0.2 = 1090.91 \times 0.2$

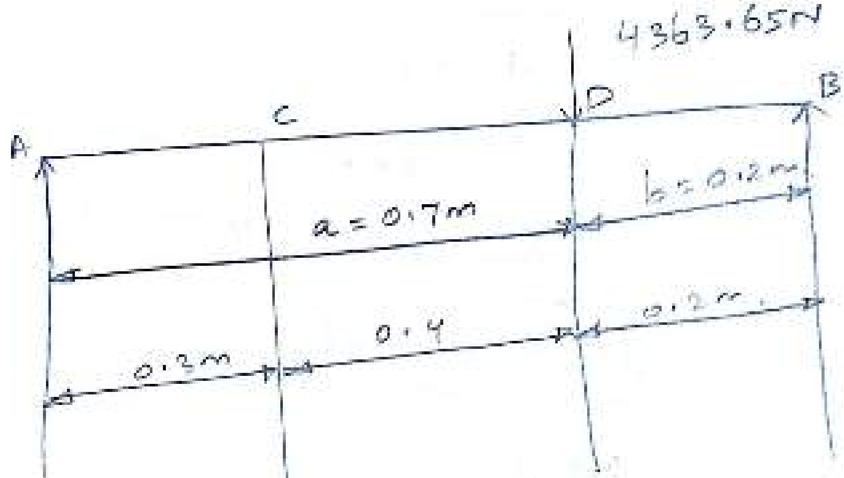
$$\boxed{M_{DV} = 218.182\ N\text{-}m}$$

<u>Horizondal Bending moment due to horizondal force.</u>

vertical force at D = $T_3 + T_4$

$F_D = 3000.01 + 1363.64$

$W_D = \boxed{F_D = 4363.65\,N}$



Take moment about 'A'

$a = 0.7\,m$

$b = 0.2\,m$

$l = 0.9\,m.$

$$R_{AH} = \frac{W \times b}{l}$$

$$= \frac{W_D \times b}{l} = \frac{4363.65 \times 0.2}{0.9}$$

$R_{AH} = 969.7\,N$

$$R_{BH} = \frac{W \times a}{l} = \frac{W_D \times a}{l} = \frac{4363.65 \times 0.7}{0.9}$$

$R_{BH} = 3393.95\,N$

moment

$M_{AH} = 0$

$M_{BH} = 0$

$M_{CH} = R_{AH} \times 0.3$

$= 969.7 \times 0.3 = 290.91\ N\text{-}m$

$M_{DH} = R_B \times 0.2$

$= 3393.95 \times 0.2$

$\boxed{M_{DH} = 678.79\ N\text{-}m}$

Resultant B.M at C

$M_C = \sqrt{M_{CV}^2 + M_{CH}^2}$

$= \sqrt{654.54^2 + 290.91^2}$

$\boxed{M_C = 716.27\ N\text{-}m}$

Resultant B.M at D

$M_D = \sqrt{M_{DV}^2 + M_{DH}^2}$

$= \sqrt{218.18^2 + 678.79^2}$

$\boxed{M_D = 712.99\ N\text{-}m}$

$M_{max} = M_C = M$

$M = 716.27\ N-m$

$\boxed{M = 716.27 \times 10^3\ N-mm}$

$T_e = \sqrt{M^2 + T^2}$

$T_e = \sqrt{(716.27 \times 10^3)^2 + (368184)^2}$

$\boxed{T_e = 805358.41\ N-mm}$

$d = \sqrt[3]{\dfrac{T_e \times 16}{\pi \times \tau_{sh}}}$

$d = \sqrt[3]{\dfrac{805358.41 \times 16}{\pi \times 42}}$

$d = 46.05\ mm.$

From PSGDB : 7.20

$\boxed{d = 50\ mm}$

$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$

$M_e = \frac{1}{2}\left[M + T_e\right]$

$M_e = \frac{1}{2}\left[716.27 \times 10^3 + 805358.41\right]$

$M_e = 760814.20$ N-mm.

