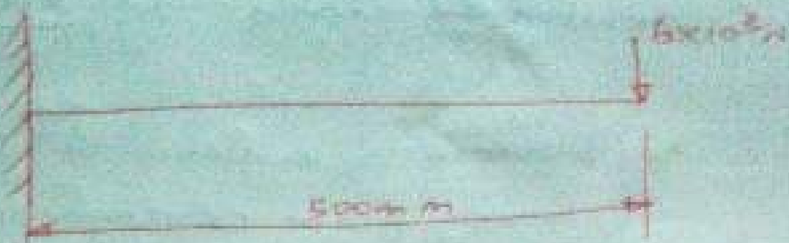
A cantilever of span 500mm carries a vertical downward load of 6 kN a free end. Assume yield value of 350mpa and factor of safety of 3. Find the economical section for the cantilever among.

a) circular cross section of diameter 'd'

b) Rectangular section of depth 'h' and width 't' with $h/t = 2$.

c) I section of depth 7t and flange width 5t where t is thickness.

Specify the dimension and cross sectional area.



given:-

Beam length $l = 500mm$

Load $P = 6kN = 6\times10^3 N$

$\sigma_y = 350 mpa$

$n = 3$

Load P causes bending stress in the cross section

Allowable stress $\sigma_b = \frac{\sigma_y}{n} = \frac{350}{3}$

$= 116.67 N/mm^2$

To find:-

Economical section

Solution:-

a) circular cross section of diameter 'd'

Maximum bending moment

$M_b = P\times l = 6\times10^3 \times 500$

$= 3\times10^6$ N-mm.

moment of Inertia $I = \frac{\pi d^4}{64}$

y = Distance of outermost layer of the beam from neutral axis.

$= \frac{\text{Diameter of the beam}}{2} = d/2$

W.K.T, bending eqn

$$\frac{M_b}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$ [Refer PSGDB 7.1]

① ② ③

equate ① & ② $= \frac{3\times10^6}{\frac{\pi d^4}{64}} = \frac{116.67}{d/2}$

$$d^3 = \frac{64\times3\times10^6}{116.67\times2\times\pi}$$

$$d^3 = 261.91\times10^3$$

$$d = 63.98\,mm$$

$$\boxed{d \approx 64mm}$$

$A = \pi/4\, d^2$

$= \pi/4 \times (64)^2$

$= 3217\,mm^2$

2) Rectangular section of depth h and width t, $h/t = 2$

$M_b = 3\times10^6$ N-mm

$$I = \frac{t h^3}{12} = \frac{1}{12} t \cdot (2t)^3 = 0.666 t^4$$

$$\boxed{I = 0.666 t^4}$$

$$\boxed{h = 2t}$$

$$y = \frac{h}{2} = \frac{2t}{2} = t \Rightarrow \boxed{y = t}$$

$h = 2 \times 34$

$\boxed{h = 68 \text{ mm}}$

sub. above value ~~to~~ to following eqn

$$\frac{M_b}{I} = \frac{\sigma_b}{y}$$

$$\frac{3 \times 10^6}{0.666 t^4} = \frac{116.67}{t}$$

$$116.67 \times 0.666 t^3 = 3 \times 10^6$$

$$t^3 = \frac{3 \times 10^6}{116.67 \times 0.666}$$

$$t = 33.798 \text{ mm}$$

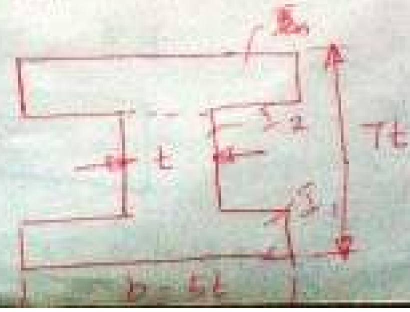
$$\boxed{t = 34 \text{mm}}$$

Beam depth $h = 2t = 2 \times 34 = 68$ mm

Cross sectional area $= h \times t = 68 \times 34$

$= 2313 \text{mm}^2$

3) 'I' section depth $7t$, width $5t$



I_1, For outer rectangle $= \frac{bh^3}{12}$

I_2 For the cut rectangle $= \frac{b_1 h_1^3}{12}$

Here $b = 5t$, $b_1 = (5t - t) = 4t$
$h = 7t$ $h_1 = (7t - t - t) = 5t$

$$I_1 = \frac{(5t)(7t)^3}{12}$$

$$I_2 = \frac{(4t)(5t)^3}{12}$$

$$I = I_1 - I_2 = \frac{1215 t^4}{12}$$

$$y = \frac{h}{2} = \frac{7t}{2} = 3.5t$$

using bending equ.

$$\frac{\sigma_b}{y} = \frac{M_b}{I}$$

$$\frac{116.67}{3.5t} = \frac{3 \times 10^6}{\left(\frac{1215}{12} t^4\right)}$$

$$t^3 = 776.15$$

$$\boxed{t = 9.19 mm}$$

Depth $h = 7t$ $= 7 \times 9.19 = 68 mm$

width $b = 5t = 5 \times 9.19 = 49mm$

Required cross section of the I section

$= bh - b_1h_1$

$= (49 \times 68) - (37 \times 47)$

$= 1593 mm^2$

$\therefore b_1 = 4t = 37mm$

$h_1 = 5t = 47mm$

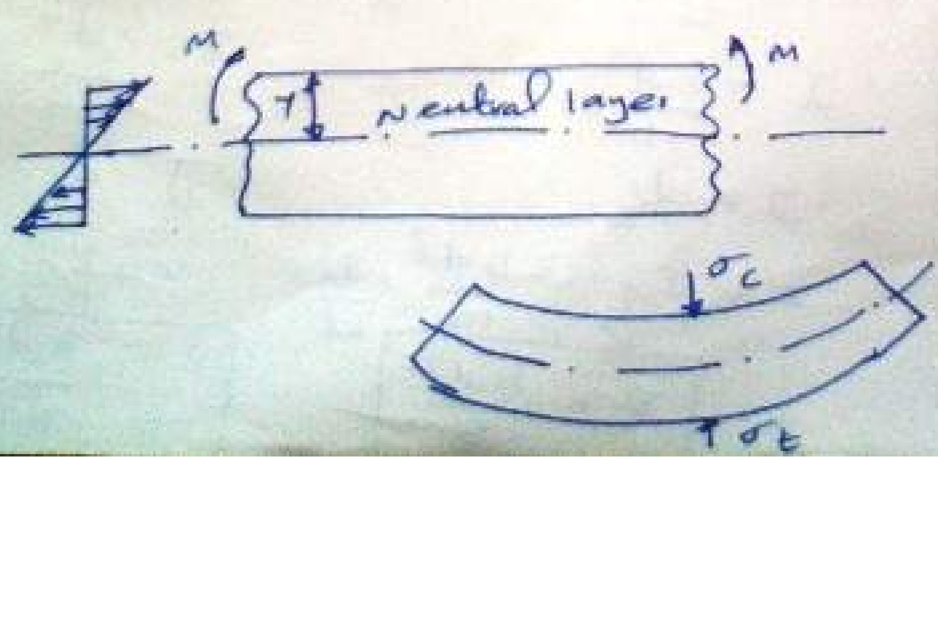
Result:-

comparing the areas of the three cross-section it is found that the I section is most economical.

most economical 'I' section cross section $= 1593 mm^2$

Bending stress in straight beam:-

The resistance offered by the internal stresses to the bending movement is called Bending stress.



Impact and shock loading

The Load acting on any machine component can be of either of these two types

1) Gradual load.
2) Suddenly applied (or) Impact (or) Shock load.

Gradual load -

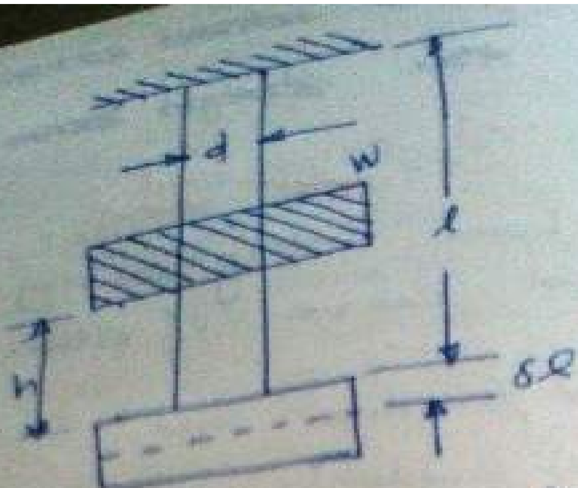
Gradual load is one which, goes on increasing over a period of time till the maximum value is reached.

Suddenly applied (or) impact (or) Shock load.

Impact (or) shock load is which is applied suddenly (or) with some initial velocity.

Ex: Punching presses, hammers.

The stress produced due to impact loads is more than the stresses produced if the same load is applied gradually.



Let us assume that a bar of diameter 'd' and length 'l' has a collar at its base.

⇒ A Load W falls from a height 'h' on the collar.

⇒ δl = deformation of the bar.

⇒ Due to falling load, energy is gained But at the same time as the weight 'W' falls down potential energy is lost.

equating these two,

K.E gained = P.E lost.

$$\frac{1}{2} \times W \times \delta l = W(h + \delta l)$$

But $W = \sigma_1 \times A$

σ_i = induced stress

A = area of the bar

$$\delta l = \frac{Wl}{AE} = \frac{\sigma_i \times l}{E}$$

$$\therefore \tfrac{1}{2} \times \sigma_i \times A \times \delta l = W(h + \delta l)$$

$$\tfrac{1}{2} \times \sigma_i \times A \times \frac{\sigma_i \times l}{E} = W\left(h + \frac{\sigma_i l}{E}\right)$$

$$\tfrac{1}{2} \times \frac{Al}{E} \sigma_i^2 = \frac{Wh}{E} + \frac{Wl}{E} \sigma_i$$

$$\tfrac{1}{2} \times \frac{Al}{E} \sigma_i^2 - \frac{Wh}{E} - \frac{Wl}{E} \sigma_i = 0$$

$$ax^2 + bx + c = 0$$

where $x = \sigma_i$

solving this quadratic eqn on σ_i,

$$\sigma_i = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2hAE}{Wl}}\right)$$

i.e $\sigma_i = \sigma + \sigma\left(1 + \sqrt{\frac{2h}{e}}\right)$

$\left(\because \frac{W}{A} = \sigma, \text{ and } \frac{\sigma}{E} = \text{Stain}(e)\right)$

impact load $\boxed{e_i = \sigma\left(1 + \sqrt{\frac{2h}{e}}\right)}$

$$\frac{\sigma^2 \times A \times l}{2E} = P(h + \delta l)$$

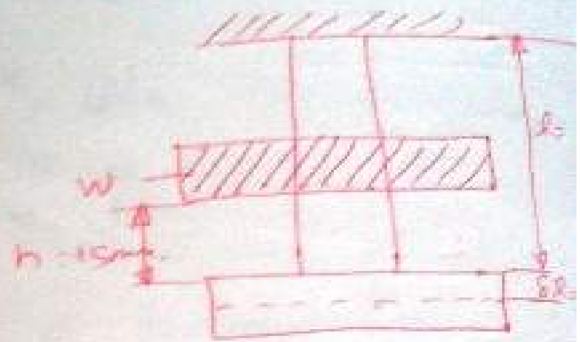
$$\sigma = P/A \left[1 + \sqrt{1 + \frac{2hAE}{Pl}}\right]$$

If $h = 0$

$$\sigma = \frac{2P}{A}$$

1.27 An unknown weight falls from a distance of 15mm on to a collar rigidly attached to the lower end of a vertical bar 2.5m long and 500mm² cross-section. The maximum instantaneous extension is 2mm. Find the corresponding stress and the value of the weight falling. $E = 2\times10^5 N/mm^2$

Given:-



$h = 15mm$

$l = 2.5m = 2500mm$

$A = 500mm^2$

$\delta l = 2mm$

$E = 2\times10^5 N/mm^2$

To find:-

Stress (σ) and weight (w)

Solution:-

$$\text{strain}(e) = \frac{\delta l}{l} = \frac{2}{2500} = 0.0008$$

strain(e) = Stress(σ) / Young's modulus(E)

$$\sigma = E\times e = 2\times10^5 \times 0.0008 = 160\ N/mm^2$$

To find out the unknown weight, equating the strain energy and the loss in potential energy

$$\tfrac{1}{2} P(\delta l) = W(h+\delta l)$$

$$P = \text{stress} \times \text{Area} = 160 \times 500$$

$$= 8\times10^4 N$$

Hence $\tfrac{1}{2} \times 8\times10^4 \times \dfrac{2}{\cancel{}} = W(15+2)$

$$8\times10^4 = W(17)$$

$$W = \frac{8\times10^4}{17}$$

$$\boxed{W = 4705.88N}$$

Result:-

Stress = 160 N/mm²

Weight: 4705.88N

A steel rod of 5cm diameter and 3m long when unloaded is suspended from one end and has a weight of 5KN, threaded to it. The weight is allowed to fall freely from a height of 2.54cm on to a head formed on the the lower end of a rod. Find the maximum stress produced in the rod, ~~Find the maximum stress produced in the rod~~ Also, find height of drop so that maximum stress does not exceed 7KN/cm² for the material of the rod modulus of elasticity is 20 KN/cm².

Given:-

$d = 5cm = 50mm$ $\quad P = 5kN = 5\times10^3 N$

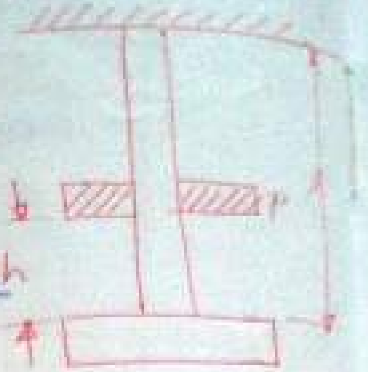
$l = 3m = 3\times10^3 mm$

$h = 2.54cm$

$E = 20 \frac{MN}{cm^2}$

$= 20\times10^6 \frac{N}{cm^2} = \frac{20\times10^6}{100}$

$= 0.2\times10^6 N/mm^2$



$\boxed{E = 0.2\times10^6 N/mm^2}$

$A = \frac{\pi d^2}{4}$

$= \frac{\pi\times50^2}{4}$

$\boxed{A = 1963.49 mm^2}$

$\sigma = 7 kN/cm^2 = 7\times10^3 N/cm^2$

$= \frac{7\times10^3}{100} N/mm^2$

$\boxed{\sigma = 70 N/mm^2}$

$$\sigma = P/A\left[1 + \sqrt{1 + \frac{2AhE}{P\cdot l}}\right]$$

$$\sigma = \frac{5\times10^3}{1963.49}\left[1 + \sqrt{1 + \frac{2\times1963.49\times2.54\times0.2\times10^6}{5\times10^3\times3\times10^3}}\right]$$

$\boxed{\sigma = 95.44 N/mm^2}$

If $\sigma = 70 N/mm^2$, what is the value of h?

$$\sigma = \frac{5\times10^3}{1963.49}\left[1 + \sqrt{1 + \frac{2\times1963.49\times h\times0.2\times10^6}{5\times10^3\times3\times10^3}}\right]$$

$$\tau_0 = 2.54\left[1 + \sqrt{1 + 52.35h}\right]$$

$$\frac{\tau_0}{2.54} = \left[1 + \sqrt{1 + 52.35h}\right]$$

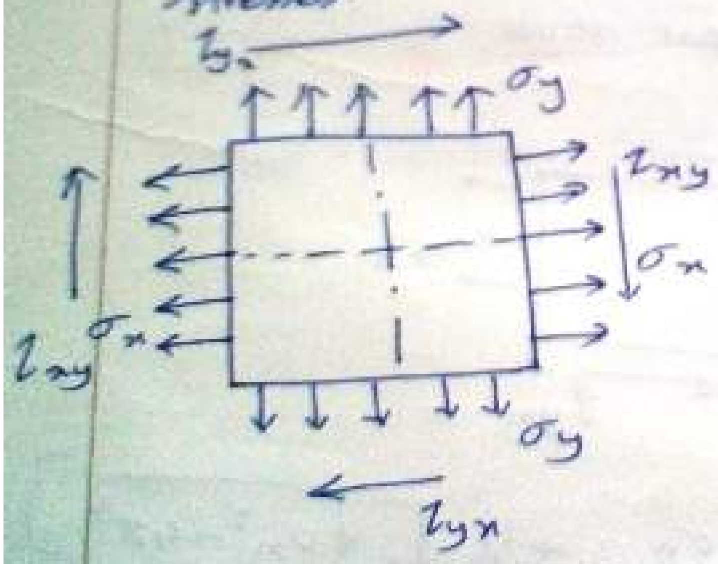
$$27.55 - 1 = \sqrt{1 + 52.35h}$$

$$26.55^2 = 1 + 52.35h$$

$$\boxed{h = 13.44\,mm}$$

PRINCIPAL STRESS:-

Principal plane is a plane in which shear stress is zero, and the direct stresses acting along these planes are known as principal stresses.



Bi-axial tensile stress coupled with a shear stress.

When shear stress is also acting in addition we have to find out maximum and minimum principal stresses.

$$\sigma_{max}(\sigma_1) = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left[\frac{\sigma_x+\sigma_y}{2}\right]^2 + \tau_{xy}^2}$$

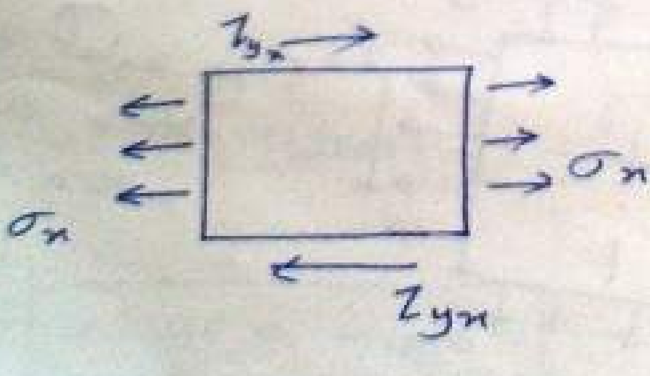
$$= \frac{1}{2}\left[(\sigma_x+\sigma_y) + \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}\right.$$

$$\sigma_{min}(\sigma_2) = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{1}{2}\left[(\sigma_x+\sigma_y) - \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}\right.$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}$$

when a member is subjected to direct stress in one plane accompanied by simple shear stress.



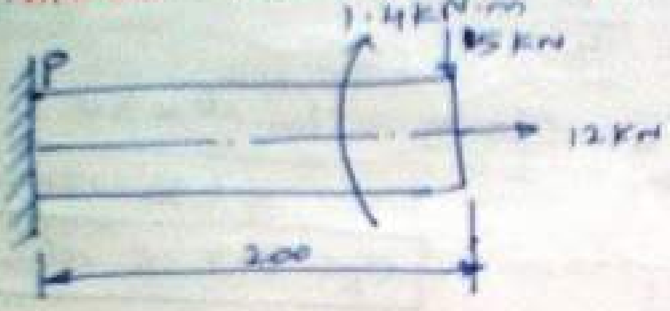
$$\sigma_{t\,max} = \frac{\sigma_x}{2} + \frac{1}{2}\left[\sqrt{\sigma_x^2 + 4\tau^2}\right]$$

$$\sigma_{t\,min} = \frac{\sigma_x}{2} - \frac{1}{2}\left[\sqrt{\sigma_x^2 + 4\tau^2}\right]$$

$$\tau_{max} = \frac{1}{2}\left[\sqrt{\sigma_n^2 + 4\tau^2}\right]$$

Tensile stress $\sigma_n = \sigma_b + \sigma_d$ compressive stress $\sigma_n = \sigma_b - \sigma_d$

A cylinder bar of 60mm diameter and 200mm long is fixed one end and other end is free and carrying a load as shown in fig with an axial load of 12 kN & downward transverse load of 5 kN and a torque of 1.4 kN·m. calculate the maximum stresses induced at point 'p' of the bar.



1) Effect of 12 kN :-

Direct Tensile stress $(\sigma_d) = P/A$

$$= \frac{12 \times 10^3}{\frac{\pi}{4} d^2}$$

$$= \frac{12 \times 10^3 \times 4}{\pi \times (200)^2}$$

$$= 4.24 \text{ N/mm}^2$$

2) Effect of 5 kN :-

Bending moment

$M_b = P \times l = 5 \times 10^3 \times 200 = 1 \times 10^6$ N.mm

$$I = \frac{\pi d^4}{64} = \frac{\pi \times 60^4}{64} = 636.17 \times 10^3 \ mm^4$$

$$\boxed{I = 636.17 \times 10^3 \ mm^4}$$

$$y = d/2 = \frac{60}{2} = 30 mm$$

$$\boxed{y = 30 mm}$$

$$\frac{M_b}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M_b}{I} \cdot y = \frac{1 \times 10^6 \times 30}{636.17 \times 10^3}$$

$$\boxed{\sigma_b = 47.15 \ N/mm^2}$$

2) Effect of Torque :- 1.4 kN.m

$$T = 1.4 \times 10^6 \ N.mm$$

$$J = \frac{\pi d^4}{32} = \frac{\pi \times 60^4}{32} = 1.27 \times 10^6 \ mm^4$$

$$\boxed{J = 1.27 \times 10^6 \ mm^4}$$

$$R = d/2 = \frac{60}{2} = 30 mm$$

$$\boxed{R = 30 mm}$$

$$\frac{T}{J} = \frac{Z}{R}$$

$$T = Z \cdot \pi/16 \ d^3$$

$$Z = \frac{T \times 16}{\pi \times d^3} = \frac{1.4 \times 10^6 \times 16}{\pi \times 60^3}$$

Z

$$\tau = \frac{T}{J} \cdot R$$

$$= \frac{1.4 \times 10^6 \times 30}{1.27 \times 10^6}$$

$$\boxed{\tau = 33.07 \ N/mm^2}$$

Direct tensile stress and indirect tensile stress caused due to bending at point 'p' can be added.

$$\sigma_x = \sigma_d + \sigma_b = 4.24 + 47.15$$
$$= 51.39 \ N/mm^2$$

Now that we have tensile stress and shear stress acting simultaneously let us apply principal stresses.

$$\sigma_{tmax} = \frac{\sigma_x}{2} + \frac{1}{2}\left[\sqrt{\sigma_x^2 + 4\tau^2}\right]$$

$$= \frac{51.39}{2} + \frac{1}{2}\left[\sqrt{51.39^2 + 4(33.07)^2}\right]$$

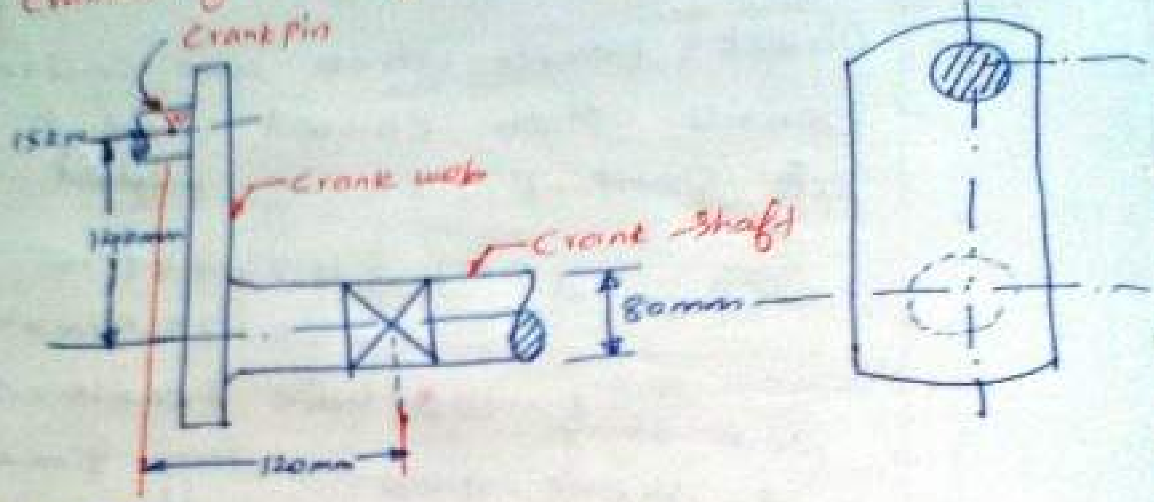
$$= 25.695 + 41.87$$

$$\boxed{\sigma_{tmax} = 67.565 \ N/mm^2}$$

$$\tau_{max} = \frac{1}{2}\left[\sqrt{\sigma_x^2 + 4\tau^2}\right]$$
$$= \frac{1}{2}\left[\sqrt{51.39^2 + 4(33.07^2)}\right]$$

$$\tau_{max} = 41.87 \ N/mm^2$$

(5) 5.15 An overhang crank with pin and shaft is shown in figure. A tangential load of 15kN acts on the crank pin. Determine the maximum principal stress and the maximum shear stress at the centre of the crankshaft bearing.