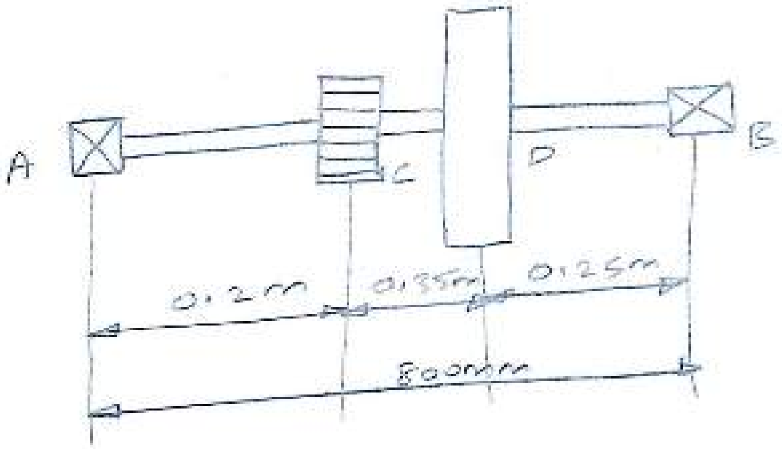
$$d = \sqrt[3]{\frac{M_e \times 32}{\pi \times \sigma_b}}$$

$$d = \sqrt[3]{\frac{760814.2 \times 32}{\pi \times 63}}$$

$d = 49.73$ mm ≈ 50 mm

From PSGDB $d = 50$ mm.

A Shaft is supported on bearings A and B 800mm between centers. A 20° straight tooth spur gear with 600mm pitch diameter is located 200mm to the right of left hand bearing A and a 700mm diameter pulley is mounted 250mm towards the left of bearing B. The gear is driven by a pinion with a downward tangential force while the pulley drives the horizontal belt having 180° angle of wrap. The pulley also serves as a flywheel and weight 2000N. The maximum belt tension is 3:1. Determine the maximum bending moment and the necessary shaft diameter if the allowable shear stress of the material is 40N/mm². The maximum belt tension is 3000N.



given:-

$\theta = 180° = 180 \times \pi / 180 = \pi = 3.14$ radian

$W = 2000N$ $\phi = 20°$

$\frac{T_1}{T_2} = 3 \Rightarrow T_1 = 3T_2$

$T_1 = 3000N$ D_p (or) $D_p = 700mm$

$R_p = 350mm$

$\tau_{sh} = 40N/mm^2$ pulley vertical load $= 2000N$

To find:-

diameter of the shaft (d)

solution:-

$T_1 = 3000N$

$T_1 = 3T_2$

$T_2 = T_1/3 = \frac{3000}{3}$

$T_2 = 1000N$

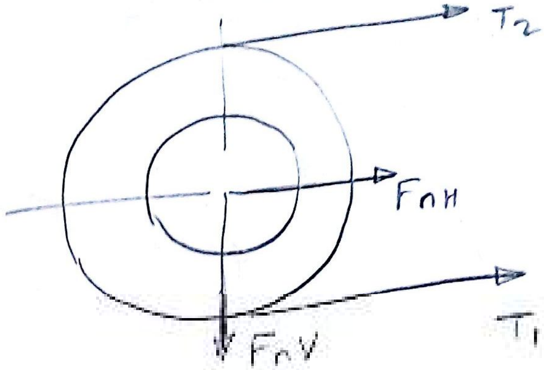
Torque on pulley (T)