~~solution~~:-

given:-

$W = 15kN = 15 \times 10^3 N$

$d = 80mm$

$y = 140mm$

$x = 120mm$

To find:-

1) The maximum principal stress
2) The maximum shear stress } at the centre of the crankshaft bearing.

Solution:-

Bending moment at the centre of the crank shaft bearing

$$M = W \times x$$

$$= 15 \times 10^3 \times 120$$

$$M = 1.8 \times 10^6 \text{ N.mm.}$$

Torque transmitted at axis of the shaft $(T) = W \times y$

$$= 15 \times 10^3 \times 140$$

$$= 2.1 \times 10^6 \text{ N.mm}$$

W.K.T bending stress due to the bending moment

$$\sigma_b = \frac{M}{Z} = \frac{M}{\frac{\pi}{32} \times d^3} \qquad Z = \frac{\pi}{32} \times d^3$$

$$= \frac{32M}{\pi d^3}$$

$$= \frac{32 \times 1.8 \times 10^6}{\pi \times (80)^3} = 35.8 \text{ N/mm}^2 = 35.8 \text{ mpa}$$

$= \cancel{\text{N/mm}^2} = \cancel{20.9 \text{ Mpa}}$

$\boxed{\cancel{\sigma_b = 20.29 \text{ mpa}}}$

$$\boxed{\sigma_b = 35.8 \text{ mpa}}$$

$$R = d/2$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{T}{J} \cdot R = \frac{T}{\frac{\pi d^4}{32}} \cdot R = \frac{T \times \overset{16}{32}}{\pi d^4} \cdot \frac{d}{2}$$

$$= \frac{\cancel{32}}{16} \quad \frac{16 \times T}{\pi d^3}$$

$$\tau = \frac{16 \times 2.1 \times 10^6}{\pi \times (80)^3}$$

$$\boxed{\tau = 20.88\ N/mm^2}$$

maximum principal stress

$$\sigma_{t\,max} = \frac{\sigma_x}{2} + \left[\sqrt{\sigma_x^2 + 4\tau^2}\right]$$

$$\because \sigma_x = \sigma_b$$

$$\sigma_{t\,max} = \frac{\sigma_b}{2} + \left[\sqrt{\sigma_b^2 + 4\tau^2}\right]$$

$$= \frac{35.8}{2} + \left[\sqrt{35.8^2 + 4(20.88)^2}\right]$$

$$= 17.9 + 27.5$$

$$\boxed{\sigma_{t\,max} = 45.4\ N/mm^2}$$

maximum principal stress

$$\tau_{max} = \frac{1}{2}\left[\sqrt{\sigma_t^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{(35.8)^2 + (4 \times 20.88^2)}\right.$$

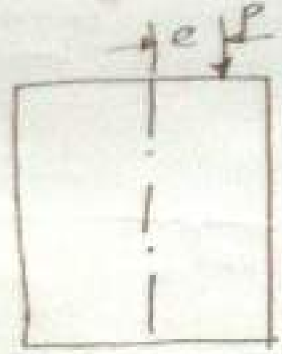
$\boxed{\tau_{max} = 27.5 \ N/mm^2}$

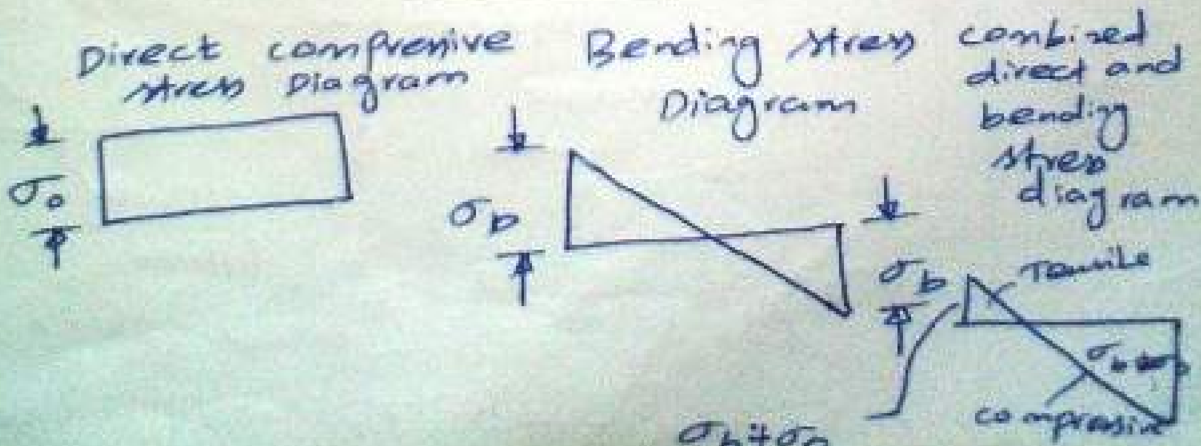
ECCENTRIC LOADING

An external load, whose line of action is ∥el but does not coincide with the centroidal axis of the m/c component, is known as an **eccentric load** (P)

⇒ The distance b/w the centroidal axis of the m/c component and the eccentric load is called **eccentricity**

⇒ It is generally denoted by e.





Formula

1) Bending stress $\sigma_b = M/Z$

$$M = P \cdot e$$

2) Direct compressive stress

$$\sigma_0 = \frac{P_1}{A} \text{ (or) } \frac{P}{A}$$

3) maximum ~~compressive~~ tensile stress

$$\sigma_{t(max)} = \sigma_b + \sigma_0$$

4) maximum ~~tensile~~ compressive stress

$$\sigma_{c(max)} = \sigma_b - \sigma_0$$

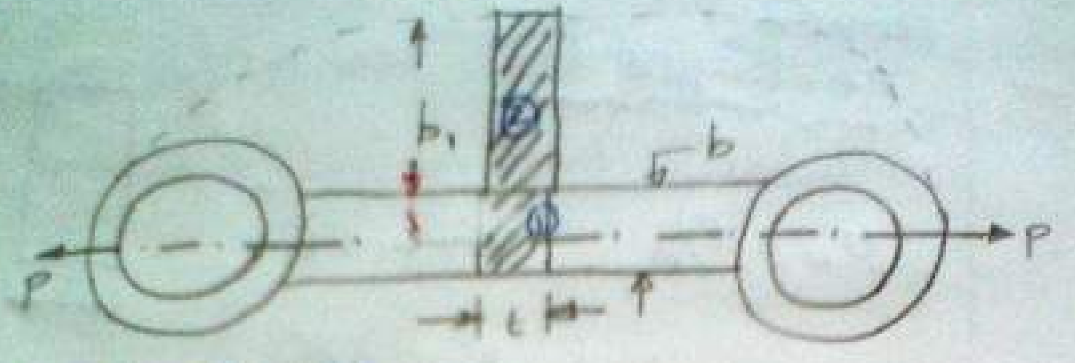
RSK Pg. 169 Ex. 5.22

A mild steel link, as shown in fig. by full lines, transmits a pull of 80 kN. Find the dimensions b and t if b = 3t. Assume the permissible tensile stress as 70 MPa. If the original link is replaced by an un-symmetrical one, as shown by dotted lines in fig., having the same thickness t, find the depth b_1, using the

given:-

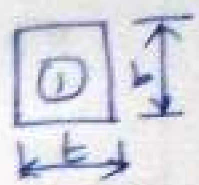
$P = 80 kN = 80 \times 10^3 N$ $b = ?$ $t = ?$

$\sigma_t = 70 MPa = 70 N/mm^2$ if $b = 3t$



Rectangle - 1

~~Rectangle 2~~



$\sigma = P/A$

$P = \sigma \times A$ — 1 $A = t \times b$

$A = t \times 3t = 3t^2 \, mm^2$

sub. the above valul ~~into~~ in the Eqn 1

$80 \times 10^3 = 70 \times 3t^2$

$$t^2 = \frac{80 \times 10^3}{70 \times 3}$$

$t = 19.5 \approx 20 mm$

$\boxed{t = 20 mm}$ — 2

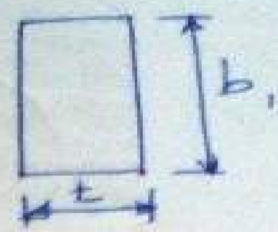
sub ② in $b = 3t$

$b = 3t$

$= 3 \times 20 = 60\,mm$

$\boxed{b = 60\,mm}$

Rectangle - ②



$\sigma_0 = P/A \qquad A = b_1 \times t$

$A = 20 b_1$

$\sigma_0 = \dfrac{80 \times 10^3}{20 b_1} = \dfrac{4000}{b_1}$ — ①

bending stress

$\sigma_b = \dfrac{M}{Z} \qquad Z = I/y \qquad I = \dfrac{bd^3}{12}$

$y = d/2$

$M_b = P \cdot e \qquad = \dfrac{bd^3}{12} \times \dfrac{2}{d} = \dfrac{bd^2}{6}$

$\boxed{Z = \dfrac{bd^2}{6}}$

$\sigma_b = \dfrac{P \cdot e}{\dfrac{20 b_1^2}{6}} \qquad e = \dfrac{b_1}{2}$

$= \dfrac{P \times b_1 \times 6}{20 \times b_1^2 \times 2}$

$= \dfrac{P \times \cancel{b_1} \times 6}{20 \times 2 \times b_1^{\cancel{2}}}$

$Z = \dfrac{t b_1^2}{6}$ — ③

$= \dfrac{20 b_1^2}{6}$ — ③

$$= \frac{180 \times 10^3 \times 16}{240 b_1} = \frac{12000}{b_1} \quad - (2)$$

$$\sigma_b = \frac{12000}{b_1} \quad - (2)$$

Total stress due to eccentric loading

$$= \sigma_b + \sigma_0 = \frac{12000}{b_1} + \frac{4000}{b_1}$$

$$\text{Total stress} = \frac{16000}{b_1}$$

$$\sigma_t = \frac{16000}{b_1}$$

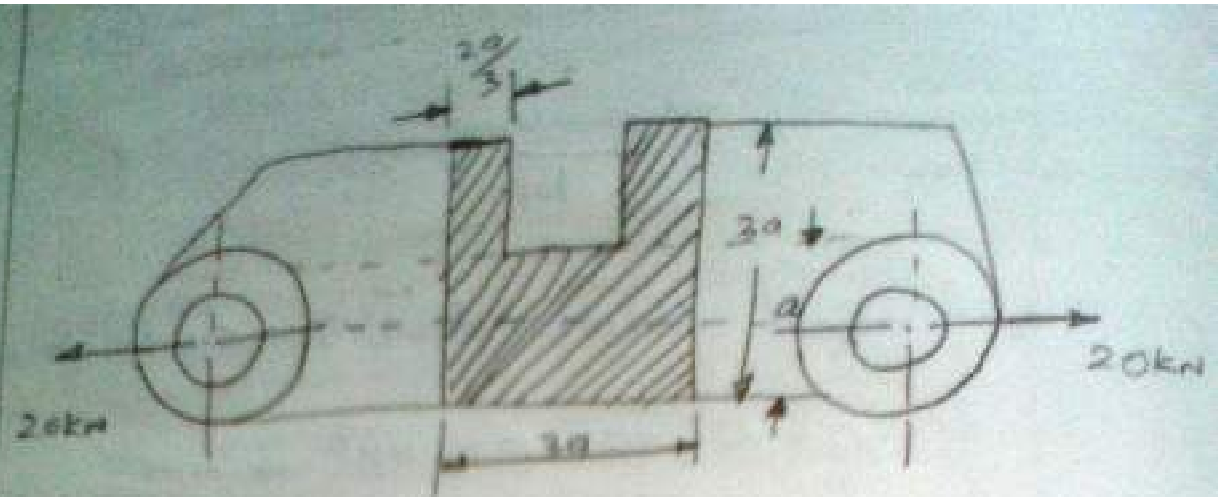
$$b_1 = \frac{16000}{\sigma_t} = \frac{16000}{70}$$

$$\boxed{b_1 = 228.5\ mm}$$

$$\boxed{b_1 = 230mm}$$

R.S.K
Pg-165
Ex: 5.27

A cast-iron link, as shown in fig is to carry a load of 20kn. If the tensile and compressive stresses in the link are not to exceed 25MPa and 80mpa respectively, obtain the dimensions of the cross-section of the link at the middle of its length.

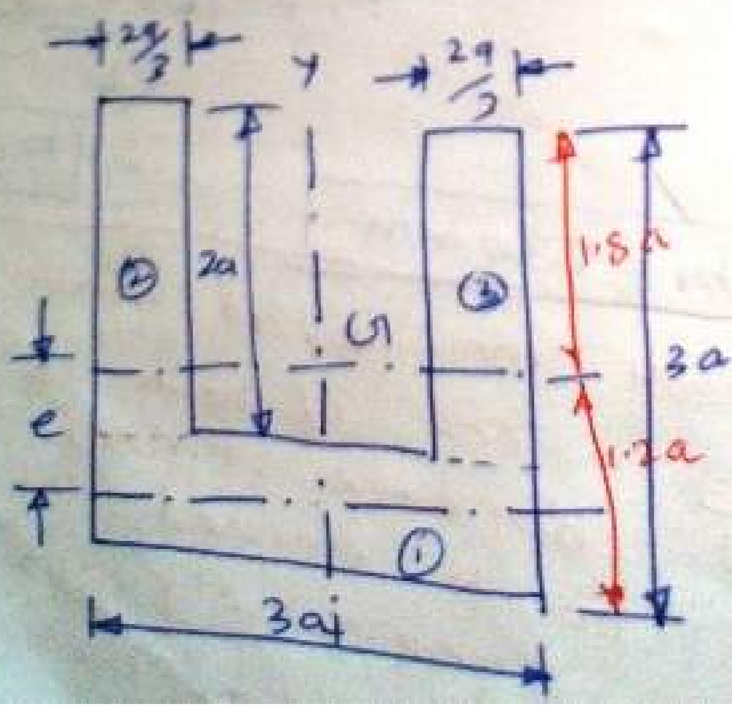


given:-

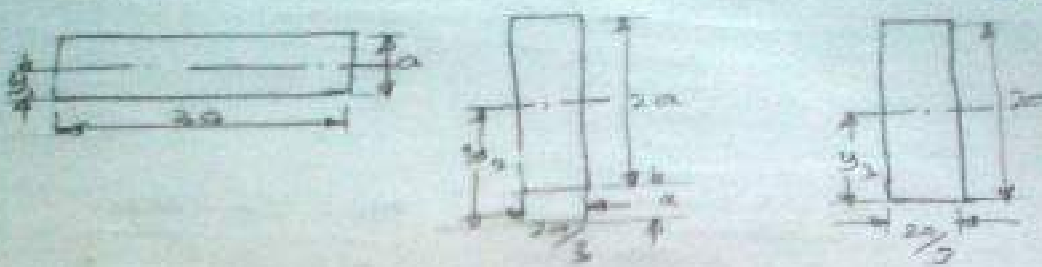
$P = 20kN = 20 \times 10^3 N$

$\sigma_{t(max)} = 25 mpa = 25 N/mm^2$

$\sigma_{c(max)} = 80 mpa = 80 N/mm^2$



Rectangle ① Rectangle ② Rectangle ③



$y_1 = a/2 = 0.5a$ $\qquad$ $y_2 = y_3 = \cancel{2}a/\cancel{2} + a = 2a$

$I = I_1 + I_2 + I_3 = I_1 + 2(I_2)$

$I_1 = I_{G1} + A_1 h_1^2$ $\qquad$ $I_{G1} = \frac{bd^3}{12} = \frac{3a(a)^3}{12} = \frac{3a^4}{12}$

$I_2 = I_{G2} + A_2 h_2^2$ $\qquad$ $I_{G2} = I_{G3} = \frac{db^3}{12}$

$I_3 = I_{G3} + A_3 h_3^2$

$= 2a\frac{(2a)^3}{3}$

$= 2a \times \frac{8a^3}{3}$

$= \frac{16a^4}{3}$

$A_1 = 3a \times a = 3a^2$

$A_2 = A_3 = 2a/3 \times 2a = \frac{4a^2}{3}$

$A = A_1 + A_2 + A_3 = 3a^2 + \frac{4a^2}{3} + \frac{4a^2}{3}$

$= 3a^2 + 2(4a^2/3) = 3a^2 + \frac{8a^2}{3}$

$\boxed{A = 5.67a^2 \, mm^2}$

$h_1 = \bar{y} \sim y_1$ (or $y_1 \sim \bar{y}$) $\qquad$ $h_2 = h_3 = \bar{y} \sim y_2$ (or $y_2 \sim \bar{y}$)

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A}$$

$$= \frac{3a^2 \times a/2 + 2(4a^2/3 \times 2a)}{5.67a^2}$$

$$\frac{\frac{3a^3}{2} + \frac{16a^3}{3}}{5.67a^2} = \frac{1.5a^3 + 5.33a^3}{5.67a^2} = 1.2a\,mm$$

$$\boxed{\bar{y} = 1.2a\,mm}$$

$h_1 = 1.2a - 0.5a$ $\quad$ $h_2 = 2a - 1.2a$

$\boxed{h_1 = 0.7a}$ $\quad$ $\boxed{h_2 = 0.8a}$

$I_1 = I_{G_1} + A_1 h_1^2$

$= \frac{3a^4}{12} + 3a \times (0.7a)^2$

$I_1 = 0.25a^4 + 1.47a^4$

$\boxed{I_1 = 1.72\,a^4\,mm^4}$

$I_2 = I_{G_2} + A_2 h_2^2$

$= \frac{16a^4}{3} + \frac{4a^2}{3} \times (0.8a)^2$

$= 0.444a^4 + \frac{4a^2}{3} \times 0.64a^2$

$= 0.444a^4 + 0.85a^4$

$\boxed{I_2 = 1.2a^4\,mm^4}$

$I = I_1 + 2(I_2)$

$= 1.72a^4 + 2\times(1.2a^4)$

$\boxed{I = 4.3a^4 \ mm^4}$

Distance of n.A from the bottom of the link

$y_t = \bar{y} = 1.2a$ mm

Distance of n.A from the top of the link

$y_c = 3a - 1.2a = 1.8a$ mm

$\boxed{y_c = 1.8a \ mm}$

Eccentricity of the load (i.e distance of n.A from the point of application of the load)

$e = 1.2a - 0.5a = 0.7mm$

$\boxed{e = 0.7a \ mm}$

W.k.T bending moment exerted on the section

$M = P.e = 20 \times 10^3 \times 0.7a = 14 \times 10^3 a$ N-mm

$\boxed{M = 14 \times 10^3 a \ N\text{-}mm}$

Tensile stress in the bottom of the link

$$\sigma_t = \frac{M}{Z_t} = \frac{M}{I/y_t} = \frac{14 \times 10^3 a}{4.3a^4/1.2a}$$

$$= \frac{14\times10^3 \times 1.2 \times a^2}{4.3 a^4}$$

$$\boxed{\sigma_t = \frac{3907}{a^2}} - ①$$

compressive stress in the top of the link

$$\sigma_c = \frac{M}{Z_c} = \frac{M}{I/y_c} = \frac{14\times10^3 a}{\frac{4.3a^4}{1.8a}}$$

$$= \frac{14\times10^3 a \times 1.8 a}{4.3 a^4}$$

$$= \frac{5860}{a^2} - ②$$

W.K.T max. tensile stress

$$[\sigma_{t(max)}] = \sigma_t + \sigma_c$$

$$= \frac{3907}{a^2} + \frac{5860}{a^2}$$

$$25 = \frac{9767}{a^2}$$

$$a^2 = \frac{9767}{25} \Rightarrow \boxed{a = 19.76\ mm}$$

w.r.t max. compressive stress

$$[\sigma_c(max)] = \sigma_c - \sigma_0$$

$$= \frac{5860}{a^2} - \frac{3530}{a^2}$$

$$80 = \frac{2330}{a^2} \Rightarrow a^2 = \frac{2330}{80}$$

$$\boxed{a = 5.4\,mm}$$

we shall take larger of two values

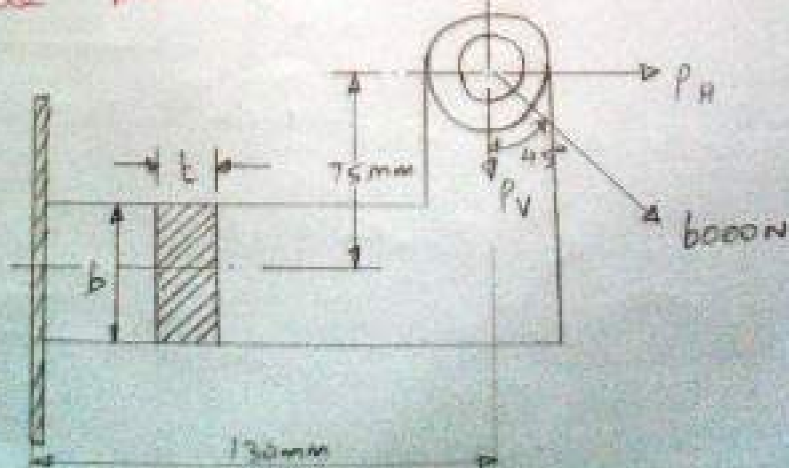
i.e $\boxed{a = 19.76\,mm}$

R.S.K

Pg-168

Ex-5.25

A mild steel bracket as shown in fig., is subjected to a pull of 6000N acting at 45° to its horizontal axis. The bracket has a rectangular section whose depth is twice the thickness. Find the cross-sectional dimensions of the bracket, if the permissible stress in the material of the bracket is limited to 60MPa.

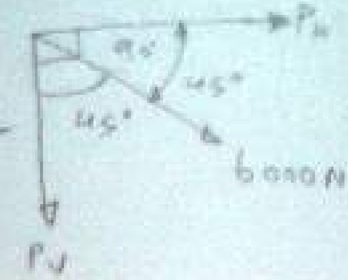


Given:-

Total tensile stress = 60 MPa

$\sigma = 60 N/mm^2$

$b = 2t$



$P_H = 6000 \times \cos 45°$

$\boxed{P_H = 4242 N}$

Solution:-

$P_V = 6000 \times \sin 45°$

$\boxed{P_V = 4242 N}$

① Direct stress

$\sigma_{ov} = \frac{P_v}{A} = \frac{4242}{2t^2} = \frac{2121}{t^2} N/mm^2$

$A = t \times b$

$= t \times 2t = 2t^2$

$\boxed{\sigma_{ov} = \frac{2121}{t^2} N/mm^2}$

② B.M due to horizontal component of load

$M_H = P_H \times 75 = 4242 \times 75 = 318150$ N-mm

$\boxed{M_H = 318150 \text{ N-mm}}$

③ B.M due to vertical component of load

$M_V = P_V \times 130 = 4242 \times 130 = 551460$ N-mm

$\boxed{M_V = 551460 \text{ N-mm}}$

Bending stress

$$\sigma_{bH} = \frac{M_H}{Z} = \frac{318150 \times 6}{4t^3}$$

$$= \frac{477225}{t^3} \text{ N/mm}^2$$

(tensile)

$$Z = \frac{bd^2}{6}$$

$$= \frac{t \times (2t)^2}{6}$$

$$= \frac{t \times 4t^2}{6}$$

$$= \frac{4t^3}{6}$$

$$\sigma_{bV} = \frac{M_V}{Z} = \frac{551460 \times 6}{4t^3}$$

$$= \frac{827190}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

Total stress

$$\sigma = \sigma_{bH} + \sigma_{bV} + \sigma_0$$

$$\frac{477225}{t^3} + \frac{827190}{t^3} + \frac{2121}{t^2} = 60$$

$$\frac{1304415}{t^3} + \frac{2121}{t^2} = 60$$

(or)

$$\frac{21740}{t^3} + \frac{35.4}{t^2} = 1$$

$$\frac{1}{t^2}\left[\frac{21740}{t}+\frac{35.4}{1}\right]=1$$

$$\frac{21740}{t}+\frac{35.4}{1}=t^2$$

$$\frac{21740+35.4t}{t}=t^2$$

$$21740+35.4t=t^3$$

$$t^3-35.4t-21740=0$$

$$t^3+0t^2-35.4t-21740=0$$

use calculator

mode - 3 times

↓

eq

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unknown → degree

2 3

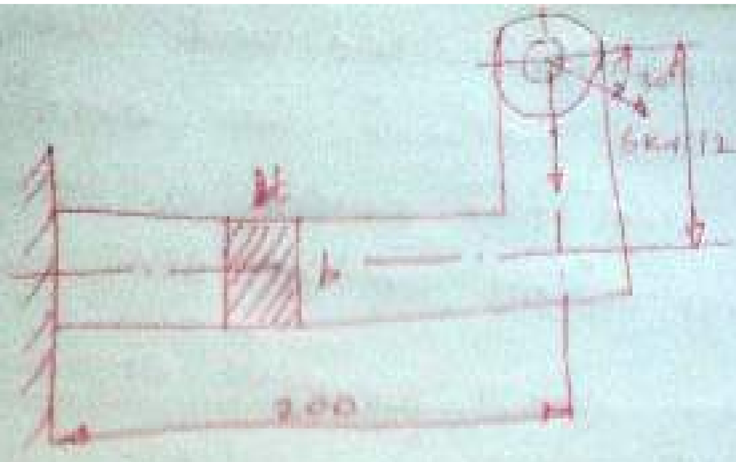
$a=1$, $b=0$ $c=-35.4$ $d=-21740$

$$\boxed{t=28.33\,mm}$$

$b=2t=2\times28.33$

$$\boxed{b=56.66\,mm}$$

A wall bracket of rectangular cross-section as shown in fig. It's subjected to a pull of 6kN acting at 30° to the horizontal. If the maximum stress induced in the bracket material is not to exceed 25N/mm² in both tension and compression design the cross-section of the bracket



Assume
($b = 2t$)

DESIGN OF CURVED BEAMS - CRANE HOOK AND C-FRAME

For the straight beam, the neutral axis of the section coincides with its centroidal axis and stress distribution in the beam is linear.