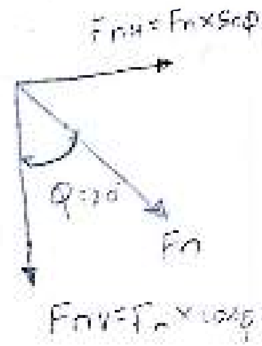
$T = (T_1 - T_2) R_{p}$

$T = (3000 - 1000)\, 350$

$T = 700000$ N-mm

$\boxed{T = 7 \times 10^5 \text{ N-mm}}$ ✓





gear

$F_t = \dfrac{2T}{D_{G}}$ (or) $\dfrac{T}{R_{G}}$

$= \dfrac{2 \times 7 \times 10^5}{600}$

$\boxed{F_t = 2333.33 \text{ N}}$ ✓

$F_n = \dfrac{F_t}{\cos\phi} = \dfrac{2333.33}{\cos 20}$

$\boxed{F_n = 2483.08 \text{ N}}$

Compare to both value

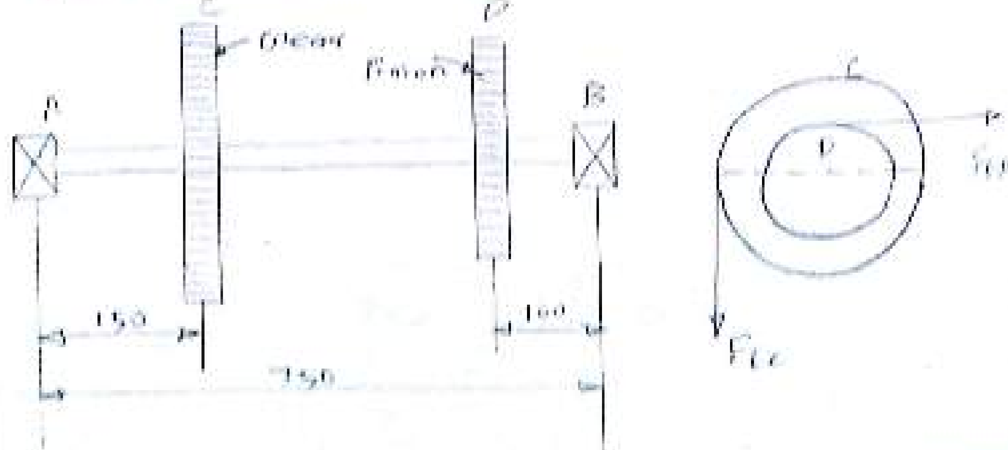
$d = 51.7\,mm \approx 56\,mm$

$\boxed{d = 56\,mm}$

From PSGDB 7.20, $d = 53\,mm$

P.S.G
Ex: 1.11
Pg 7.28

A steel solid shaft transmitting 15 kW at 200 rpm is supported on two bearing 750mm apart and has two gears keyed to it. The pinion having 30 teeth of 6mm module is located 100mm to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 6mm module is located 150mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear, determine the diameter of the shaft.



given:-

$P = 15KW = 15\times10^3 W$

$N = 200rpm$

$AB = 750mm$

$T_D = 30$

$m_D = 5mm$

$BD = 100mm$

$T_C = 100,\ m_C = 5mm$

$AC = 150mm$

$\tau = 54Mpa = 54 N/mm^2$.

To find:- Diameter of the shaft.

solution:-

W.K.T Torque transmitted by the shaft

$$T = \frac{P\times60}{2\pi N} = \frac{15\times10^3\times60}{2\times\pi\times200}$$

$$T = 716\ N\text{-}m$$

$$\boxed{T = 716\times10^3\ N\text{-}mm}$$

W.K.T, diameter of gear

= No. of teeth on gear × module.

$$D_C = T_C\times m$$

$$= 100\times5 = 500mm$$

$$r_C = \frac{500}{2} = 250mm \Rightarrow R_C = 250mm.$$

$D_D = T_D \times m = 30 \times 5 = 150\,mm$

$$R_D = \frac{150}{2} = 75\,mm$$

$$\boxed{R_D = 75\,mm}$$

assuming that torque at c and d is same, therefore tangential force on the gear c, acting downwards,

$$F_{tc} = \frac{T}{R_c} = \frac{716 \times 10^3}{250} = 2864\,N$$

$$\boxed{W_c = 2864\,N}$$

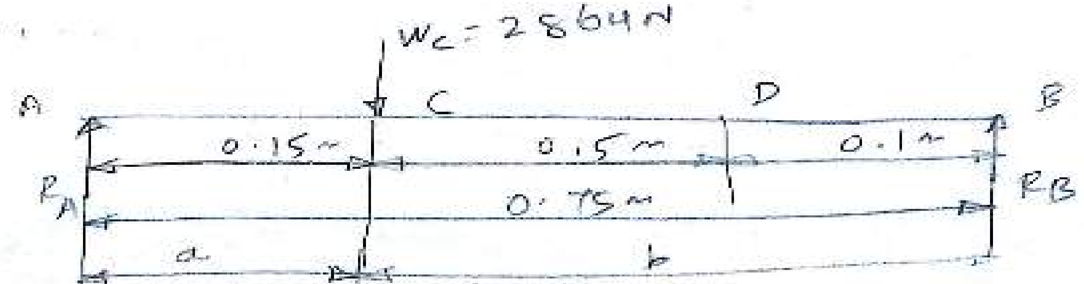
tangential force on the pinion D,

$$F_{tD} = \frac{T}{R_D} = \frac{716 \times 10^3}{75} = 9546.67$$

$$F_{tD} = 9547\,N$$

$$\boxed{W_D = 9547\,N}$$

vertical bending moment due to vertical force:



$W_C = 2864\,N$

$a = 0.15\,m$

$b = 0.6\,m$

$R_A = \frac{W_C \times b}{l}, \quad R_B = \frac{W_C \times a}{l}$

Take moment about 'A'

$R_{AV} = \frac{2864 \times 0.6}{0.75} = 2291.2\,N, \quad \boxed{R_A = 2291.2\,N}$

$R_{BV} = \frac{2864 \times 0.15}{0.75} = 572.8\,N, \quad \boxed{R_B = 572.8\,N}$

Bending moment at A & B

$M_{AV} = 0$

$M_{BV} = 0$

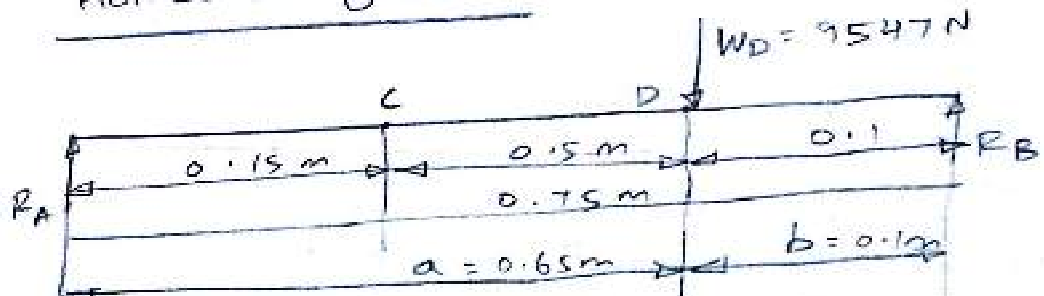
$M_{CV} = R_{AV} \times 0.15 = 2291.2 \times 0.15$

$\boxed{M_{CV} = 343.68\,N\text{-}m}$

$M_{DV} = R_{BV} \times 0.1 = 572.8 \times 0.1$

$\boxed{M_{DV} = 57.28\,N\text{-}m}$

Horizontal bending moment due to horizontal force:



Take moment about 'A')

$W_D = 9547 N$
$a = 0.65 m$
$b = 0.1 m$

$R_A = \frac{W_D \times b}{l}$

$R_B = \frac{W_D \times a}{l}$

$R_{AH} = \frac{9547 \times 0.1}{0.75} = 1272.93 \Rightarrow \boxed{R_{AH} = 1273 N}$

$R_{BH} = \frac{9547 \times 0.65}{0.75} = 8274 N \quad \boxed{R_{BH} = 8274 N}$

Bending moment at A & B

$M_{AH} = 0$

$M_{BH} = 0$

$M_{CH} = R_{AH} \times 0.15 = 1273 \times 0.15$

$M_{CH} = 190.95 N\text{-}m$

$M_{DH} = R_{BH} \times 0.1 = 8274 \times 0.1 = 827.4 N\text{-}m$

$\boxed{M_{CH} = 190.95 \ N\text{-}m}$

$\boxed{M_{DH} = 827.4 \ N\text{-}m}$

Resultant B.M at C

$M_C = \sqrt{(M_{CH})^2 + (M_{CV})^2}$

$= \sqrt{(190.95)^2 + (343.68)^2}$

$\boxed{M_C = 393.16 \ N\text{-}m}$

Resultant B.M at D

$$M_D = \sqrt{(M_{DH})^2 + (M_{DV})^2}$$

$$= \sqrt{(827.4)^2 + (57.28)^2}$$

$$\boxed{M_D = 829.38 \text{ N-m}}$$

compare to both value max. B.M is

$M_{max} = M = M_D = 829.38 \text{ N-m} = 829.38 \times 10^3 \text{ N-mm}$

Equivalent twisting moment (T_e)

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(829.38 \times 10^3)^2 + (716 \times 10^3)^2}$$

$$T_e = 1095685.714 \text{ N-mm}$$

$$\boxed{T_e = 1096 \times 10^3 \text{ N-mm}}$$

$$d = \sqrt[3]{\frac{T_e \times 16}{\pi \times \tau_{sh}}} = \sqrt[3]{\frac{1095685.714 \times 16}{\pi \times 54}}$$