But in the case of curved beams, the neutral axis of the cross-section is shifted towards the centre of curvature of the beam causing a non-linear distribution of stress

It may be noted that the neutral axis lies between the centroidal axis and the centre of curvature and always occurs within the curved beam.

Bending stress general expression

$$\sigma_b = \frac{M}{Ae}\left[\frac{y}{R_n - y}\right]$$

M - Bending moment
A - Area of cross-section
e - Distance from centroidal axis to the neutral axis ($R - R_n$)
R - Radius of curvature of the centroidal axis
R_n - Radius of curvature of the neutral axis.
y - Distance from the neutral axis to the fibre under consideration

If the section is symmetrical such as a circle, Rectangle, I-beam with equal flange, then the maximum bending stress will always occurs at the inside fibre.

If the section is unsymmetrical, then the maximum Bending stress may occur at either the inside fibre to the outside fibre

⇒ The maximum bending stress at the inside fibre is given by

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i}$$

y_i = Distance from the neutral axis to the inside fibre $R_n - R_i$

R_i = Radius of curvature of the inside fibre.

⇒ The maximum bending stress at the outside fibre is given by

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o}$$

y_o = Distance from the neutral axis to the outside fibre = $R_o - R_n$

R_o = Radius of curvature of the outside fibre.

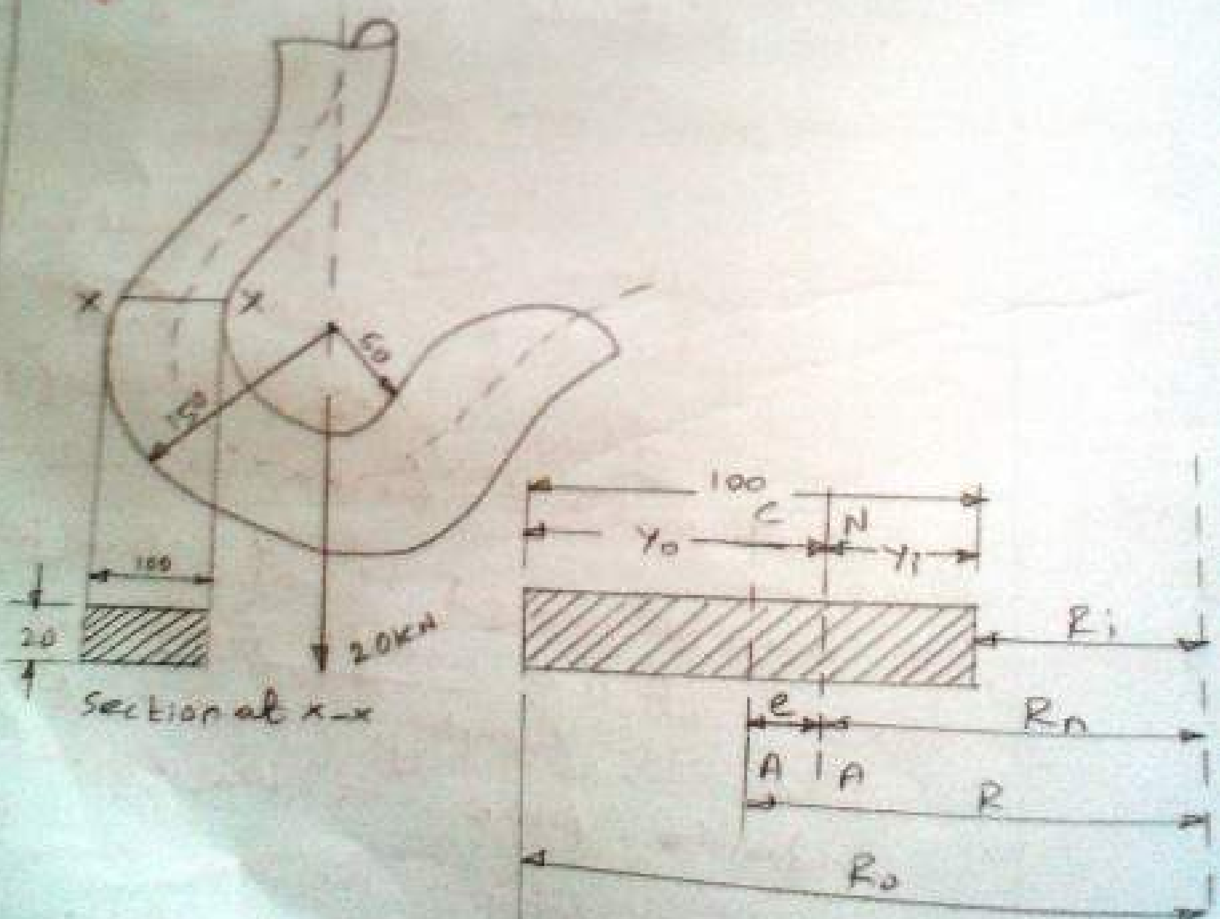
⇒ It may be noted that the bending stress at the inside fibre is tensile while the bending stress at the outside fibre is compressive.

⇒ If the section has an axial load in addition to bending, then the axial (or) direct stress (σ_d) must be added algebraically to the bending stress, in order to obtain the resultant stress in the section.

Resultant stress $\sigma = \sigma_d \pm \sigma_b$.

" inside fibre = $\sigma_d + \sigma_b$;
outside fibre = $\sigma_d - \sigma_b$

The crane hook carries a load of 20kN as shown in fig. The section at x-x is rectangular whose horizontal side is 100mm. Find the stresses in the inner and outer fibres at the given section.



Given:-

$W = 20\,kN = 20 \times 10^3\,N$

$R_i = 50\,mm$

$R_o = 150\,mm$

$h = 100\,mm$

$b = 20\,mm$

Solution:-

$A = b \times h = 20 \times 100 = 200\,mm^2$

Radius of curvature of the neutral axis

$$R_n = \frac{h}{\ln\left(\frac{R_o}{R_i}\right)}$$

$$R_n = \frac{100}{\ln\left(\frac{150}{50}\right)} = \frac{100}{1.099} = 91.02\,mm$$

$$\boxed{R_n = 91.02\,mm}$$

Radius of curvature of the centroidal axis

$$R = R_i + h/2 = 50 + \frac{100}{2} = 100\,mm$$

$$\boxed{R = 100\,mm}$$

Distance b/w the centroidal and neutral axis

$$e = R - R_n$$

$$= 100 - 91.02 = 8.98\,mm$$

$$\boxed{e = 8.98\,mm}$$

Distance b/w the load and centroidal axis

$x = R = 100\,mm$

Bending moment about centroidal axis

$M = W \times x = 20 \times 10^3 \times 100\,mm$

$\boxed{M = 2 \times 10^6\ N\text{-}mm}$

Direct tensile stress

$\sigma_D = \frac{W}{A} = \frac{20 \times 10^3}{2000} = 10\,N/mm^2$

$\boxed{\sigma_t = 10\,MPa}$

$Y_i = R_n - R_i = 91.02 - 50 = 41.02\,mm$

$\boxed{Y_i = 41.02\,mm}$

$Y_o = R_o - R_n$

$= 150 - 91.02 = 58.93\,mm$

$\boxed{Y_o = 58.98\,mm}$

max. bending stress at inside fibre

$\sigma_{bi} = \frac{M \cdot Y_i}{A \cdot e_i R_i}$

$= \frac{2 \times 10^6 \times 41.02}{2000 \times 8.98 \times 50} = 91\,N/mm^2$

$\boxed{\sigma_{bi} = 91\,MPa}$

max. bending stress at outside fibre

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o} = \frac{2\times10^6 \times 58.98}{2000\times8.98\times150}$$

$\sigma_{bo} = 44\ N/mm^2$

$\boxed{\sigma_{bo} = 44\ MPa}$

Resultant stress at the inside fibre

$= \sigma_D + \sigma_{bi} = 10 + 91 = 101\ MPa$ (tensile)

Resultant stress at the outside fibre

$= \sigma_D - \sigma_{bo} = 10 - 44 = -34\ MPa$

$= 34\ MPa$ (comp.)

The frame of a punch press is shown in fig. Find the stresses at the inner and outer surface at section x-x of the frame if W = 5000N. [M/J 2014]

given:-

$W = 5000N$

$b_i = 18mm$

$b_o = 6mm$

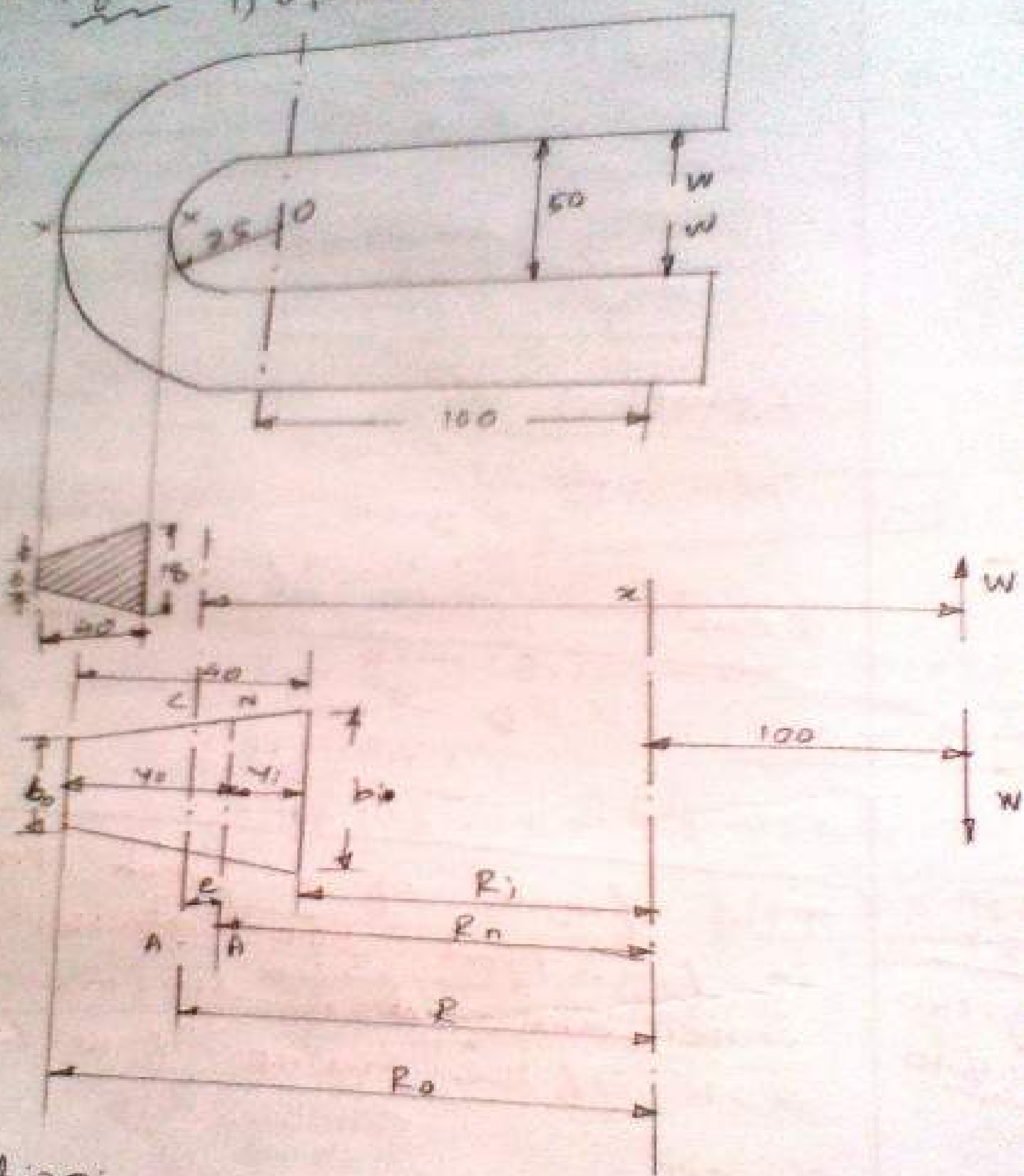
$h = 40mm$

$R_i = 25mm$

$R_o = 25+40$

$R_o = 65mm$

To find:- 1) σ_i , 2) σ_o , 3) Resultant stress



Solution:-

$$A = \frac{1}{2}(b_o + b_i)\,h$$

$$= \frac{1}{2}(6 + 18)\,40$$

$$\boxed{A = 480\,mm^2}$$

$$R_n = \frac{\left(\frac{b_i + b_o}{2}\right)h}{\left[\left(\frac{b_i R_o - b_o R_i}{h}\right)\ln\left(\frac{R_o}{R_i}\right)\right] - (b_i - b_o)}$$

$$= \frac{\left(\frac{18+6}{2}\right)\times 40}{\left[\left(\frac{18\times 65 - 6\times 25}{40}\right)\ln\left(\frac{65}{25}\right)\right] - (18-6)}$$

$$= \frac{480}{(25.5 \times 0.9555) - 12} = 38.82\,mm$$

$$\boxed{R_n = 38.82\,mm}$$

$$R = R_i + \frac{h(b_i + 2b_o)}{3(b_i + b_o)}$$

$$R = 25 + \frac{40(18 + 2\times 6)}{3(18+6)}$$

$$\boxed{R = 41.67\,mm}$$

$$e = R - R_n = 41.67 - 38.82$$

$$\boxed{e = 2.85\,mm}$$

$$x = 100 + R = 100 + 41.67$$

$$\boxed{x = 141.67\,mm}$$

$M = W \times x = 5000 \times 141.67$

$\boxed{M = 708350 \text{ N-mm}}$

$\sigma_D = \frac{W}{A} = \frac{5000}{480} = 10.42 \text{ N/mm}^2$

$\boxed{\sigma_D = 10.42 \text{ MPa}}$

$Y_i = R_n - R_i = 38.82 - 25$

$\boxed{Y_i = 13.82 \text{ mm}}$

$Y_o = R_o - R_n = 65 - 38.82$

$\boxed{Y_o = 26.18 \text{ mm}}$

$\sigma_{bi} = \frac{M \cdot Y_i}{A \cdot e \cdot R_i} = \frac{708350 \times 13.82}{480 \times 2.85 \times 25}$

$\boxed{\sigma_{bi} = 286.23 \text{ N/mm}^2}$

$\boxed{\sigma_{bi} = 286.23 \text{ MPa}}$

$\sigma_{bo} = \frac{M \cdot Y_o}{A \cdot e \cdot R_o} = \frac{708350 \times 26.18}{480 \times 2.82 \times 65}$

$\sigma_{bo} = 210.77 \text{ N/mm}^2$

$\boxed{\sigma_{bo} = 210.77 \text{ MPa}}$

∴ Resultant stress on the inner surface

$= \sigma_D + \sigma_{bi}$

$= 10.42 + 286.23$

$= 296.65\ N/mm^2$ (or) 296.65 mpa (Tensile)

Resultant stress on the outer surface

$= \sigma_D - \sigma_{bo}$

$= 10.42 - 210.77$

$= -200.35\ N/mm^2 = -200$ mpa

$= 200$ mpa (compressive)

A c-clamp is subjected to a maximum load of w, as shown in fig. If the maximum tensile stress in the clamp is limited to 140mpa, find the value of load w.

[N/D 2012]

Given:-

$\sigma_{t(max)} = 140$ Mpa $= 140\ N/mm^2$

$R_i = 25mm$

$R_o = 25 + (22+3)$

$R_o = 50mm$

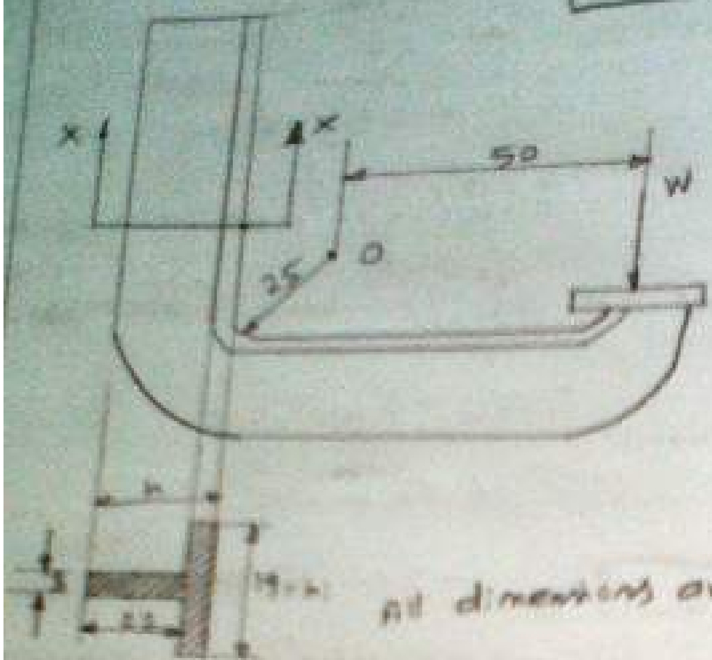
$b_i = 19$

$t_i = 3mm$ $h = 25$

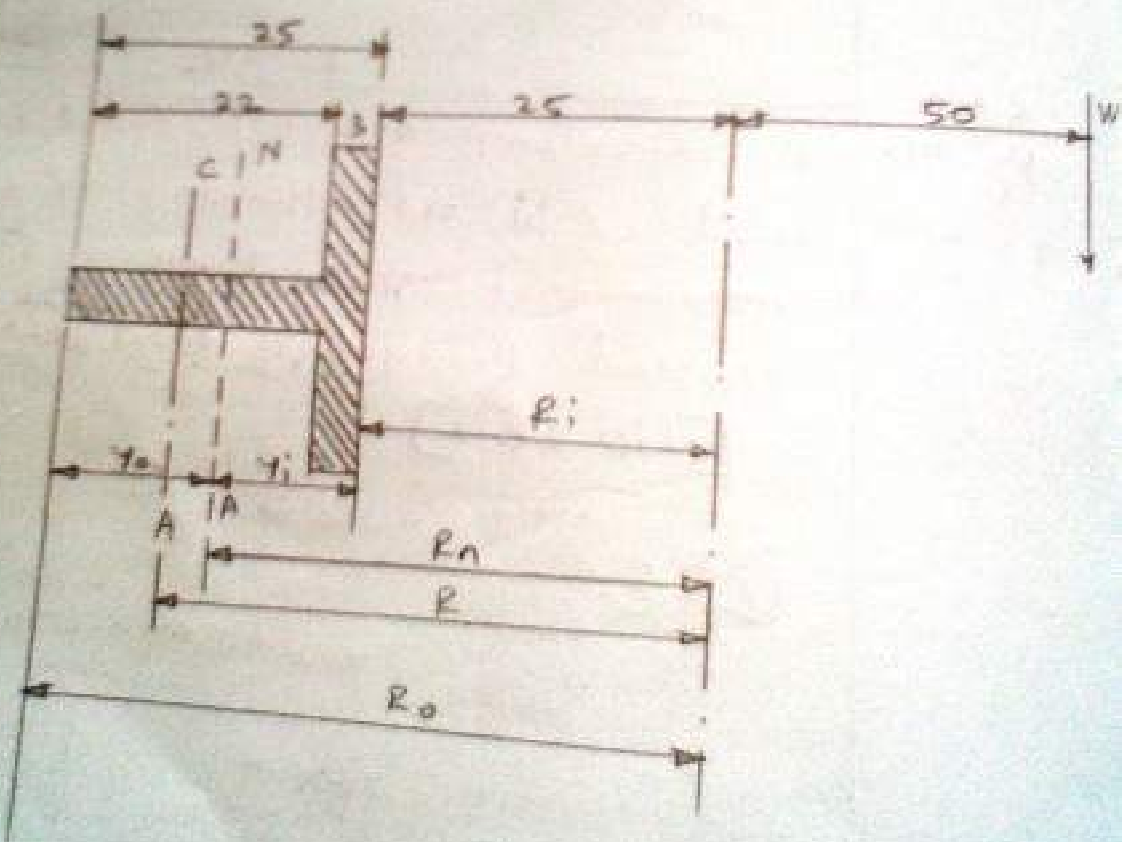
$t = 3$ mm

$A = 3\times22 + 3\times19 = 123\ mm^2$

$\boxed{A = 123\ mm^2}$



All dimensions are in mm



$$R_n = \frac{t_i(b_i - t) + t \times h}{(b_i - t)\ln\left(\frac{R_i + t_i}{R_i}\right) + t \times \ln\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{3 \times (19-3) + 3 \times 25}{(19-3)\ln\left(\frac{25+3}{25}\right) + 3 \times \ln\left(\frac{50}{25}\right)}$$

$$= \frac{123}{16 \times 0.113 + 3 \times 0.693}$$

$$\boxed{R_n = 31.64\,mm}$$

$$R = R_i + \frac{\frac{1}{2}h^2 \times t + \frac{1}{2}t_i^2(b_i - t)}{h \times t + t_i(b_i - t)}$$

$$= 25 + \frac{\frac{1}{2} \times 25^2 \times 3 + \frac{1}{2} \times 3^2(19-3)}{25 \times 3 + 3(19-3)}$$

$$R = 25 + \frac{937.5 + 72}{75 + 48}$$

$$R = 25 + 8.2 \Rightarrow \boxed{R = 33.2\ mm}$$

$$e = R - R_n = 33.2 - 31.64$$

$$\boxed{e = 1.56\ mm}$$

$$x = 50 + R = 50 + 33.2$$

$$\boxed{x = 83.2\,mm}$$

$M = W \times x$

$$\boxed{M = 83.2W \; N\text{-}mm}$$

$y_i = R_n - R_i$

$= 31.64 - 25 = 6.64mm$

$$\boxed{y_i = 6.64mm}$$

Direct stress

$$\sigma_D = \frac{W}{A} = \frac{W}{123} = 0.008W \; N/mm^2$$

$$\boxed{\sigma_D = 0.08 \; N/mm^2}$$

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i} = \frac{83.2W \times 6.64}{123 \times 1.56 \times 25}$$

$$\boxed{\sigma_{bi} = 0.115W \quad N/mm^2}$$

max. tensile stress

$$\sigma_{t(max)} = \sigma_D + \sigma_{bi}$$

$$= 0.008W + 0.115W$$

$$140 = 0.123W$$

$$W = \frac{140}{0.123} = 1138N$$

$$\boxed{W = 1138N}$$

$Y_o = R_o - R_n = 50 - 31.64 = 18.36\ mm$

$$\boxed{Y_o = 18.36 mm}$$

$$\sigma_{bo} = \frac{M \cdot Y_o}{A \cdot e \cdot R_o} = \frac{83.2W \times 18.36}{123 \times 1.56 \times 50}$$

$$\boxed{\sigma_{bo} = 0.16W}\ N/mm^2$$

$$\sigma_{t(max)} = \sigma_t - \sigma_{bo}$$
$$= 0.008W - 0.16W$$
$$= -0.152W \ N/mm^2$$
$$= 0.152W \ N/mm^2 \text{ (compressiv}$$

STRESS CONCENTRATION:-

In most of the engineering components stress distribution is not unifo stress distribution will be uniform only, when there is no change in cross-sec

Irregularity of the stress distribut due to abrupt changes of form is called stress concentration.

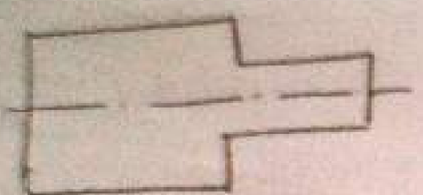
Stress raisers:-

Sudden changes in cross section and material discontinuity is referred as stress raisers.

Ex: Holes, notches, ~~fillets~~, steps, threads, etc.

How to minimise stress concentration

Suitably modifying the shape of the components, stress concentration near the stress raiser can be minimised.



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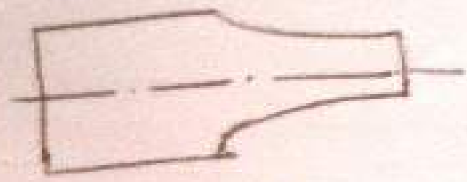
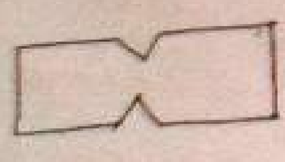


Prefered



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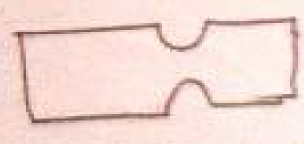


preferred.