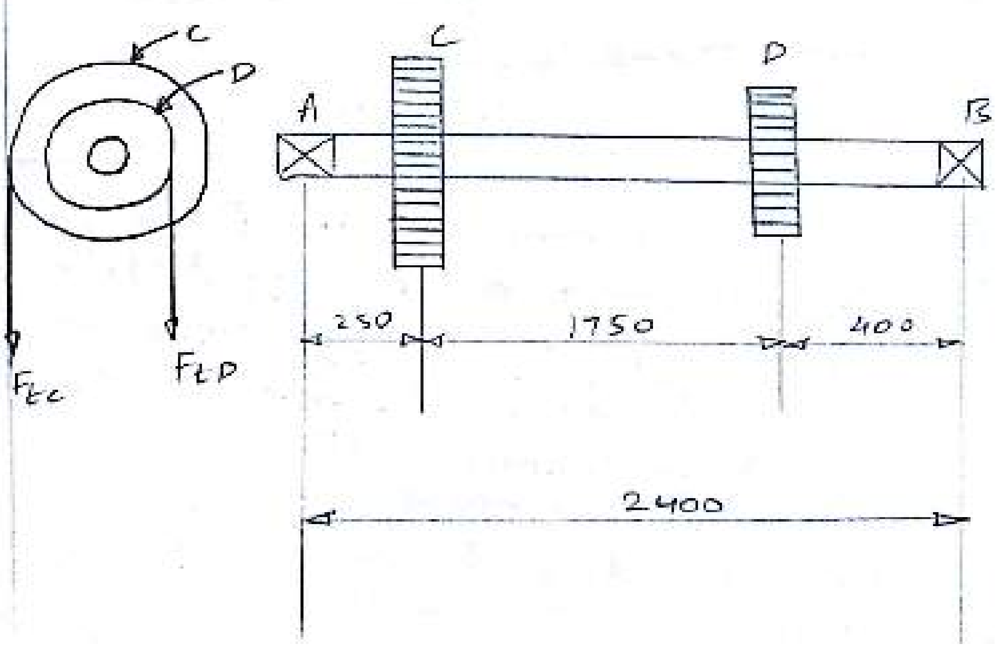
$$\boxed{d = 46.93 \text{ mm}}$$

From PSGDB 7.20

std. diameter $\boxed{d = 50 \text{ mm}}$

- A horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250mm and 400mm respectively from the centre line of the left and right bearings. The pitch diameter of the gear C is 600mm and that of gear D is 200mm. The distance between the centre line of the bearings is 2400mm. The shaft transmits 20KW at 120rpm. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure F_{tC} of the gear C and F_{tD} of the gear D act vertically downwords.

Find the diameter of the shaft, if the working stress is 100 Mpa in tension and 56 Mpa in shear. The gear C and D weighs 950N and 350N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively.



Given:-

$AC = 250mm$

$BD = 400mm$

$D_C = 600mm, R_C = 300mm$

$D_D = 200mm, R_D = 100mm$

$AB = 2400mm, P = 20kW = 20\times10^3 W$

$N = 120rpm, \sigma_t = 100Mpa = 100 N/mm^2$

$\tau = 56mpa = 56 N/mm^2, W_C = 950N$

$W_D = 350N, k_b = 1.5, k_t = 1.2$

To find:-

Diameter of the shaft.

Solution:-

W.K.T $T = \frac{P\times60}{2\pi N} = \frac{20\times10^3\times60}{2\times\pi\times120}$