Stress concentration factor (K_t):

$$K_t = \frac{\text{maximum stress induced due to stress concentration}}{\text{stress caused without considering the stress concentration}}$$

$K_t = \dfrac{\text{maximum stress } (\sigma_{max})}{\text{nominal stress } (\sigma)}$

$\sigma_{max} = K_t \times$ stress at net section

concentration in ductile and brittle materials

In static loading, ductile material can yield plastically to relieve the stress concentration effect to some extent. In brittle materials, this is not possible.

⇒ But in brittle materials, so, stress concentration is more serious brittle materials.

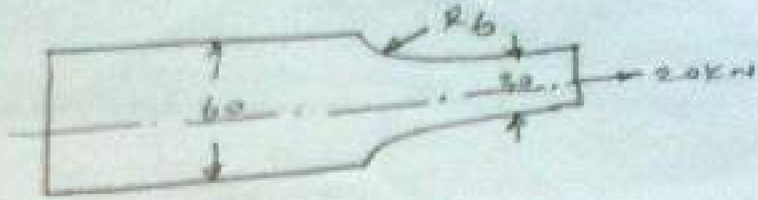
In cyclic loading, stress concentration effect is serious both in ductile and brittle materials.

Notch sensitivity (q):

This is defined as the degree to which the actual stress concentration effect compares with the theoretical stress concentration effect.

Fatigue stress concentration factor (K_f):

Taking stress concentration into account find the maximum stress induced when a tensile load of 20kN is applied to a stepped shaft of diameters 60mm and 30mm with fillet radius of 6mm.



Given:-

$P = 20kN$

$D = 60mm$

$d = 30mm$

$r = 6mm$

To find:-

$\sigma_{max} = ?$

Solution:-

From PSGDB Pg.No: 7.11

$r/d = \frac{6}{30} = 0.2$

$D/d = \frac{60}{30} = 2$

$K_t = 1.5$

$A = \pi/4 \, d^2$

$= \pi/4 (30)^2$

$\boxed{A = 706.86 \, mm^2}$

$K_t = \frac{\sigma_{max}}{\sigma}$

$\sigma = P/A = \frac{20 \times 10^3}{706.86}$

$\sigma_{max} = K_t \cdot \sigma$

$\sigma = 28.29 \, N/mm^2$

$= 1.5 \times 28.29 = 42.44 \, N/mm^2$

Design for variable loading :- [Problems]

Formulas:-

Fluctuating stress

1) mean (or) average stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2) Reversed stress component (or) alternating (or) variable stress.

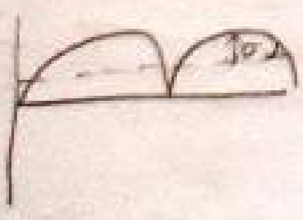
$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

3) For repeated loading, the stress varies from maximum to zero (i.e $\sigma_{min} = 0$) in each cycle.

$$\sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

4) stress ratio $= R = \frac{\sigma_{max}}{\sigma_{min}}$

Repeated stress:-



$\sigma_m = \frac{\sigma_{max}}{2}$

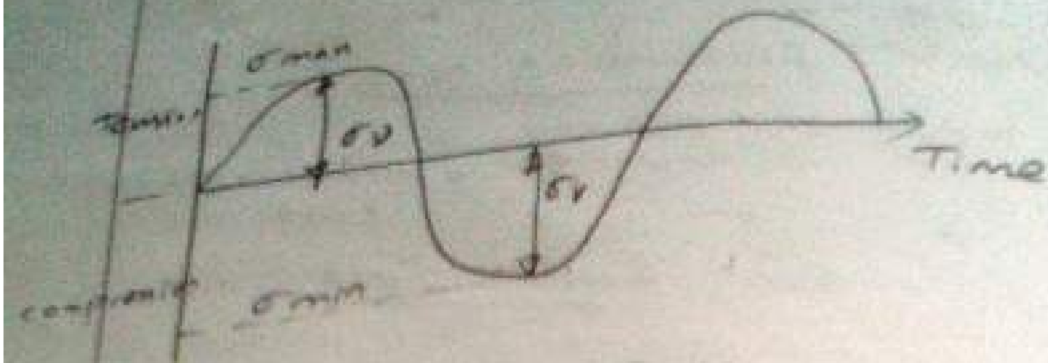
$\sigma_{min} = 0$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\boxed{\sigma_m = \frac{\sigma_{max}}{2}}$$

$\sigma_v = \sigma_m$.

Completely reversed stress:-



$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$= \frac{\sigma_{max} + (-\sigma_{max})}{2}$$

$$\boxed{\sigma_m = 0}$$

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_{max} - (-\sigma_{max})}{2}$$

$$= 2\sigma_{max}/2$$

$$\boxed{\sigma_v = \sigma_{max}}$$

FOS (For fatigue loading)

$$FOS = \frac{\text{Endurance limit stress}}{\text{Design stress}} = \frac{\sigma_e}{\sigma_d}$$

Goodman method :-

$$\frac{1}{F \cdot S} = \frac{\sigma_m}{\sigma_u} = \frac{\sigma_v}{\sigma_e}$$

A Goodman line is used when the design is based on ultimate strength may be used for ductile or brittle material

$$\frac{1}{F \cdot S} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times k_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

k_l = Load correction factor

k_b = Load factor for reversed bending load.

K_{sur} = Surface finish factor

K_{sz} = size factor

Solderberg method :-

This design is based on yield strengt

$$\frac{1}{F_{OS}} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times k_f}{\sigma_e \cdot k_l \times K_{sur} \times K_{sz} \times k_e}$$

Gerber method :-

A prabolic curve drawn between endurance limit (σ_e) and ultimate tensile strength (σ_u).

$$\frac{1}{Fos} = \left(\frac{\sigma m}{\sigma u}\right)^2 Fos + \frac{\sigma v \times kf}{\sigma e}$$

combined stresses

combined stresses refers to a combination of axial, bending and shear stress acting simultaneously on a machine component.

1) variable bending and variable axial stress.
2) variable bending with variable shear stress
3) variable axial with variable shear stress.

Gerber method

$$\frac{1}{F.s} = \left(\frac{\sigma m}{\sigma u}\right)^2 F.s + \frac{\sigma v \times kf}{\sigma e}$$

equivalent bending stress

$$\sigma_{eq} = \sigma m + \frac{\sigma a \times \sigma y \times kf}{\sigma e \times Ksur \times Ksz}$$

equivalent shear stress

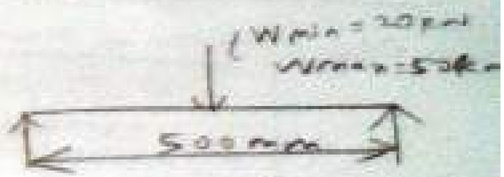
$$\tau_{es} = \tau m + \frac{\tau v \tau y\, kf}{\tau e \times Ksur \times Ksz}$$

max. equivalent shear stress

$$\tau es(max) = \frac{\tau y}{F.s} = \frac{1}{2}\sqrt{(\sigma eq)^2 + 4(\tau es)^2}$$

A circular bar of 500mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20kN and a maximum value of 50kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by:

1) ultimate strength of 650MPa
2) yield strength of 500MPa
3) endurance strength of 350MPa.



given:-

$l = 500mm$

$W_{min} = 20kN = 20\times10^3 N$

$W_{max} = 50kN = 50\times10^3 N$

$FOS = 1.5$

$K_{sz} = 0.85$

$K_{sur} = 0.9$

$\sigma_u = 650MPa = 650N/mm^2$

$\sigma_y = 500MPa = 500N/mm^2$

$\sigma_e = 350MPa = 350N/mm^2$

$$M = \frac{Wl}{4}$$

(Load acting at mid span)

To find:-

Diameter of bar (d)

solution:-

max. bending moment

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4}$$

$$\boxed{M_{max} = 6250 \times 10^3 \text{ N-mm}}$$

min. bending moment

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{20 \times 10^3 \times 500}{4}$$

$$\boxed{M_{min} = 2500 \times 10^3 \text{ N-mm}}$$

mean bending moment

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2}$$

$$= 4375 \times 10^3 \text{ N-mm}$$

$$\boxed{M_m = 4375 \times 10^3 \text{ N-mm}}$$

variable bending moment

$$M_v = \frac{M_{max} - M_{min}}{2}$$

$$= \frac{6250 \times 10^3 - 2500 \times 10^3}{2}$$

$$\boxed{M_v = 1875 \times 10^3 \text{ N-mm}}$$

section modulus

$$z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

$$\boxed{z = 0.0982 d^3 \text{ mm}^3}$$

mean or avg bending stress

$$\sigma_m = \frac{M_m}{z} = \frac{4375 \times 10^3}{0.0982 d^3}$$

$$\boxed{\sigma_m = \frac{44.5 \times 10^6}{d^3} \text{ N/mm}^2}$$

variable bending stress

$$\sigma_v = \frac{M_v}{z} = \frac{1875 \times 10^3}{0.0982 d^3}$$

$$\boxed{\sigma_v = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2}$$

w.k.t according to the good man's formula.

$$\frac{1}{F_{os}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times k_f}{\sigma_e \times k_{sur} \times k_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5\times10^6}{d^3\times650} + \frac{19.1\times10^6\times1}{d^3\times350\times0.9\times0.85}$$

Assume $K_6 = 1$

$$0.666 = \frac{68\times10^3}{d^3} + \frac{71\times10^3}{d^3} = \frac{139\times10^3}{d^3}$$

$$d^3 = 139\times10^3\times1.5$$

$$d^3 = 209 = \boxed{d = 59.3\,mm}$$

w.k.t according to the soderberg's formula

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v\times K_6}{\sigma_e\times K_{sur}\times K_{sz}}$$

Assume $K_6 = 1$

$$\frac{1}{1.5} = \frac{44.5\times10^6}{d^3\times500} + \frac{19.1\times10^6\times1}{d^3\times350\times0.9\times0.85}$$

$$= \frac{89\times10^3}{d^3} + \frac{71\times10^3}{d^3}$$

$$= \frac{160\times10^3}{d^3}$$

$$d^3 = 160\times10^3\times1.5 = 240\times10^3$$

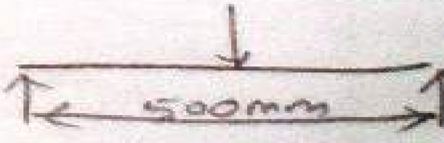
$$d = 62.14\,mm.$$

Taking larger value of above two diameter

$\boxed{d = 62.1\,mm}$

A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4P. The span of the beam is 500mm and its cross-section is circular with a diameter of 60mm. Taking for the beam material an ultimate stress of 700MPa, a yield stress of 500MPa, endurance limit of 330MPa for reversed bending and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and surface finish factor of 0.9.

given:-



$W_{min} = P$

$W_{max} = 4P$

$d = 60mm$ — $d = 60mm$

$l = 500mm$

midspan

$M = \frac{W \times l}{4}$

$\sigma_u = 700\,MPa = 700\,N/mm^2$

$\sigma_y = 500\,MPa = 500\,N/mm^2$

Taking larger value of above two diameter

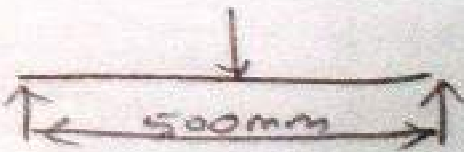
$\boxed{d = 62.1\,mm}$

A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4P. The span of the beam is 500mm and its cross-section is circular with a diameter of 60mm. Taking for the beam material an ultimate stress of 700MPa, a yield stress of 500MPa, endurance limit of 330MPa for reversed bending and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and surface finish factor of 0.9.

given:-

$W_{min} = P$

$W_{max} = 4P$



$d = 60mm$

$d = 60mm$

$l = 500mm$

midspan

$M = \frac{W \times l}{4}$

$\sigma_u = 700\,MPa = 700\,N/mm^2$

$\sigma_y = 500\,MPa = 500\,N/mm^2$

$\sigma_e = 330\,\text{mpa} = 330\,N/mm^2$

$F.S = 1.3$

$K_{sz} = 0.85$

$K_{sur} = 0.9$

Solution:-

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{4P \times 500}{4} = 500P\ N\text{-}mm$$

$$\boxed{M_{max} = 500P\ N\text{-}mm}$$

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{P \times 500}{4} = 125P\ N\text{-}mm$$

$$\boxed{M_{min} = 125P\ N\text{-}mm}$$

mean(or) avg bending moment

$$M_m = \frac{M_{max} + M_{min}}{2}$$

$$= \frac{500P + 125P}{2} = 312.5P\ N\text{-}mm$$

$$\boxed{M_m = 312.5P\ N\text{-}mm}$$

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{500P - 125P}{2} = \frac{375P}{2}$$

$$\boxed{M_v = 187.5P \ \text{N-mm}}$$

<u>mean(or) avg bending stress</u>

$$\sigma_m = \frac{M_m}{Z} = \frac{312.5P}{21.21\times10^3}$$

$$Z = \pi/32 \, d^3 = \pi/32 \times (60)^3$$

$$Z = 21.21\times10^3 \ \text{mm}^3$$

$$\boxed{\sigma_m = 0.0147P \ \text{N/mm}^2}$$

variable bending stress

$$\sigma_v = \frac{M_v}{Z} = \frac{187.5P}{21.21\times10^3} = 0.0088P \ \text{N/mm}^2$$

$$\boxed{\sigma_v = 0.0088 \ \text{N/mm}^2}$$

w.k.T according to the goodman's formula.

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147P}{700} + \frac{0.0088P}{330 \times 0.9 \times 0.85}$$ Assume $k_f = 1$

$$= 2.1 \times 10^{-5}P + 3.49 \times 10^{-5}P$$

$$P(2.1 \times 10^{-5} + 3.49 \times 10^{-5})$$

$$= P(5.59 \times 10^{-5})$$

$$P = \frac{1}{1.3 \times (5.59 \times 10^{-5})} = 13760N = 13.76 kN$$

$$\boxed{P = 13.76 kN}$$

w.k.t according to soderberg's formula

$$\frac{1}{F.S} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times k_f}{\sigma_e \times k_{sur} \times k_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147P}{500} + \frac{0.0088P \times 1}{330 \times 0.9 \times 0.85}$$ assume $k_f = 1$

$$= (2.94 \times 10^{-5} + 3.49 \times 10^{-5})P$$

$$= 6.43P$$

$$P = \frac{1}{1.3 \times 6.43} \Rightarrow P = 11970 N$$

$$\boxed{P = 11.97 kN}$$

we know take large value of this two,

$\therefore \boxed{P = 18.76\ KN}$

A hot rolled steel shaft is subjected to a torsional moment that varies from 330N-m clockwise to 110N-m counterclockwise and an applied bending moment at a critical section varies from 440N-m to -220N-m. The shaft is of uniform cross-section and no keyway is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of 550MN/m² and a yield strength of 410 MN/m². Take the endurance limit as half the ultimate strength, factor of safety of 2, size factor of 0.85 and a surface finish factor of 0.62.

Given:-

$T_{max} = 330\ N\text{-}m$ (clockwise)

$T_{min} = 110\ N\text{-}m$ (counterclockwise)

$= -110\ N\text{-}m$ (clockwise)

$M_{max} = 440\ N\text{-}m$

$M_{min} = -220\ N\text{-}m$

$\sigma_u = 550\ MN/m^2 = 550 \times 10^6\ N/m^2$

$\sigma_y = 410\ MN/m^2 = 410 \times 10^6\ N/m^2$

$\sigma_e = \frac{1}{2}\sigma_u = 275 \times 10^6\ N/m^2$

$F.S = 2$, $K_{sz} = 0.85$ $K_{sur} = 0.62$

To find:-

The required shaft diameter

Solution:-

W.K.T mean torque

$$T_m = \frac{T_{max} + T_{min}}{2}$$

$$= \frac{330 + (-110)}{2} = 110\ N\text{-}m$$

$$\boxed{T_m = 110\ N\text{-}m}$$

Variable torque

$$T_v = \frac{T_{max} - T_{min}}{2}$$

$$= \frac{330 - (-110)}{2} = 220\ N\text{-}m$$

$$\boxed{T_v = 220\ N\text{-}m}$$

mean shear stress

$$\tau_m = \frac{16}{\pi d^3} \times T_m$$

$$\tau_m = \frac{16 \times 110}{\pi d^3} = \frac{560}{d^3} \ N/m^2$$

variable shear stress

$$\tau_v = \frac{16}{\pi d^3} \times T_v$$

$$= \frac{16 \times 220}{\pi d^3} = \frac{1120}{d^3} \ N/m^2$$

Assumption $\boxed{\tau_e = 0.55 \times \sigma_e}$ $\boxed{\tau_y = 0.5 \times \sigma_y}$

$$\tau_e = 0.55 \times 275 \times 10^6 \qquad \tau_y = 0.5 \times 410 \times 10^6$$

$$\tau_e = 151.25 \times 10^6 \ N/m^2 \qquad = 205 \times 10^6 \ N/m^2$$

W.K.T equivalent shear stress

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

$$= \frac{560}{d^3} + \frac{1120 \times 205 \times 10^6 \times 1}{d^3 \times 151.25 \times 10^6 \times 0.62 \times 0.85}$$

$= \frac{560}{d3} + \frac{2880}{d3} = \frac{3440}{d3}$ N/m² —①

mean (or) avg bending moment

$M_m = \frac{M_{max} + M_{min}}{2} = \frac{440 + (-220)}{2}$

$\boxed{M_m = 110 \text{ N-m}}$

Variable Bending moment

$M_v = \frac{M_{max} - M_{min}}{2} = \frac{440 - (-220)}{2}$

$= 330 \text{ N-m}$

section modulus

$Z = \pi/32\, d3 = 0.0982 d3 \text{ m}^3$

mean bending stress

$\sigma_m = \frac{M_m}{Z} = \frac{110}{0.0982 d3} = \frac{1120}{d3}$ N/m²

$\boxed{\sigma_m = \frac{1120}{d3} \text{ N/m}^2}$

Variable bending stress

$$\sigma_v = \frac{M_v}{Z} = \frac{330}{0.0982 d^3} = \frac{3360}{d^3} \text{ N-m.}$$

$$\sigma_{eq} = \sigma_m + \frac{\sigma_v \times \sigma_y \times k_{fb}}{\sigma_e \times k_{su1} \times k_{sz}}$$

Assume $k_{fb} = 1$

$$= \frac{1120}{d^3} + \frac{3360 \times 410 \times 10^6 \times 1}{d^3 \times 275 \times 10^6 \times 0.85 \times 0.62}$$

$$= \frac{1120}{d^3} + \frac{9506}{d^3} = \frac{10626}{d^3} \text{ N/m}^2$$

W.K.T the max. equivalent shear stress

$$\tau_{es} = \frac{\tau_y}{F.S} = \frac{1}{2}\sqrt{(\sigma_{eqb})^2 + 4(\tau_{es})^2}$$

$$\frac{205 \times 10^6}{2} = \frac{1}{2}\sqrt{\left(\frac{10626}{d^3}\right)^2 + 4\left(\frac{3440}{d^3}\right)^2}$$

$$205 \times 10^6 \times d^3 = \sqrt{113 \times 10^6 + 4(11.84 \times 10^6)}$$

$$d^3 = \frac{12.66 \times 10^3}{205 \times 10^6}$$

$d = 39.5$ mm or 40. $\Leftarrow$ $d = 40$ mm $\quad d^3 = 6.17 \times 10^{-5} = 0.0395$ m

A pulley is keyed to a shaft midway b/w two bearings. The shaft is made of cold drawn steel for which the ultimate strength is 550 MPa and the yield strength is 400 MPa. The bending moment at the pulley varies from −150 N-m to 400 N-m as the torque on the shaft varies from −50 N-m to 150 N-m. obtain the diameter of the shaft for an indefinite life. The stress concentration factors for the key way at the pulley in bending and in torsion are 1.6 and 1.3 respectively.

Take the following values.

Factor of safety = 1.5

Load correction factors 1.0 in bending &
0.6 in torsion

size effect factor = 0.85

surface effect factor = 0.88.

given: -

$\sigma_u = 550$ mpa $= 550$ N/mm^2

$\sigma_y = 400$mpa $= 400$ N/mm^2

$M_{min} = -150$ N-m

$M_{max} = 400$N-m

$T_{min} = -50$ N-m

$T_{max} = 150$N-m

$K_{fb} = 1.6$

$K_{fs} = 1.3$

$F.s = 1.5$

$K_b = 1$

$K_s = 0.6$

$K_{sz} = 0.85$

$K_{sur} = 0.88$

To find: -

Diameter of the shaft

solution: -

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{400 + (-150)}{2} = 125 \text{ N-m}$$

$$\boxed{M_m = 125 \times 10^3 \text{ N-mm}}$$

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{400 - (-150)}{2}$$

$$= 275 \text{ N-m} = 275 \times 10^3 \text{ N-mm}$$

$$\boxed{M_v = 275 \times 10^3 \text{ N-mm}}$$

Section modulus

$$Z = \frac{\pi}{32} d^3 = 0.0982 d^3 \ mm^3$$

Mean bending stress

$$\sigma_m = \frac{M_m}{Z} = \frac{125 \times 10^3}{0.0982 d^3} = \frac{1273}{d^3}$$

$$\boxed{\sigma_m = \frac{1273 \times 10^3}{d^3}}$$

Variable bending stress

$$\sigma_v = \frac{M_v}{Z} = \frac{275 \times 10^3}{0.0982 d^3} = \frac{2800 \times 10^3}{d^3}$$

$$\boxed{\sigma_v = \frac{2800 \times 10^3}{d^3}}$$

assume ← $\sigma_{eb} = \sigma_e = \frac{\sigma_u}{2} = \frac{550}{2} = 275 \ N/mm^2$

$$\sigma_{reb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_f}{\sigma_e \times K_{sz1} \times K_{sz}}$$

$$= \frac{1273 \times 10^3}{d^3} + \frac{2800 \times 10^3 \times 400 \times 1.6}{d^3 \times 275 \times 0.88 \times 0.85}$$

$$= \frac{1273 \times 10^3}{d^3} + \frac{8712 \times 10^3}{d^3} = \frac{9985 \times 10^3}{d^3} \, N/mm^2$$

mean Torque

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{150 + (-50)}{2} = 50 \, N\text{-}m$$

$$\boxed{T_m = 50 \times 10^3 \, N\text{-}mm}$$

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{150 - (-50)}{2} = 100 \, N\text{-}m$$

$$\boxed{T_v = 100 \times 10^3 \, N\text{-}mm}$$

mean shear stress

$$\tau_m = \frac{16 \, T_m}{\pi d^3} = \frac{16 \times 50 \times 10^3}{\pi d^3} = \frac{255 \times 10^3}{d^3} \, N/mm^2$$

$$\boxed{\tau_m = \frac{255 \times 10^3}{d^3} \, N/mm^2}$$

$$\tau_v = \frac{16 \, T_v}{\pi d^3} = \frac{16 \times 100 \times 10^3}{\pi d^3} = \frac{509 \times 10^3}{d^3} \, N/mm^2$$

Assumption $\boxed{\tau_e = \sigma_e \times K_s} = 275 \times 0.6 = 165 \, N/mm^2$

Assuming $\tau_y = 0.5 \sigma_y = 0.5 \times 400 = 200 \, N/mm^2$

$$\boxed{\tau_y = 200 \, N/mm^2}$$

W.K.T equivalent shear stress

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{f0}}{\tau_e \times K_{sa1} \times K_{s2}}$$

$$= \frac{255\times10^3}{d^3} + \frac{509\times10^3\times200\times1.3}{d^3\times165\times0.88\times0.85}$$

$$= \frac{255\times10^3}{d^3} + \frac{1072\times10^3}{d^3} = \frac{1327\times10^3}{d^3}\ N/mm^2$$

maximum equivalent shear stress

$$\tau_{es(max)} = \frac{\tau_y}{F.s} = \frac{1}{2}\sqrt{(\sigma_{eq})^2 + 4(\tau_{es})^2}$$

$$\frac{200}{1.5} = \frac{1}{2}\left(\sqrt{\left(\frac{9985\times10^3}{d^3}\right)^2 + 4\left(\frac{1327\times10^3}{d^3}\right)^2}\right)$$

$$= \frac{7650500}{d^3} \qquad \frac{9985\times10^3}{d^3} + \frac{2\times(1327\times10^3)}{d^3}$$

$$= \frac{7650\times10^3}{d^3} \qquad = \frac{9985\times10^3}{d^3} + \frac{2654\times10^3}{d^3}$$

$$d^3 = \frac{7650\times10^3\times1.5}{200}$$

$$= 57375$$

$$\boxed{d = 45mm}$$

$$d = 38.56 \approx 40mm \Rightarrow \boxed{d = 40mm}$$