$T = 1591 N-m$

$\boxed{T = 1591\times10^3 N-mm}$

Tangential force acting at gear C

$F_{tC} = \frac{T}{R_C} = \frac{1591\times10^3}{300} = 5303 N$

$\boxed{F_{tC} = 5303N}$

Total load acting downwards on the shaft at C

$= F_{tc} + W_c = 5303 + 950$

$= 6253\,N$

tangential force acting at gear D

$F_{tD} = \frac{T}{R_D} = \frac{1591 \times 10^3}{100}$

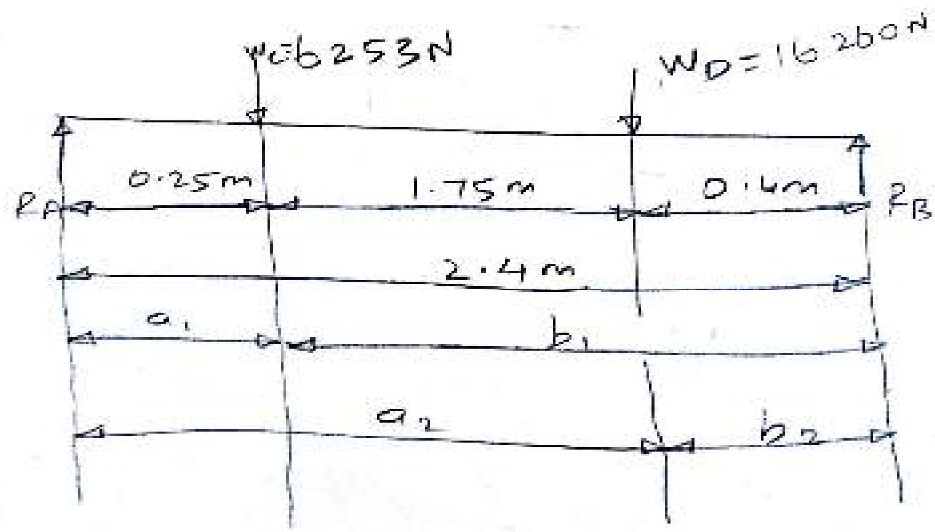
$\boxed{F_{tD} = 15910\,N}$

Total load acting downwards on the shaft at D

$= F_{tD} + W_D = 15910 + 350$

$= \underline{16260\,N}$

Assume the shaft as a simply supported beam,



$W_C = 6253 N$

$a_1 = 0.25 m$

$a_2 = 2 m$

$W_D = 16260 N$

$b_1 = 2.15 m$

$b_2 = 0.4 m$

Take moment about 'A'

$$R_A = \frac{W_C \times b_1}{l} + \frac{W_D \times b_2}{l}$$

$$= \frac{6253 \times 2.15}{2.4} + \frac{16260 \times 0.4}{2.4}$$

$$\boxed{R_A = 8311 N}$$

$$R_B = \frac{W_C \times a_1}{l} + \frac{W_D \times a_2}{l}$$

$$= \frac{6253 \times 0.25}{2.4} + \frac{16260 \times 2}{2.4}$$

$$\boxed{R_B = 14201 N}$$

Bending moment at A & B

$M_A = 0$

$M_B = 0$

$M_C = R_A \times 0.25 = 6311 \times 0.25$
$= 2077.75$

$M_D = R_B \times 0.4 = 14201 \times 0.4$
$= 5680.4 N$

Maximum bending moment

$M_{max} = M - M_D$

$M = 5680.4 \ N-m$

$T = 1591 \ N-m$

$T_e = \sqrt{(K_b \times M)^2 + (K_t \times T)^2}$

$T_e = \sqrt{(1.5 \times 5680.4)^2 + (1.2 \times 1591)^2}$

$T_e = 8731.87 \ N-m$

$\boxed{T_e = 8731.87 \times 10^3 \ N-mm}$

Equivalent Twisting moment (T_e)

$T_e = \frac{\pi}{16} d^3 \times \tau_{sh}$

$d = \sqrt[3]{\frac{T_e \times 16}{\pi \times \tau_{sh}}}$

$= \sqrt[3]{\frac{8731.87 \times 10^3 \times 16}{\pi \times 56}}$

$\boxed{d = 92.60 \ mm}$

Equivalent bending moment

$$M_e = \frac{1}{2}\left[(K_b \times M) + \sqrt{(K_b \times M)^2 + (K_t \times T)^2} \right.$$

$$= \frac{1}{2}\left[(K_b \times M) + T_e \right]$$

$$M_e = \frac{1}{2}\left[(1.5 \times 5680.4) + 8731.87 \right]$$

$M_e = 8626$ N-m

$M_e = 8626 \times 10^3$ N-mm

Equivalent bending moment (M_e)

$$M_e = \frac{\pi}{32} \times \sigma_b \times d^3$$

$$d = \sqrt[3]{\frac{M_e \times 32}{\pi \times \sigma_b}}$$

$$d = \sqrt[3]{\frac{8626 \times 10^3 \times 32}{\pi \times 100}}$$

$d = 95.78$ mm

compare to both value

$d = 95.78$ mm $\simeq 100$ mm.

$\boxed{d = 100\text{mm}}$